Computation of Solar Radiation From Sky Cover

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A procedure for estimating global solar radiation from sky cover is developed from the records of 47 stations in the United States with long periods of radiation observations during the 10-year period, March 1961 through February 1971. It fits a general parabolic equation of the form $Y = B + (1 - B)(1 - N)^P$ to the observations, where Y is the observed global solar radiation divided by clear sky radiation and N is the sky cover. The variables B (the point at which the parabola crosses the y axis) and P (a variable parameter less than 1.0) are selected to minimize the sum of the errors $(Y - Y_{\text{calc}})^2$, where Y_{calc} is the calculated value of Y. The equation $Y = B + (1 - B)(1 - N)^{0.61}$ is selected as most representative, and the B values in this equation that minimize the sum of the errors squared for the individual stations are shown in Figure 1. The average absolute error of the 5306 data points is 1.18 MJ m^{-2}, or 7% of the average observed radiation. Because of the uncertainties of the observed global solar radiation and observed sky cover, the procedure should be used with caution, particularly for periods of less than 1 month.

INTRODUCTION

Engineers and hydrologists are frequently faced with the problem of estimating global solar radiation for a point where no measurements of radiation are available. Due to the limited number of stations in the United States where global solar radiation is measured it usually is necessary to estimate the radiation from percent of possible sunshine or sky cover. *Hamon et al.* [1954] developed a graphical method for converting percent of possible sunshine into daily values of incident solar radiation for stations between 25°N and 50°N. *Lamoreaux* [1962] used this relation in his work of adapting modern evaporation formulae for use with computers. *Baker and Haines* [1969] attempted to expand the solar radiation network through correlation of radiation data with frequently observed climatological parameters in the north-central region and Alaska. The original purpose of the present study was to make solar radiation estimates from sky cover for the revision of the evaporation maps of the United States [*Kohler et al.*, 1959]. When mean monthly observed solar radiation for the 10-year period, March 1961 through February 1971, was compared to the solar radiation computed from the Hamon-Weiss-Wilson relation, the computed solar radiation nearly always exceeded the observed values. Another study has been made to determine the adjustment factors to be applied to the Hamon-Weiss-Wilson relation to eliminate this bias (E. S. Thompson, unpublished manuscript, 1974).

A review of previous work relating sky cover to solar radiation showed that as early as 1919, *Kimball* [1919] found an almost straight-line relationship between solar radiation and average monthly values of cloudiness and sunshine duration. *Sivkov* [1968] suggested an equation of the form

$$S_{\text{rad}} = (1 - N)C_{\text{rad}} \tag{1}$$

where S_{rad} is observed solar radiation, C_{rad} is theoretical solar radiation for clear sky, and N is sky cover.

E. S. Thompson (unpublished manuscript, 1974) modified this equation to account for the transmission of solar energy through the clouds and reflection from these clouds by using a transmission-reflection coefficient, X_K. The equation then becomes

$$S_{\text{rad}} = [1 - N + X_K(N)]C_{\text{rad}} \tag{2}$$

and

$$X_K = 1/N(S_{\text{rad}}/C_{\text{rad}} + N - 1) \tag{3}$$

However, due to the large uncertainties in the observed data, with variations for different instruments and calibration of equipment, linear interpolation of X_K between data points would be very questionable.

PARABOLIC FIT METHOD

It is assumed that a parabolic curve of the general form

$$Y = B + (1 - B)(1 - N)^P \tag{4}$$

gives a good fit to the observed data points, relating the percent of clear sky solar radiation Y (the observed radiation divided by clear sky radiation) with N (the sky cover). Here P is a variable parameter less than 1.0, and B is the point where the parabola crosses the y axis. Clear sky solar radiation is calculated by the Hamon-Weiss-Wilson relation using 100% sunshine values. The necessary information on how the Hamon-Weiss-Wilson relation is used to obtain estimated clear sky solar radiation is included in the appendix. A computer program (CFIT) was written to find B and P values in (4) which provide a minimum of $\sum(Y - Y_{\text{calc}})^2$. The B and P values are checked by this program over a range of values by increments. The procedure to determine B and P values is as follows.

