Reprinted from the
Proceedings of the Symposium on
Modeling Techniques
ASCE/San Francisco, Calif./September 3-5, 1975

FLOW2D: A TWO-DIMENSIONAL FLOW MODEL FOR FLOOD PLAINS AND ESTUARIES

by

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ABSTRACT

In a variety of flood plains and estuaries a one-dimensional analysis of the water movement is inappropriate. For these cases a Two-Dimensional Flow Model (FLOW2D) has been developed. FLOW2D numerically solves the equations of continuity and momentum in finite element form to predict stages and discharges throughout the area of interest. The continuity equation relates the rate of change in water surface elevation to the net rate of instantaneous flow into a segment. The momentum equation is used in a reduced form where it expresses the balance between gravitational and frictional forces. The friction slope is computed through Manning's Equation.

A flood plain or estuary is divided into a number of segments. The input data required are: minimum, median, and maximum elevations within each segment; segment size; cross-section descriptions for the boundaries between segments; Manning's n for each of these cross-sections; and flow or water surface hydrographs at the external boundaries. Most of these data are readily available to a practicing engineer from topographic maps, aerial photographs, or field surveys.

A unique feature of this model which seeks to minimize the cost of simulation runs, while maintaining strict control of the accuracy of the solutions, is a variable time step. Through the use of filter/prediction techniques, the model internally reduces the time step when sharp changes in water surface elevations are occuring. In this manner a more "detailed" description of these changes is obtained. On the other hand, where no such variations are present, the model increases the time step so as to minimize the cost of the solution.

This model has been used to estimate flood elevations in a variety of flood plains in Puerto Rico. In those basins where observed flood

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elevations were available, the simulated results were in close agreement. Earlier versions of this model were used to model flood/tide flows into Lake Pontchartrain in New Orleans, and tidal variations inside Boston Harbor.

INTRODUCTION

The simulation of flows and/or stages in wide and irregular flood plains and estuaries frequently requires a two-dimensional representation of the flow processes be used. In such cases a one-dimensional model may significantly underestimate the stages or flows in certain portions of the area being modeled while overestimating those in other portions. This paper presents the theory, a solution procedure, and a practical application of a two-dimensional model, FLOW2D, which has been used in a number of practical, engineering problems.

THEORY

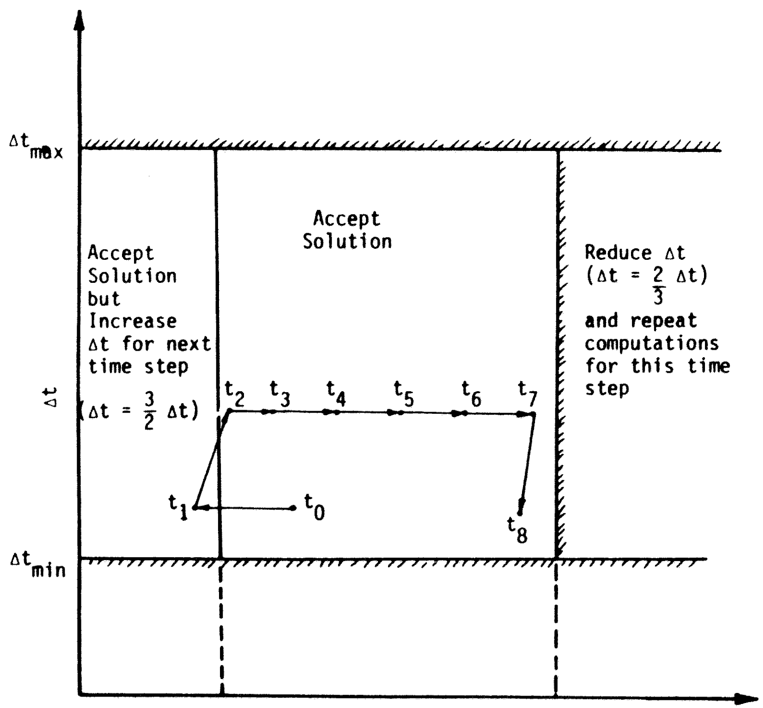
The model is based on the theory of unsteady open channel flow. The study area is divided into segments as shown in Figure 1. The net rate of instantaneous flow into a segment is related to the rate of change in water surface elevation in that segment through the continuity equation:

$$\underbrace{SA_i \frac{dH_i}{dt}}_{\substack{\text{rate of}\\ \text{accumulation}\\ \text{of water in}\\ \text{segment } i}} = \underbrace{-\sum_{j \epsilon J_i} Q_{ij} + q_i}_{\substack{\text{net rate of}\\ \text{inflow to}\\ \text{segment } i}} \qquad (1)$$

where:
- H_i = water surface elevation at the centroid of segment i (ft)
- SA_i = surface area in segment i (ft^2) which may be a function of H_i, i.e., $SA_i = SA_i(H_i)$
- Q_{ij} = flow from segment i to segment j (ft^3/sec)
- q_i = flow into segment i from outside the study area (ft^3/sec)
- t = time (sec)
- J_i = set of segments that are adjacent to i

Equation (1) assumes the (net) inflow rate to a segment can be related to the rate of change in elevation at the centroid of the element.

