

SUPPRESSION OF WAVES IN SLOTTED-WALLED CHANNEL^a

Errata

The following corrections should be made to the original paper:

- Page 399, line 9: Should read (915 mm) instead of (915)
- Page 403, paragraph 2, line 3: Should read "beach" instead of "bench"
- Page 404, paragraph 1, line 15: Should read (0.75 m/s) instead of (75 m/s)
- Page 405, Fig. 5, caption: Should read "Wave Amplitude on Each Side of Screen (1 in. = 25.4 mm; 1 ft = 0.305 m)
- Page 407, paragraph 1, line 5: Should read (61 cm) instead of (6 cm)
- Page 407, line 2 from bottom: Should read "Eqs. 5 and 7" instead of Eqs. 4 and 6"
- Page 408, Eq. 14: Should read $(\sigma - kV)^2$ instead of $(\sigma - KV)^2$
- Page 408, line 1 after Eq. 14: Should read "Eqs. 12 and 14" instead of "Eqs. 11 and 13"
- Page 408, line 1 after Eq. 15: Should read "Eq. 15" instead of "Eq. 14"

FINITE ELEMENT SOLUTION OF SAINT-VENANT EQUATIONS^b

Discussion by Danny L. Fread,³ M. ASCE

The authors have made a needed contribution to the literature with their presentation of a finite element solution of the one-dimensional unsteady flow equations. The following comments are concerned with the authors' "case 2" application, wherein they have made some statements in their comparison of the finite element and four-point finite difference techniques which require some further consideration.

First, the authors state that the downstream stage hydrograph generated by the finite element method using 12-hr time steps lies closer to the curve based on a weighted four-point finite difference "box" scheme using 0.5-hr time steps than to the curve based on the same finite difference solution using 12-hr time steps. This, they suggest, indicates that the finite element method is somewhat

^aMarch, 1976, by Alfred M. Binnie (Proc. Paper 12006).

^bJune, 1976, by Richard L. Cooley and Syed Afaq Moin (Proc. Paper 12213).

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more accurate than the finite difference method for this particular example. This is a questionable conclusion, since the authors have used a downstream boundary condition in the finite element solution different from that used in the finite difference solution. The authors used a single-value rating curve based on Manning's equation, with the channel bottom slope used as the energy slope. Their solution, particularly at the downstream boundary, can be different from that obtained using a loop rating curve based on the Manning equation with a variable energy slope to account for unsteady nonuniform flow as used in the finite difference solution. This is indicated in the authors' Fig. 5, where,

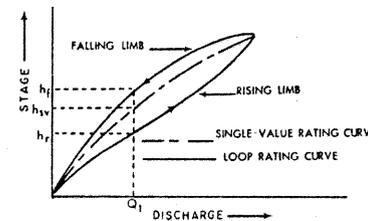


FIG. 7.—Typical Single-Value and Loop Rating Curves

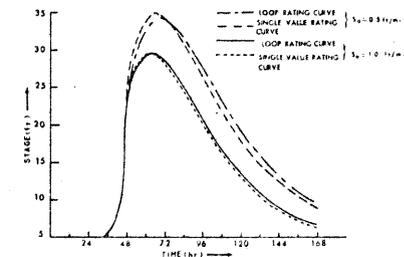


FIG. 8.—Effect of Channel Slope and Downstream Boundary Rating Curve on Computed Downstream Hydrographs

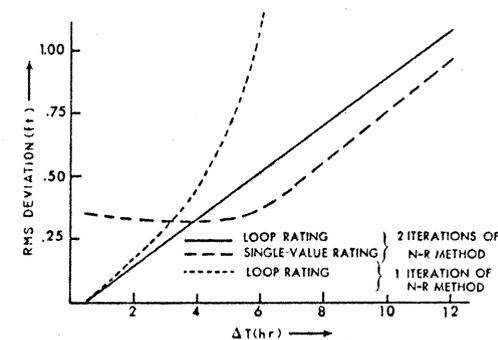


FIG. 9.—Effect of Time Step on Root Mean Square Deviation of Downstream Hydrographs for Loop and Single-Value Rating Curves (Loop Rating Curve with 0.5-hr Time Steps Used as Standard for Comparison)

on the rising limb of the stage hydrograph, the finite element solution using 12-hr time steps exceeds that of the finite difference solution using 12-hr time steps and, on the falling limb, the relationship is reversed. This relationship is identical to that which would result when using the two different boundary conditions, as indicated in Fig. 7 by the relative magnitudes of the stages, h_r , h_f , and h_{sv} , associated with the rising limb (loop rating), falling limb (loop rating), and single-value rating, respectively.