Average monthly percent of clear sky solar radiation versus average monthly sky cover is first plotted for the 10-year period, March 1961 through February 1971, for each station so that a reasonable low value of B can be estimated. Here DB is the incremental increase in B, BN is the maximum value of B desired for each trial (cannot be greater than clear sky radiation), DP is the incremental increase in P, and PN is the maximum value of P. The first trials use $DB = 0.1$, $DP = 0.1$, and $PN = 1.0$. The results of the first trials are then used as a guide in choosing B, DB, BN, P, DP, and PN for the second trials. Two trials are sufficient to obtain values of B and P to two decimal places.

Additional information calculated by CFIT is based on the best fit B and P values and is calculated in order as follows:

- $\sum Z^2$ sum of errors squared, equal to $\sum(Y - Y_{\text{calc}})^2$;
- $\sum Z^2/(n_{\text{dp}})$ conditional variance (Y on X), where n_{dp} is the number of data points;
- S_y standard error of estimate (Y on X), equal to the square root of the conditional variance;
- $\bar{A}_E$ average absolute error, equal to $\sum(\text{abs})(y_o -$

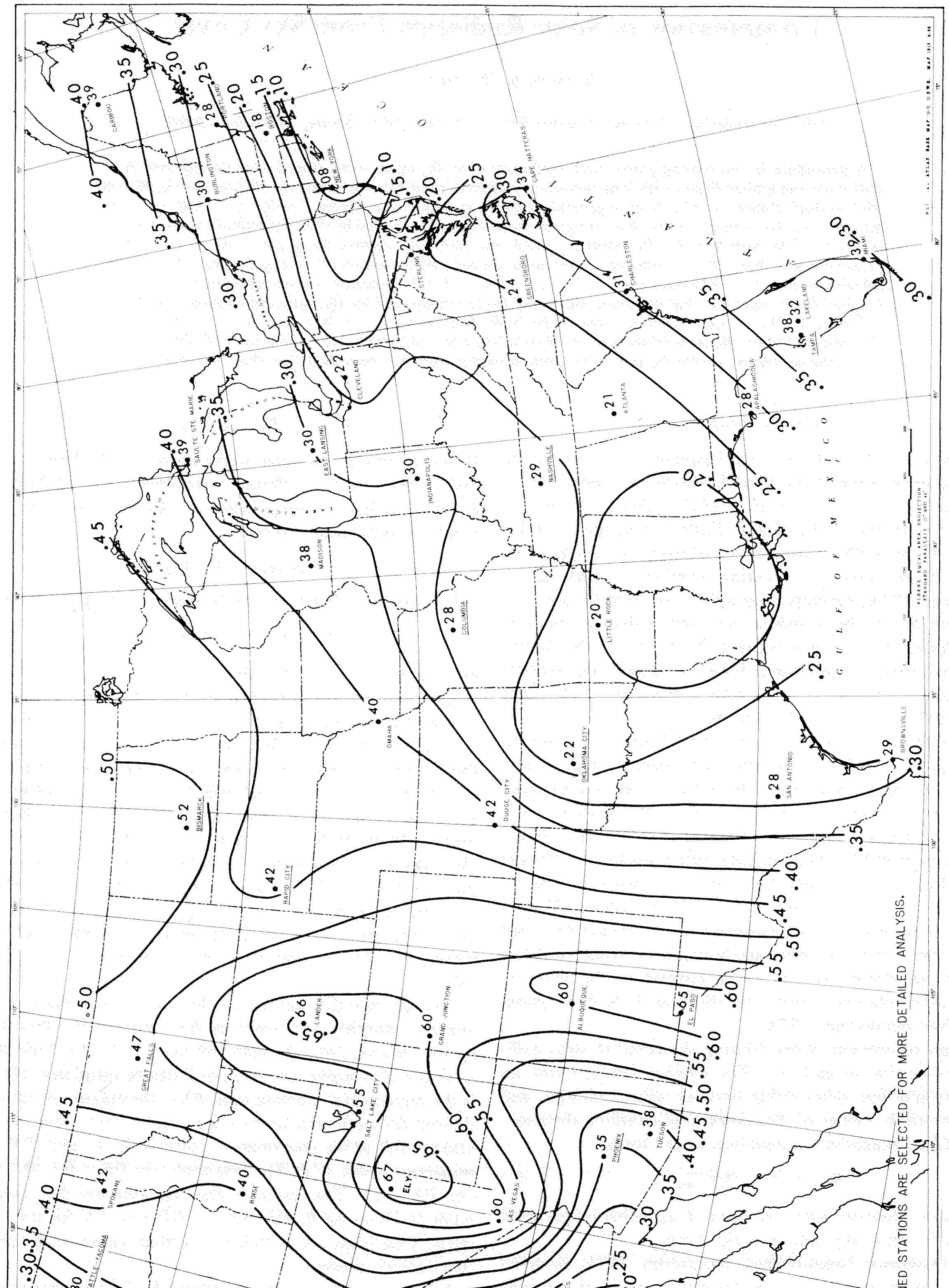

Fig. 1. B values in the equation $Y = B + (1 - B)(1 - N)^{0.61}$ when B only is selected that minimize the sum of the errors squared for the individual stations (method D).

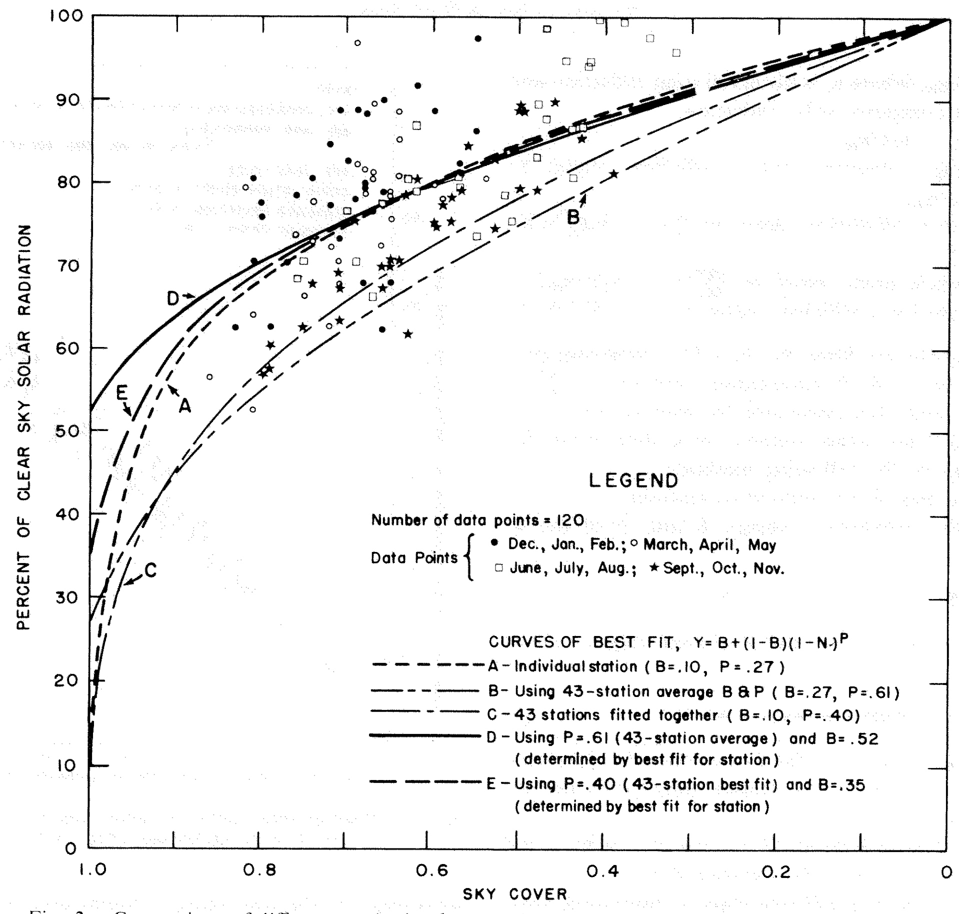

Fig. 2. Comparison of different methods of computing solar radiation, Bismarck, North Dakota.