The flow between segments is assumed to be governed also by the momentum equation for unsteady flow. We further assume that the terms $\frac{\partial v}{\partial t}$ and $v\frac{\partial v}{\partial x}$ are negligible with respect to the other terms, thereby reducing the momentum equation to a balance between gravitational and frictional forces. In other



Maximum Value of $|\theta_i|$ over the Study Area

Figure 2: Policy for Adjusting Time Step Δt

Boundary Conditions

Certain elements are defined as boundary segments because they are located where the user would like to specify a water surface elevation (e.g., tide levels). When the "neighboring" segment (j) is such a "water surface boundary" segment, i.e., one at which the user has alread specified the elevations, there is no row in the $\underline{A}$ matrix for this segment and anytime this segment appears in the set J_i for adjacent segments, the corresponding values of $\gamma_{ij}HF_j$ are subtracted from element b_i (see Equation (20)) on the rhs of Equation (12c).

PRACTICAL APPLICATION

FLOW2D has been applied to a variety of flood plains and estuaries to determine the discharges and stages that would result from selected inflows and/or tide levels. A typical application of this model will be described in detail here. The objective of this study was to estimate t flood stages that would result in the Cibuco River Flood Plain (Puerto Rico) as a consequence of flood inflows from the upstream catchment and storm tides in the Atlantic Ocean. The Cibuco Flood Plain covers an area of roughly 12.3 mi^2 (31.9 km^2). Its upstream inflows are the Cibuco River and its major tributary the Indio River. Both of these rivers rise in the center of the island of Puerto Rico and flow north to the Atlantic Ocean. Their total drainage area is approximately 107 m (718 km^2). The terrain is generally mountainous with steep slopes, unti the rivers arrive in the coastal area where a wide and flat flood plain exists. The outlet from this area is a narrow mouth to the Atlantic Oce A series of small hills and sand dunes prevent flows to the ocean except through the mouth of the Cibuco, even during high tides and flood stages (Figure 3). The narrow mouth constrains the flow to the ocean, therefor storing the large runoff volume events in the flood plain area.

The flood plain was divided into relatively small segments (about 20 to 150 acres each) in order to obtain flood levels at many points throughout the area. The total number of segments used was 87 as shown in Figure 3. The segment data was obtained from U.S.G.S. Quadrangle Map and from field cross-section for the key segments such as the main chann For each segment the following data was required: total surface area, minimum elevation at any point, maximum elevation, and the elevation bel which one-half of the total area occurs. For each boundary between two elements a cross-section description was required. This data was also obtained from U.S.G.S. Quadrangle Maps and field inspection. Specifical the following data were required for each cross-section: distance from the cross-section to the centroid of the upstream and downstream segment a table of elevations and top widths of the available flow area through the cross-section; and a roughness coefficient (Manning's n).