The difference in the hydrographs due to the type of rating curve used at the downstream boundary is sensitive to the channel bottom slope and the

rate of change of the hydrograph. For the particular hydrograph examined by the authors, which has a time to peak of 48 hr and a 0.5-hr time step, the root mean square deviation between the two hydrographs (see Fig. 8) is 0.358 ft (0.109 m) when the channel bottom slope, S_o , is 1.0 ft/mile (0.0001894 ft/ft); however, when the bottom slope is reduced to 0.5 ft/mile (0.0000947 ft/ft), the root mean square deviation increases to 1.002 ft (0.306 m). A more complete presentation of the effects of slope and rate of change of flow on hydrographs determined from single-value and loop rating curves is given by the writer (22, pp. 30-35).

The computed downstream hydrograph is sensitive to the size of time step. Using the loop rating curve with 0.5-hr time step as the standard for comparison, the root mean square deviations of other downstream hydrographs with different size time steps are shown in Fig. 9. The root mean square deviation associated with the single-value rating curve is less than that of the loop rating curve for time steps in excess of 4 hr. The root mean square values for 12-hr time steps are 1.094 ft (0.334 m) and 0.971 ft (0.296 m) for the loop and single-value rating curves, respectively. Thus, by using the single-value rating curve with 12-hr time steps, as the authors have done, the computed downstream hydrograph generated by the four-point finite difference scheme deviates less from the standard hydrograph than that computed with 12-hr time steps and a loop rating curve. This indicates a compensating effect of the deviations arising from the use of a single-value rating rather than a loop rating boundary condition and the deviations due to numerical truncation errors associated with the time step size. The compensating effect resulted in, at least a portion of, the apparent increase in accuracy, which the authors observed; therefore, it is questionable if the "case 2" example illustrates the superior accuracy of the finite element method as concluded by the authors.

Second, the authors state that the Newton-Raphson method for solving the system of equations associated with the weighted four-point finite difference "box" scheme is generally a more time-consuming process than the predictor-corrector method used in the authors' finite element solution. The essential difference in computational time required by the two solution techniques is the number of times the system of equations is solved at each time step. The number of iterations made in the Newton-Raphson procedure can be controlled through either the selected convergence tolerance criteria or the specified number of iterations. Since the predictor-corrector method requires two complete solutions of the system of equations at each time step, the writer controlled the Newton-Raphson method to iterate twice at each 12-hr time step, and the generated hydrograph had a root mean square deviation from the standard hydrograph of 1.094 ft (0.334 m). When as many as 10 iterations were used at each time step, the root mean square value remained the same. Thus, solving the system of equations more than twice does not produce an increase in the accuracy; therefore, in this example, the Newton-Raphson method is comparable to the predictor-corrector method in computational requirements.

The authors mention that instability was encountered when only the predictor part of the scheme was used. The writer also observed this when the Newton-Raphson method was applied only once each time step. However, when the time step was reduced to 6 hr, the solution became stable, although the root mean square value was twice (see Fig. 9) that obtained when the Newton-Raphson

method was applied twice at each 6-hr time step. The application of only the predictor part or the single iteration Newton-Raphson method is essentially equivalent to linear finite difference schemes, e.g., Chen, et al. (21). The stability and accuracy of each of the aforementioned time-saving solution techniques is dependent on the size of time step used in relation to the degree of nonlinearity of the flow over the time step interval, which, in the example, is appreciable for a time step of 12 hr.

Also, the exponent associated with the natural base e in Eq. 32 should be $[(1 - t/\tau)][1/(\gamma - 1)]$.

APPENDIX.—REFERENCES

21. Chen, Y. H., et al., "Transport of Material by Unsteady Flow," *Unsteady Flow in Open Channels*, Vol. I, K. Mahmood, and V. Yevjevich, eds., Water Resources Publications, Fort Collins, Colo., 1975, pp. 313-366.
22. Fread, D. L., "A Dynamic Model of Stage-Discharge Relations Affected by Changing Discharge," *NOAA Technical Memorandum NWS HYDRO-16*, Office of Hydrology, National Weather Service, National Oceanic and Atmospheric Administration, Silver Spring, Md., Nov., 1973.