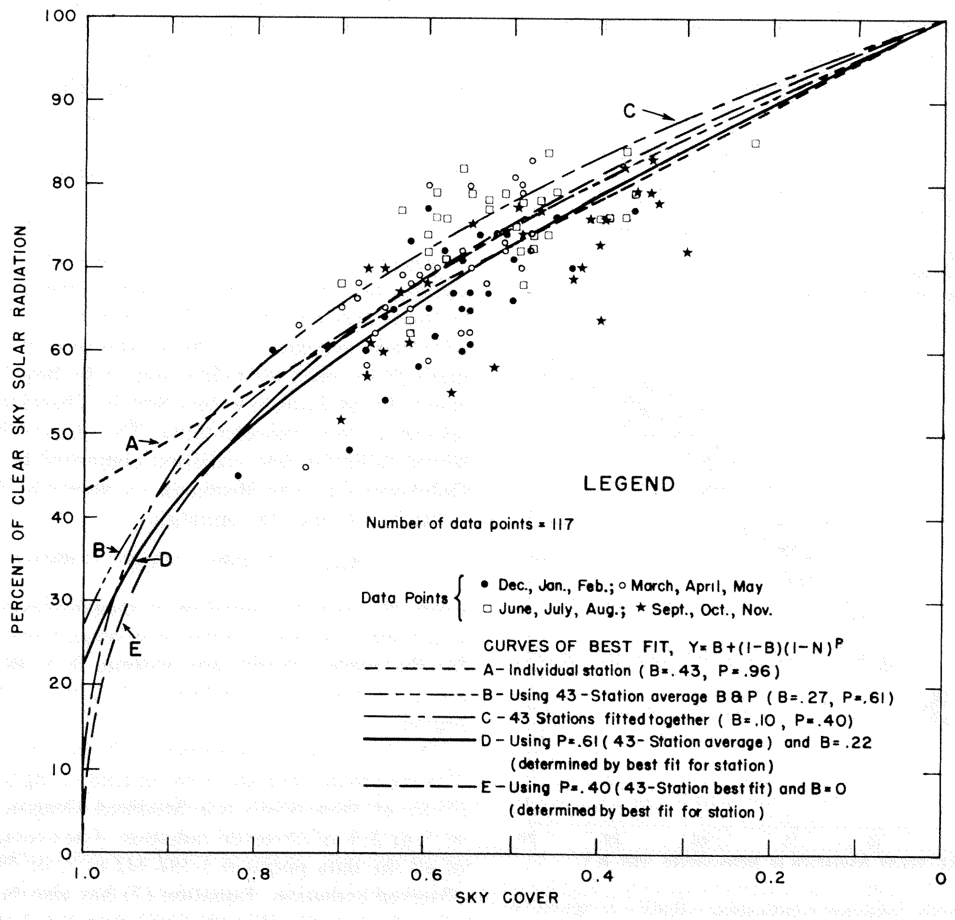

Fig. 3. Comparison of different methods of computing solar radiation, Oklahoma City, Oklahoma.

$y_c)/n_{dp}$, where y_o is observed solar radiation and y_c is computed solar radiation;

Bias $= \sum(y_o - y_c)/n_{dp}$;

$\bar{y}_o$ average observed solar radiation, equal to $\sum\bar{y}_o/n_{dp}$;

σ standard deviation, equal to $\{[y_o{}^2 - \bar{y}_o(\sum y_o)]/n_{dp}\}^{1/2}$;

S standard error, equal to $[\sum(y_o - y_c)^2/n_{dp}]^{1/2}$;

r correlation coefficient, equal to $(1 - S^2/\sigma^2)^{1/2}$.

Forty-seven stations are fitted by the CFIT computer program, and the values of B, P, conditional variance, S_y, $\sum Z^2$, y_o, $\bar{A}_E$, Bias, S, σ, and r are computed for each station.

Variables B and P and other statistics were determined for 15 selected stations by the following methods.

Method A is the best fit for individual stations.

Method B is the arithmetic average B and P of the 43 selected stations.

$$Y = 0.27 + 0.73(1 - N)^{0.61} \tag{5}$$

Method C is the best fit for the combined data of all 43 selected stations.

$$Y = 0.10 + 0.90(1 - N)^{0.40} \tag{6}$$

Method D is the average P of the 43 stations ($P = 0.61$) and B from best fit of the individual station data (B values are shown in Figure 1).