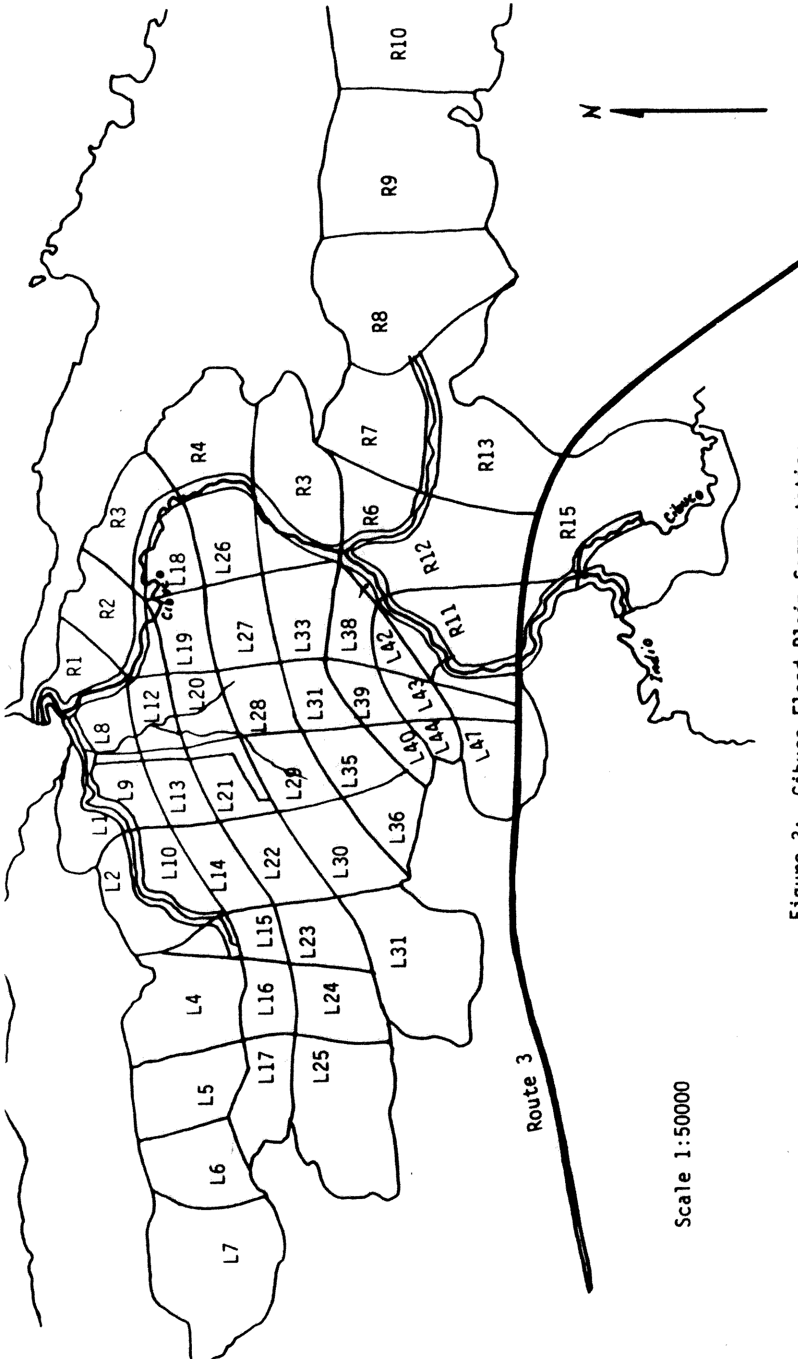


Figure 3: Cibuco Flood Plain Segmentation

$$a_{ik} = -\gamma_{ij} \qquad \text{when } k = j,\ j \epsilon J_i \tag{19a}$$

$$\phantom{a_{ik}} = 0 \qquad \text{when } k = j,\ j \not\epsilon J_i \tag{19b}$$

$$a_{ii} = -\sum_{j \epsilon J_i} \beta_{ij} + SA_i/\Delta t \tag{19c}$$

$$b_i = H_i{*}\ SA_i/\Delta t + -\sum_{j \epsilon J_i} \alpha_{ij} + q_i^{+} \tag{20}$$

The resulting set of linear simultaneous equations can be solved for all the water surface elevations at the next time step H_i^+. In FLOW2D a Gauss-Jordan elimination procedure is used to solve for these variables.

An important limitation of this model is that N simultaneous equations must be solved at each time step if there are N segments in the study area. The practical impact of this limitation is partly offset by the capability to have irregularly shaped segments and segments of unequal size. Also a computational policy internally varies Δt to achieve desired accuracy without excess cost.

The solution procedure yields values of H_i^+ which differ from the forecast values HF_i by the amount $|\theta_i|$. If the largest value of $|\theta_i|$ from all segments i exceeds a user-selected tolerance level, Δt is internally reduced by a factor of two-thirds and the computations are repeated again. On the other hand, if the largest value of $|\theta_i|$ is less than a user selected minimum, the time step is increased by a factor of two-thirds for the next step. This policy is illustrated in Figure 2.

A minimum and maximum Δt are also specified. The maximum Δt usually represents the time step at which the user would like to see the results, even if the solution procedure could be solved in much larger steps. The minimum Δt sets the lower limit beyond which the user feels the solution procedure could become too costly. If the model internally determined that it needs to go below this limit, it will so indicate and terminate the execution. The user can restart the solution procedure if he so desires and set a lower minimum bound for Δt.

Figure 2 also shows how the values of Δt might change during the first few time steps of an event. Experience has shown Δt to be reduced to a few seconds when water levels are rapidly changing and to be increased to several hours during hydrograph recessions when water levels are slowly changing. The effect of this Δt policy is for Δt to seek optimum levels which may vary from time to time. Experience has shown the solution cost to be a small fraction of the cost of using a fixed Δt. The actual variations in Δt during a practical application of the model will be presented later.

$$\hat{H} = (1-\omega_2)\,\hat{H} + \omega_2 H^+ \tag{14}$$

where $\hat{H}$ is computed at the end of a time step and represents an estimate of the true solution H^* to Equation (5) at the beginning of the next time step. Also define as:

$$\dot{\hat{H}} = (1-\omega_1)\dot{\hat{H}}^- + \omega_1\left[\frac{H^+-H}{\Delta t}\right] \tag{15}$$

where $\dot{H}$ is computed at the end of a time step. Finally, let:

$$HF = \hat{H} + \dot{\hat{H}}\,\Delta t \tag{16}$$

Equations (14) and (15) tend to filter-out high frequency noise in H^+ and $\frac{H^+-H}{\Delta t}$. In the limit as ω_1 and ω_2 approach unity, Equations (14), (15), and (16) approach Equation (13). Values of ω_1 and ω_2 nearest zero tend to filter out most noise. If ω_1 and ω_2 are set equal to ze), the expression for Q_{ij} will always be linearized about the initial values of H_i and Q_{ij}.

Solution Procedure

When Equation (5) is substituted into Equation (6), a system of simultaneous linear equations is formed. The value of SA_i should depend, however, on H_i^+. To avoid high frequency variations in SA_i, the following is used:

$$SA_i = \frac{SA_i(H_i) + SA_i(HF_i)}{2} \tag{17}$$

This should be adequate, provided δH is sufficiently small in Equation (8).

The set of simultaneous equations can be structured as:

$$\underline{\underline{A}}\cdot\underline{H}^+ = \underline{B} \tag{18}$$

where:

$\underline{\underline{A}} = \{a_{ik}\}$

$\underline{H}^+ = \{H_i^+\}$

$\underline{B} = \{b_i\}$

and there terms are:

The Manning's friction coefficients for overbank flow were estimated from examination of soil and land use maps of the flood plain. Three general surface roughness classifications with "n" values of 0.045 0.060, and 0.110 were defined from these observations. For streamflow a "n" of 0.030 or 0.035 was assumed. Prior studies have shown that the flood levels estimated with this model are, in general, relatively insensitive to variation of the roughness parameter over the range of roughne coefficient values that might be appropriate in this area.

All of the events simulated for this phase of the study were analyz with a Manning's "n" of 0.045 for sugar cane areas. This value reflects "cropped sugar cane." The resulting flood elevations in the flood plain remote from the streams are in general higher than if sugar cane at its full growth were assumed, i.e., use a Manning's "n" of 0.11. Such a condition is the more critical case for the evaluation of the changes to flood elevations in the basin resulting from the development (and removal from the flood plain) of the proposed project location. A sensitivity ru was carried out where a Manning's n of 0.21 was used for the sugar cane areas. The resulting maximum water surface elevations were not significa different from those obtained with a value of 0.045 for the elements on the sides of the flood plain, but were somewhat higher for the main strean segments which are bordered on each side by sugar cane areas.

The system inputs to the model to simulate a flood event were a set of boundary conditions applied as forcing functions to the appropriate flood plain segments. Discharge hydrographs derived through the M.I.T. Catchment Model were applied to certain upstream segments to represent the flood inflows. Water surface elevation hydrographs are applied to th ocean boundary segments to represent the behavior of a storm tide. Final rainfall hydrographs are input to all segments to reflect the storm event over the flood plain.

An initial steady state flow condition for the system was achieved with a base flow of 1,000 cfs in both the Cibuco and Indio Rivers and a tidal level set at 1.0 foot. All of the flood plain simulation runs were started from the steady state flow condition by applying to the appropria segments, boundary condition hydrographs for the desired events.

Although the Two-Dimensional Flow Model has been tested and verifie in previous studies, a verification run was carried out to demonstrate the applicability of the model to this particular flood plain, and to verify the data used. To this end, the storm of December 12, 1965 (hydrographs are shown in Figure 4) was input to the flood plain model.

A contour map of the flood levels attained by this event as estimate by the U.S. Geological Survey (1971) is shown in Figure 5. The water surface elevations obtained by the simulation of this event through a rainfall runoff model of the upstream catchment (the M.I.T. Catchment Model) and the FLOW2D Model are shown in Figure 6.

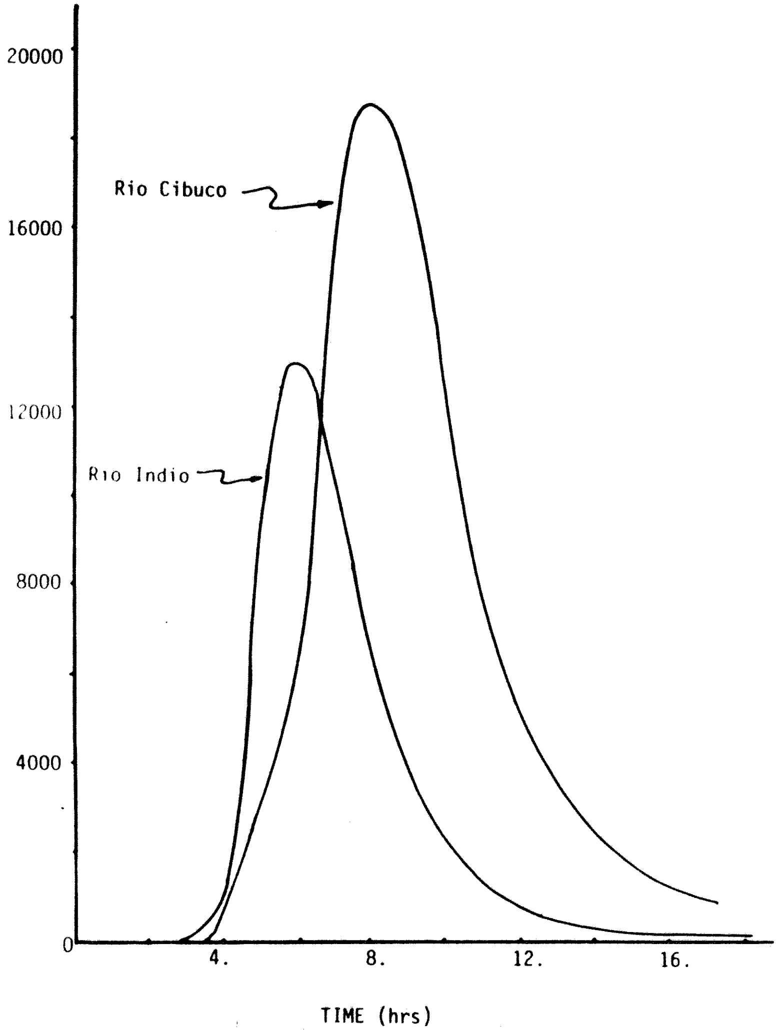


Figure 4: Simulated Discharge Hydrographs

$$\gamma_{ij} = \frac{Q_{ij}(HF_i, HF_j + \Delta H_j) - Q_{ij}(HF_i\ HF_j)}{\Delta H_j} \qquad (12b)$$

$$\alpha_{ij} = Q_{ij}(HF_i, HF_j) - \beta_{ij}HF_i - \gamma_{ij}HF_j \qquad (12c)$$

Equation (7) will hold if δH_i and δH_j are sufficiently small.