Method E is the best fit P for all 43 stations ($P = 0.40$) and B determined by best fit of individual station data.

Comparisons of these five different ways of computing solar radiation, presented in Figures 2 and 3, show that there is very little difference between method A and method D.

Bismarck, North Dakota, and Oklahoma City, Oklahoma, are also fitted for best fit B and P by seasons, because it appears from the data points of Figures 2 and 3 that separate equations for the four seasons might give better results than one equation per station (method A). Use of separate seasonal equations reduces the average absolute error, bias, and standard error slightly for the fall and winter seasons only, but the correlation coefficients are much higher for the lumped data due to the greater range of the values covered. When the average absolute errors, bias, and standard errors of the four seasons are combined, the means are about the same as those obtained by using one equation per station.

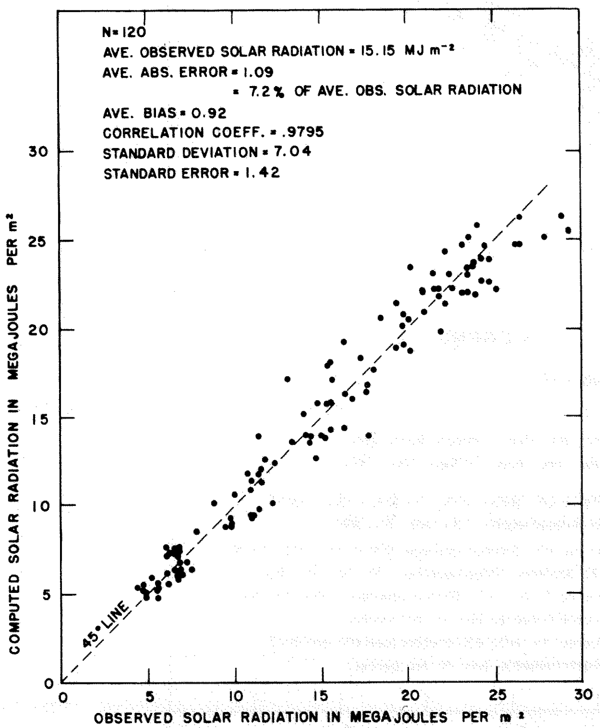

Fig. 4. Observed solar radiation versus solar radiation computed by method D, Bismarck, North Dakota.

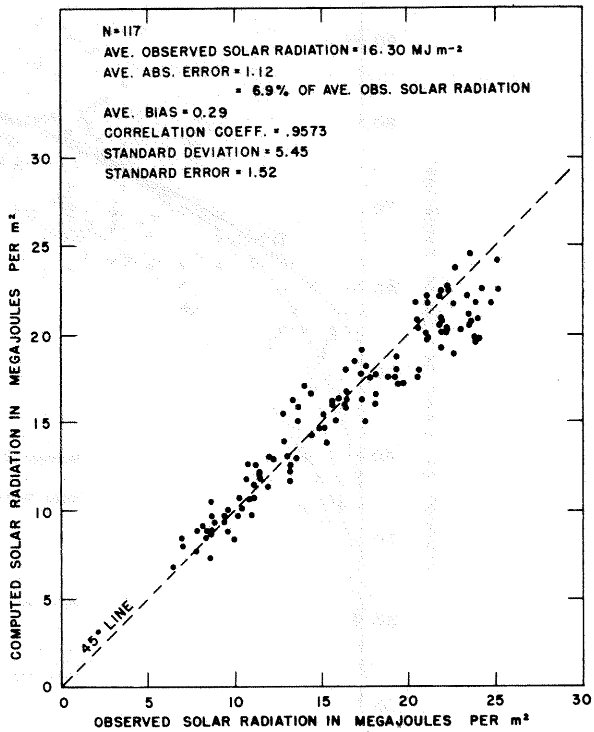

Fig. 5. Observed solar radiation versus solar radiation computed by method D, Oklahoma City, Oklahoma.