Forecast Procedure

The forecast procedure recognizes that computed values of H^+ differ from the true solution H^* to the original differential equations as given by Equation (4), and the errors form a stochastic process. These errors also depend in a very complex way on the forecast procedure used to obtain HF_i. A systematic analysis of all of the equations being used would reveal a very intricate feedback structure, which must not be permitted to become unstable. The "best" general way to deal with this issue is unclear, mainly because stability properties of non-linear solution procedures are data dependent. By trial and error, many different approaches were tried until the following stable scheme emerged.

Let H_i denote an estimate of the true solution H_i^*. It is assumed that the spectrum of H_i^* does not include very high frequencies relative to the time step Δt. Any such high frequency components of H_i^+, are therefore assumed to be caused by a spurious error component induced by the solution procedure.

An obvious possible forecast procedure would be a simple linear extrapolation using the two previous solutions, H and H^-:

$$HF = H + \frac{H-H^-}{\Delta t^-}\Delta t \qquad (13)$$

where Δt^- is the time between H^- and H and Δt is the time between H and H^+. Since the H values may contain a high frequency error term, the values of $\frac{H-H^-}{\Delta t^-}$ will be very sensitive to these errors, amplifying relative to the true values that would obtain if H^* were known. The high-frequency elements in $\frac{H-H^-}{\Delta t^-}$ are introduced by Equation (13) into HF; they are passed on to the coefficients α, β and γ by Equation (12); and they are fed back to H^+ when Equation (6) is substituted into Equations (5). Our experience has shown that the high frequency errors may not adequately be damped out during this process when Equation (13) is used to compute HF. The procedure usually became unstable.

One procedure that tends to damp-out high frequency errors is as follows. Define $\hat{H}$ as:

The Taylor Series expansion of Q_{ij} is:

$$Q_{ij}(H_i^+, H_j^+) = Q_{ij}(HF_i + \delta H_i,\ HF_j + \delta H_j) \tag{9a}$$

$$= Q_{ij}(HF_i,\ HF_j) + \frac{\partial Q_{ij}}{\partial H_i}\bigg|_{HF_i} \cdot \delta H_i + \frac{\partial Q_{ij}}{\partial H_j}\bigg|_{HF_j} \cdot \delta H_j + \dots \tag{9b}$$

The derivatives may be computed as:

$$\frac{\partial Q_{ij}}{\partial H_i}\bigg|_{HF_i} \approx \frac{Q_{ij}(HF_i + \Delta H_i,\ HF_j) - Q_{ij}(HF_i,\ HF_j)}{\Delta H_i} \tag{10a}$$

$$\frac{\partial Q_{ij}}{\partial H_i}\bigg|_{HF_j} \approx \frac{Q_{ij}(HF_i,\ HF_j + \Delta H_j) - Q_{ij}(HF_i,\ HF_j)}{\Delta H_j} \tag{10b}$$

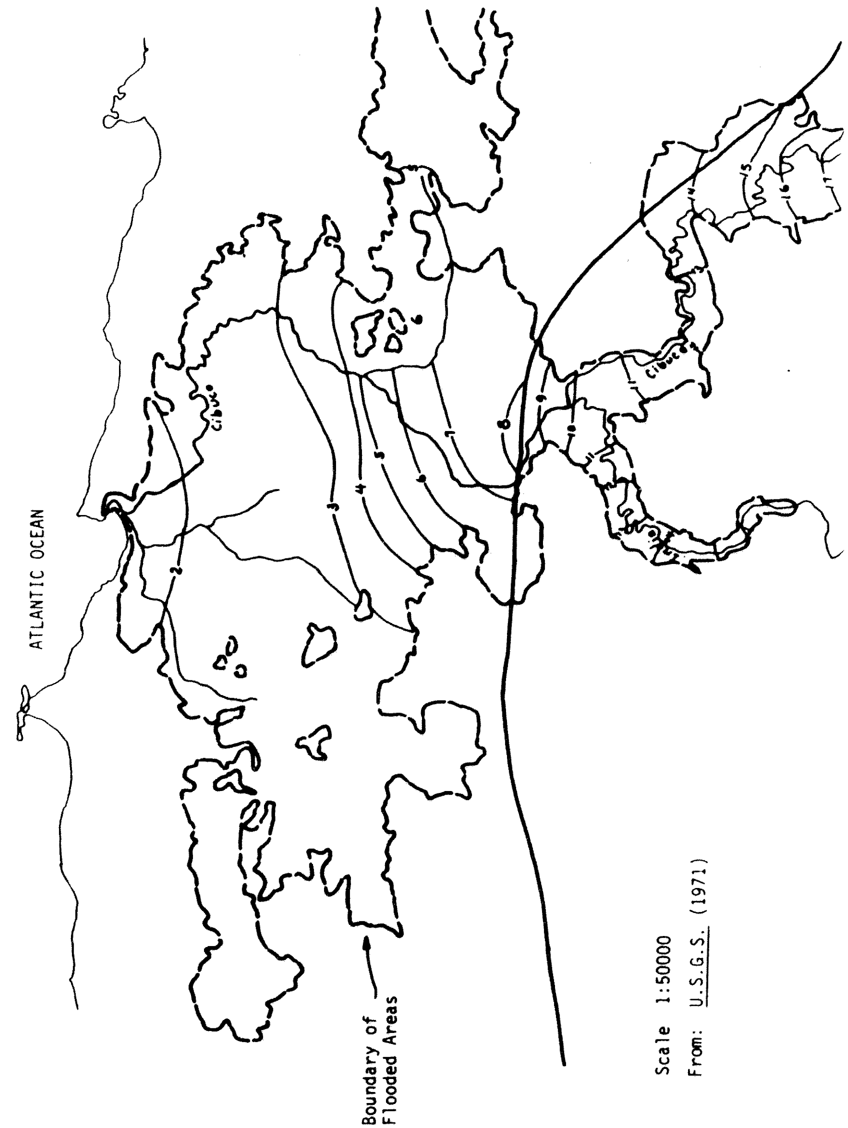
where ΔH_i and ΔH_j are forecast changes in H_i and H_j.