RESULTS AND DISCUSSION

Inspection of the graphs and statistics of the 15 selected stations for which B and P are computed by the five different methods indicates that method D, which holds $P = 0.61$ (the arithmetic average P of the 43 stations) constant and determines B for each individual station by best fit, gives the best results for all stations except Seattle. Therefore method D was selected as most representative. The relationships between observed radiation and radiation computed by this method for Oklahoma City and Bismarck are shown in Figures 4 and 5.

Method D uses the equation

$$S_{rad} = C_{rad}(B) + C_{rad}(1 - B)(1 - N)^{0.61} \tag{7}$$

where S_{rad} is solar radiation in megajoules per square meter, C_{rad} is clear sky solar radiation in megajoules per square meter for the proper month and station, N is sky cover in hundredths, and B is the coefficient for a station determined from Figure 1.

The 47 stations used to derive (7) include 5306 data points. The maximum absolute error in computing solar radiation by (7) for all these points is at Medford, Oregon, and is 0.827 MJ m^{-2}, or 31% of observed radiation. The average absolute error of all the data points is 1.184 MJ m^{-2}, or 7% of the average observed radiation. Equation (7) has also been tested with 1 year of mean monthly observed data for three stations which

TABLE 1*a*. Results of Testing Mean Monthly Observed Data Using (7), for Santa Maria, California (Latitude, 34.9°; Elevation, 71.93 m), for 1971

	Observed Radiation, MJ m^{-2}	Sky Cover	Computed Radiation, MJ m^{-2}	Error,* MJ m^{-2}
Jan.	11.72	0.47	10.33	1.38
Feb.	15.44	0.39	14.18	1.26
March	19.87	0.39	17.99	1.88
April	23.22	0.40	21.17	2.05
May	23.56	0.53	20.21	3.35
June	27.11	0.35	24.60	2.51
July	26.99	0.29	25.02	1.97
Aug.	25.23	0.29	23.18	2.05
Sept.	21.30	0.21	21.34	−0.04
Oct.	16.78	0.24	17.32	−0.54
Nov.	12.34	0.38	12.30	0.04
Dec.	9.46	0.44	9.67	−0.21
Average	19.42	0.37	18.11	

Here B = 0.10, A0 = 23.0321, A1 = −8.9366, A2 = −1.0477, A3 = −0.0724, B1 = 0.5527, and B2 = −0.1284. Average absolute error, 1.44; Bias = 1.31; average percent absolute error, 7.4%; and maximum error, 3.35, or 17.2%.

*Error is computed by subtracting the computed radiation from the observed radiation.

are not used in developing the procedure. The results of the tests, presented in Tables 1*a*, 1*b*, and 1*c*, show an average absolute error of from 4 to 9%. Several months of daily observed data are also used to test (7) and show an average error of about 17%. However, the maximum error of these daily data is 1.128 MJ m^{-2}, or 57% of the observed radiation. The errors in solar radiation computed from observed daily sky cover are much greater than those computed from mean monthly sky cover due to the nature of the sky cover observations. It is recommended that mean monthly values of sky cover be used when this technique is used to compute solar radiation.

CONCLUSIONS

The procedure that fits the general parabolic equation and uses (7) gives a very reasonable estimate of solar radiation from mean monthly sky cover observations. If the technique is used for computing from daily values, the limitations and the errors expected should be recognized.

TABLE 1*b*. Results of Testing Mean Monthly Observed Data Using (7), for Midland, Texas (Latitude, 31.9°; Elevation, 869.98 m), for 1971

	Observed Radiation, MJ m^{-2}	Sky Cover	Computed Radiation, MJ m^{-2}	Error,* MJ m^{-2}
Jan.	13.97	0.40	13.76	0.21
Feb.	17.57	0.39	17.11	0.46
March	22.80	0.30	21.92	0.88
April	25.14	0.37	24.60	0.54
May	25.14	0.53	24.22	0.92
June	25.60	0.52	24.89	0.71
July	26.69	0.35	26.44	−0.25
Aug.	21.59	0.60	21.67	−0.08
Sept.	19.16	0.40	21.46	−2.30
Oct.	15.40	0.51	17.20	−1.80
Nov.	13.43	0.43	14.60	−1.17
Dec.	10.42	0.60	11.30	−0.88
Average	19.74	0.45	19.93	

Here B = 0.45, A0 = 23.9078, A1 = −8.1274, A2 = −1.1268, A3 = −0.0849, B1 = 0.5402, and B2 = −0.0837. Average absolute error, 0.85; Bias = −0.019; average percent absolute error, 4.3%; and maximum error, −2.30, or 11.6%.