Now let:

$$\hat{Q}_{ij} = Q_{ij}(HF_i,\ HF_j) + \frac{Q_{ij}(HF_i + \Delta H_i,\ HF_j) - Q_{ij}(HF_i,\ HF_j)}{\Delta H_i}(H_i^+ - HF_i) + \frac{Q_{ij}(HF_i,\ HF_j + \Delta H_j) - Q_{ij}(HF_i,\ HF_j)}{\Delta H_j}(H_j^+ - HF_j) \tag{11}$$

This is a linear expression in H_i^+ and H_j^+ and is the same as Equation (6) which was sought, provided that:

$$\beta_{ij} = \frac{Q_{ij}(HF_i + \Delta H_i,\ HF_j) - Q_{ij}(HF_i,\ HF_j)}{\Delta H_i} \tag{12a}$$



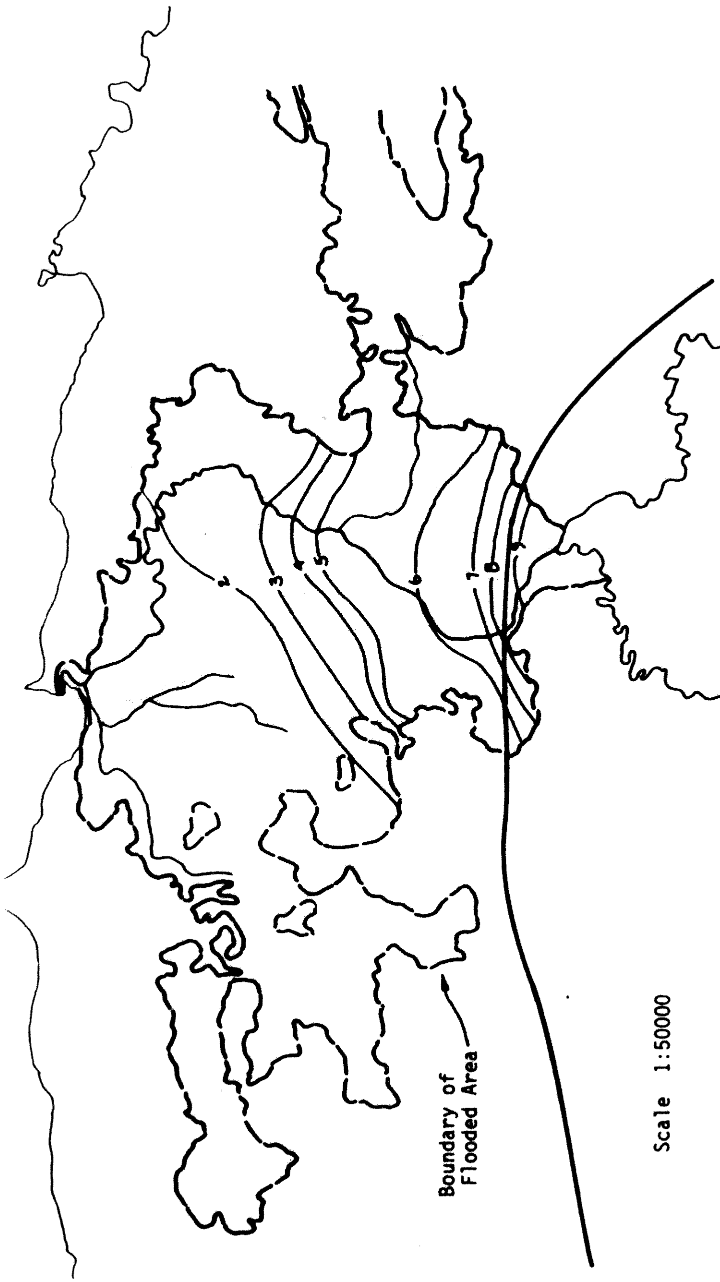


Figure 6: Simulated Flood Contours for Event of Dec. 11, 1965

Non-linear Difference Equation

A backward difference scheme is used to transform Equation (4) into the difference equation:

$$\frac{H_i^+ - H_i}{\Delta t} = \frac{-\sum\limits_{j \epsilon J_i} Q_{ij}(H_i^+, H_j^+) + q_i^+}{SA_i} \tag{5}$$

where: H_i = water surface elevation in segment i at time t (ft)
H_i^+ = water surface elevation in segment i at time t + Δt (ft)
H_j = water surface elevation in segment j at time t (ft)
H_j^+ = water surface elevation in segment j at time t + Δt (ft)
q_i^+ = exogenous inflow rate to segment i at time t + Δt(ft^3/sec)

Linearization of $Q_{ij}(H_i^+, H_j^+)$

The objective at this point is to develop a linear relation of the form:

$$\hat{Q}_{ij} = \alpha_{ij} + \beta_{ij}H_i^+ + \gamma_{ij}H_j^+ \tag{6}$$

such that:

$$\hat{Q}_{ij} \simeq Q_{ij}(H_i^+, H_j^+) \tag{7}$$

so that the estimate $\hat{Q}_{ij}$ may be used in Equation (5) in place of $Q_{ij}(H_i^+, H_j^+)$.

One way to do this is to forecast the values of H_i^+ and H_j^+ and expand the function Q_{ij} in a Taylor Series about the forecast point. This assumes the forecast is sufficiently close to the correct values to assure that Equation (7) holds. The forecast procedure will be discussed later.

Let:

$$H_i^+ = HF_i + \delta H_i \tag{8a}$$

$$H_j^+ = HF_j + \delta H_j \tag{8b}$$

where HF_i and HF_j are forecasts and δH_i and δH_j are forecast errors. A procedure to assure δH_i and δH_j are within acceptable bounds will also be discussed later.

words, the friction slope is assumed equal to the slope of the water surface, $S_f = S_w$. The relation between friction slope, flow rate, flow area, and hydraulic radius is assumed to be given by the Manning Formula:

$$Q = \frac{1.49}{n} AR^{2/3} S_f^{1/2} \quad (2)$$

This equation is evaluated at the cross-section between segments. It is assumed that the water surface is essentially piecewise linear between segments so the water surface slope is assumed to be equal to the average slope of the water surface between the segments; and the water surface elevation at the cross-section is given be linear interpolation between the water surface elevations at the centroids of the segments. Thus:

$$Q_{ij} = \frac{1.49}{n} AR^{2/3} \left[\frac{H_i - H_j}{DIST_{ij}}\right]^{1/2} \quad (3)$$

Note that A and R also depend on H_i and H_j so that Equation (3) defines Q_{ij} as a non-linear function $Q_{ij}(H_i,H_j)$ of H_i and H_j.

When Equation (3) is substituted into Equation (1), a system of simultaneous ordinary non-linear differential equations is formed:

$$\frac{dH_i}{dt} = \frac{-\sum\limits_{j \epsilon J_i} Q_{ij}(H_i,H_j) + q_i}{SA_i(H_i)} \quad (4)$$

SOLUTION OF THE SIMULTANEOUS ORDINARY NON-LINEAR DIFFERENTIAL EQUATIONS

There are several steps required to solve Equation (4) for all segments in the study area. First, the equations must be re-written as non-linear difference equations. Next, the non-linear expression for Q_{ij} must be linearized. Then, a procedure for using this linearization to solve the non-linear difference equations is needed. Finally, one or more segments must be denoted as boundary segments where water surface elevation hydrographs are given for the duration of the simulation period. Inflow hydrographs from streams flowing into the study area and/or from precipitation over the area are applied directly to the appropriate segments through the variable q_i.

In general, very good agreement is shown between these two estimate of the flood elevations. The trend or slope of the water surface (down from the Rio Cibuco at the junction with the Rio Indio towards the ocean is similar in both figures. Both contour plots show flooding in Vega Baja. More specifically, the U.S.G.S. estimated a peak flood elevation at Central San Vicente of 5.6 meters. The flood plain model predicted a elevation of approximately 5.3 meters. This elevation would have been higher if we had included the dam on the Rio Cibuco. This dam was not included because of lack of information about it available at the curren time. This dam was demolished shortly after the 1965 event and therefor it did not enter into any other simulation runs for this study. U.S. estimates are slightly higher (by less than 0.5 meters) near the mouth o the Rio Cibuco. On the other hand, the flood elevations from the FLOW2D are slightly higher near the eastern and western corners of the flood plain.

The simulation of this historical event was carried out for 24 hour The initial time step was set at 6 minutes, and the minimum and maximum Δt allowed were 1 second and 1 hour. For those periods where large discharges were occuring (Figure 4), the model selected time step was generally in the range of 5 to 15 minutes. For very brief portions of the simulation, it carried out the solution procedure at steps as small as 6 seconds or as long as 1 hour.