*Error is computed by subtracting the computed radiation from the observed radiation.

TABLE 1*c*. Results of Testing Mean Monthly Observed Data Using (7), for Sterling, Virginia (Latitude, 39.0°; Elevation, 85.34 m), for 1971

	Observed Radiation, MJ m^{-2}	Sky Cover	Computed Radiation, MJ m^{-2}	Error,* MJ m^{-2}
Jan.	7.49	0.72	7.32	0.17
Feb.	9.37	0.68	10.25	−0.88
March	15.27	0.66	13.89	1.38
April	22.09	0.46	20.58	1.51
May	19.54	0.63	19.58	−0.04
June	22.26	0.66	19.66	2.60
July	22.89	0.59	20.42	2.47
Aug.	22.76	0.50	20.04	2.72
Sept.	15.23	0.69	14.02	1.21
Oct.	9.08	0.72	10.79	−1.71
Nov.	8.28	0.65	8.91	0.63
Dec.	5.77	0.75	6.28	−0.51
Average	15.00	0.64	14.31	

Here B = 0.24, A0 = 21.7752, A1 = −10.0178, A2 = −0.8661, A3 = −0.0636, B1 = 0.5615, and B2 = −0.1640. Average absolute error, 1.32; Bias = 0.69; average percent absolute error, 8.8%; and maximum error, 2.72, or 18.1%.

*Error is computed by subtracting computed radiation from observed radiation.

Application of the values of B from Figure 1 when N = 1 leads to inconsistent values of estimated radiation for different areas, since (7) reduces to

$$S_{rad} = C_{rad}(B) \qquad (8)$$

Equation (7) is not valid for high values of N; and while these high values of N will never actually occur on a monthly basis, an arbitrary value of N = 0.88 has been set as the upper range of N in this equation.

APPENDIX

Hamon et al. [1954] combined relations between insolation received at the earth's surface and (1) percent of possible sunshine, (2) latitude, and (3) time of year graphically in their Figure 5 to provide a working chart for estimating values of global solar radiation.

In this paper, this graphical relation is modified for solution by computer (E. S. Thompson, unpublished manuscript, 1974). No attempt is made here to explain how the following equations are obtained, as that is beyond the scope of this paper.

Clear sky solar radiation is calculated by the equation

$$C_{rad} = \mathrm{A0} + \mathrm{A1}\cos x + \mathrm{A2}\cos 2x + \mathrm{A3}\cos 3x + \mathrm{B1}\sin x + \mathrm{B2}\sin 2x \qquad (9)$$

where C_{rad} is global solar radiation for 100% sunshine (clear sky radiation) and A0–B2 are the latitude coefficients given in Table 2 and

$$x = 2\pi(\mathrm{DAY})/365 \text{ rad} \qquad (10)$$

in which π = 3.1416 and DAY is computed for the fractional part of the year at the beginning of the month as follows.

January and April through August

$$\mathrm{DAY} = \mathrm{T} + \mathrm{TA} + 10 \qquad (11)$$

February and September

$$\text{DAY} = \text{T} + \text{TA} + 11 \quad (12)$$

March

$$\text{DAY} = \text{T} + \text{TA} + 9 \quad (13)$$

October

$$\text{DAY} = \text{T} + \text{TA} + 8 \quad (14)$$

November

$$\text{DAY} = \text{T} + \text{TA} + 6 \quad (15)$$

December

$$\text{DAY} = \text{T} + \text{TA} + 7 \quad (16)$$

in which TA is day of the month (as shown in the tabulation below for monthly data) and

$$\text{T} = 30(\text{MO} - 0.99999)^{1.00503} \quad (17)$$

where MO is month of the year.

Month	TA
Jan.	17
Feb.	14
March	15
April	15
May	13
June	9
July	16
Aug.	16
Sept.	15
Oct.	15
Nov.	14
Dec.	12

The following is an example based on a requirement to compute August clear sky global solar radiation for Sterling, Virginia, latitude of 39°.