Overall these results show very good agreement, considering the pos errors in rainfall measurement, the fact that we have no data on the com historic hydrograph, and the field flood elevation estimates for any his event.

FLOW2D has also been successfully applied to the Loiza/Herrera and Mameyes flood plains in Puerto Rico; as a model of flood/tide flows into Lake Pontchartrain in New Orleans; and to model tidal variations inside Boston Harbor.

CONCLUSIONS

A practical method is offered for analysis of two-dimensional flow in flood plains, rivers or estuaries. The study area is segmented into a number, N, of irregularly shaped, unequal-in-size segments. Since N simultaneous non-linear differential equation must be solved it is important to keep N as small as possible. Values of N as large as 100 are practical to use, however. Numerical procedures are offered for efficient solution of the differential equations while preserving desired solution accuracy. These procedures cause the time step, Δt, to vary in a more or less optimal way during the solution. A number of applications of the model have been made. Where historical data were available, excellent agreement between computed and observed water levels and discharges were obtained. An example application to a flood plain in Puerto Rico is presented.

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ACKNOWLEDGEMENTS

A version of the model presented here was developed y at M.I.T. under the support of the Urban Systems Laboratory. developments of the model were carried out by John C. Schaake, Jr. private consulting contracts; and at Resource Analysis, Inc. ntract with Flavio Acaron and Associates of San Juan, Puerto he example presented in the last section was carried out by ms under contract to Highland Realty, Inc. of Rio Piedras, ico.

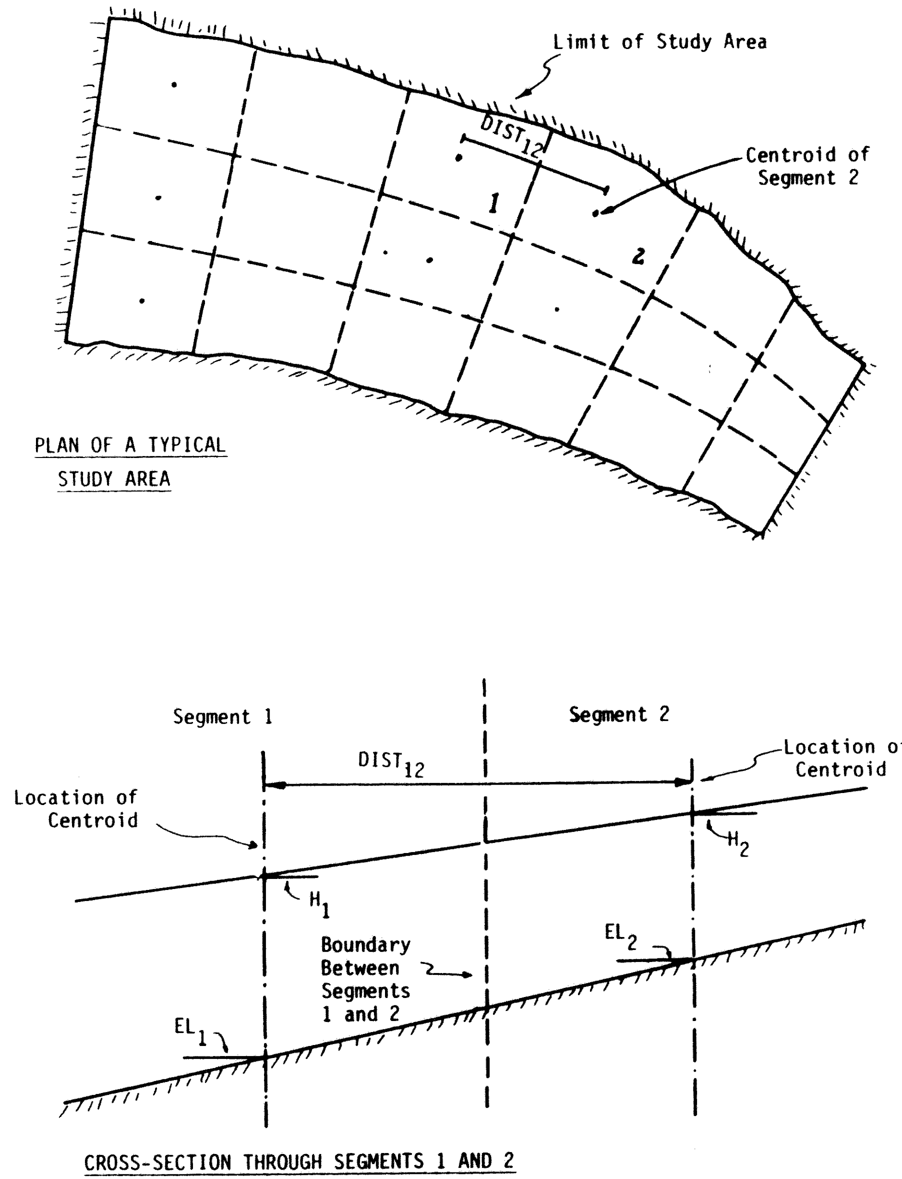

Figure 1: Decomposition of a Study Area into Segments