From Table 2

$$A0 = 21.7752 \qquad A1 = -10.0178$$

$$A2 = -0.8661 \qquad A3 = -0.0636$$

$$B1 = 0.5615 \qquad B2 = -0.1640$$

From above tabulation (for August)

$$\text{TA} = 16$$

By (17)

$$\text{T} = 30(8 - 0.99999)^{1.00503} = 212.07$$

By (11)

$$\text{DAY} = 212.07 + 16 + 10 = 238.07$$

By (10)

$$x = 6.2832(238.066/365) = 4.09812 \text{ rad}$$

By (9)

$$C_{\text{rad}} = 21.7752 + (-10.0178) \cos (4.09812) + (-0.8661) \cos 2(4.09812) + (-0.0636) \cos 3(4.09812) + 0.5615 \sin (4.09812) + (-0.1640) \sin 2(4.09812)$$

$$C_{\text{rad}} = 21.7752 - 10.0177(-0.57636) - 0.8661(-0.33562) - 0.0636(0.96324) + 0.5615(-0.81720) - 0.1640(0.94200)$$

$$C_{\text{rad}} = 21.7752 + 5.7739 + 0.2907 - 0.0613 - 0.4589 - 0.1545 = 27.1651 \text{ MJ m}^{-2}$$

Hamon-Weiss-Wilson's Figure 5 graphically gives 649 Ly, which is 27.1542 MJ m^{-2}.

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TABLE 2. Hamon-Weiss-Wilson Latitude Coefficients

Latitude, deg	A0	A1	A2	A3	B1	B2
25	25.7805	−6.1852	−1.1368	−0.1326	0.4954	0.0845
26	25.5211	−6.4731	−1.1502	−0.1243	0.5038	0.0544
27	25.2584	−6.7593	−1.1585	−0.1163	0.5113	0.0268
28	24.9906	−7.0429	−1.1623	−0.1088	0.5180	0.0008
29	24.7195	−7.3241	−1.1606	−0.1017	0.5243	−0.0234
30	24.4433	−7.6032	−1.1535	−0.0954	0.5301	−0.0460
31	24.1638	−7.8801	−1.1418	−0.0895	0.5356	−0.0669
32	23.8793	−8.1550	−1.1251	−0.0845	0.5406	−0.0858
33	23.5915	−8.4274	−1.1033	−0.0799	0.5452	−0.1025
34	23.2986	−8.6730	−1.0761	−0.0757	0.5494	−0.1172
35	23.0024	−8.9659	−1.0443	−0.0720	0.5531	−0.1297
36	22.7011	−9.2320	−1.0071	−0.0686	0.5565	−0.1402
37	22.3969	−9.4960	−0.9652	−0.0665	0.5590	−0.1498
38	22.0873	−9.7579	−0.9184	−0.0649	0.5607	−0.1577
39	21.7752	−10.0177	−0.8661	−0.0636	0.5615	−0.1640
40	21.4576	−10.2755	−0.8088	−0.0632	0.5615	−0.1686
41	21.1371	−10.5307	−0.7460	−0.0632	0.5611	−0.1715
42	20.8112	−10.7838	−0.6786	−0.0636	0.5602	−0.1728
43	20.4815	−11.0349	−0.6063	−0.0644	0.5590	−0.1724
44	20.1472	−11.2842	−0.5284	−0.0657	0.5573	−0.1703
45	19.8091	−11.5311	−0.4456	−0.0682	0.5552	−0.1665
46	19.4669	−11.7759	−0.3577	−0.0715	0.5527	−0.1611
47	19.1205	−12.0185	−0.2648	−0.0753	0.5498	−0.1540
48	18.7698	−12.2587	−0.1665	−0.0795	0.5468	−0.1452
49	18.4155	−12.4968	−0.0632	−0.0841	0.5439	−0.1381
50	18.0560	−12.7328	−0.0452	−0.0891	0.5406	−0.1326

These coefficients are used to compute solar radiation in megajoules per square meter. To get solar radiation in langleys, multiply all these coefficients by 23.9066 and round off to two decimals.

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