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COMPUTATION OF STAGE-DISCHARGE RELATIONSHIPS AFFECTED BY UNSTEADY FLOW¹

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ABSTRACT: The dynamic relationship between stage and discharge which is unique to a particular flood for a selected station along the river can be determined via a mathematical model based on the complete one-dimensional equations of unsteady flow, i.e., the equations for the conservation of mass and momentum of the flood wave, and the Manning equation which accounts for energy losses. By assuming the bulk of the flood wave moves as a kinematic wave, the need for spatial resolution of the flood can be eliminated, and only the time variation of either the discharge or stage at the selected station is necessary for the computation of the other. The mathematical model can be used in river forecasting to convert the forecast discharge hydrograph into a stage hydrograph which properly reflects the unique dynamic stage-discharge relationship produced by the variable energy slope of the flood discharge. The model can be used also in stream gaging to convert a recorded stage hydrograph into a discharge hydrograph which properly accounts for the effects of unsteady flow. The model is applied to several observed floods at selected stations along the Lower Mississippi, Red, and Atchafalaya Rivers. The root mean square errors between observed and computed discharges are in the range of 3 to 7 percent, values well within the accuracy of the observations. A simple, easily-applied graphical procedure is also provided for estimating the magnitude of the effect of the unsteady flow on stage-discharge ratings. As a general rule, the dynamic effect may be significant if the channel bottom slope is less than 0.001 ft/ft (about 5 ft/mi) when the rate of change of stage is greater than about 0.10 ft/hr.

(KEY TERMS: open channel flow; unsteady flow equations; stage-discharge ratings; loop ratings.)

INTRODUCTION

Hydrologists are frequently concerned with the conversion of flood discharges at selected locations along a river into corresponding stages or vice versa. This is accomplished via a known relationship between stage and discharge which is applicable for a particular location. Unfortunately, the stage-discharge relationship is not always a simple, single-valued relationship, i.e., a single value of stage does not correspond to a single value of discharge nor does the relationship remain constant for each flood. Such a condition is prevalent during floods in rivers having mild channel bottom slopes and is manifested as a

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loop in the rating curve, the graphical representation of stage as a function of the discharge as shown in Fig. 1. The loop is a manifestation of a hysteresis effect resulting from the variable energy slope associated with the dynamic inertia and pressure forces of the unsteady flood discharge. It will be termed "dynamic loop" herein to distinguish it from those loops which are attributed to alluvial bed form changes [Simons, 1961], variable backwater effects, channel storage, and return of overbank flow [Corbett, 1943].

The dynamic loop has received attention in the literature, e.g., Linsley, *et al.* [1949], Corbett [1943], and Henderson [1966]; however, the proposed methods of computing the dynamic effect in stage-discharge relations are either over simplified or lack generality. This paper presents a general and powerful mathematical model based on the complete one-dimensional equations of unsteady flow for computing either stage or discharge when the temporal variation of the other is specified (observed or predicted). The model is then tested to see if it can reproduce some observed stage-discharge relationships severely affected by the dynamics of an unsteady flood discharge.

THEORY

A unique, dynamic stage-discharge relation for a particular location along a channel can be determined via a mathematical model based on the complete one-dimensional equations of unsteady flow and the Manning equation, which accounts for energy losses due to the resistance of the channel boundary. These equations are derived in several references, e.g., Chow [1959] and Henderson [1966], and are simply stated herein as:

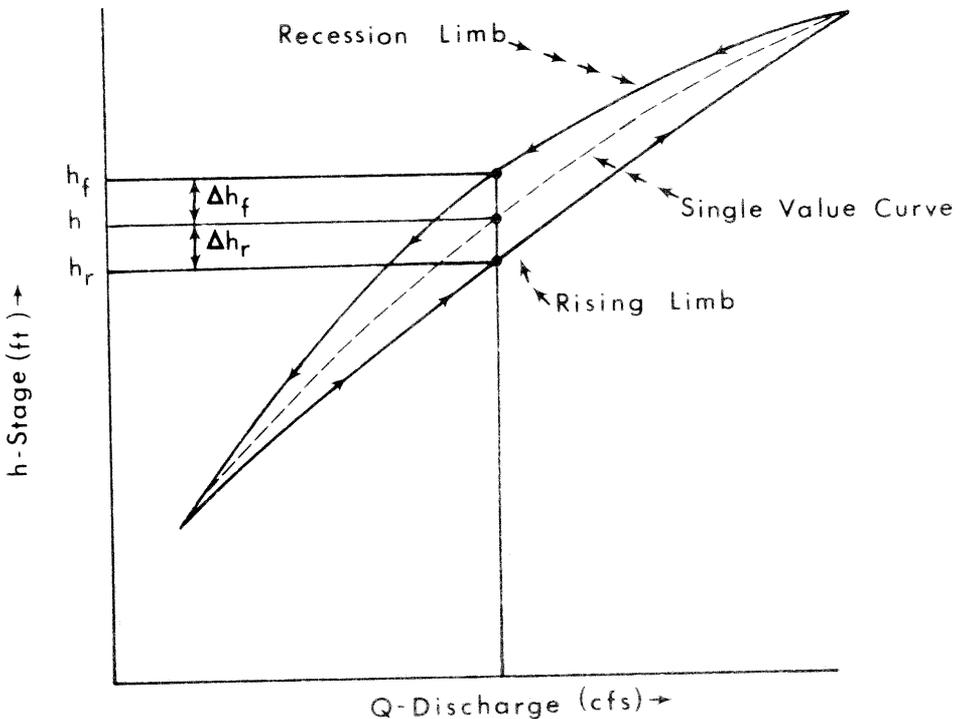


Figure 1. Stage-Discharge Relation with Hysteresis Loop Due to Changing Discharge.

$$A \frac{\partial V}{\partial x} + V \frac{\partial A}{\partial x} + B \frac{\partial h}{\partial t} = 0 \quad (1)$$

$$\frac{\partial V}{\partial t} + V \frac{\partial V}{\partial x} + g \left(\frac{\partial y}{\partial x} + S - S_0 \right) = 0 \quad (2)$$

and the Manning equation,

$$Q = \frac{1.486}{n} A R^{2/3} S^{1/2} \quad (3)$$

In the above equations, x is the distance along the channel, in ft; t is the time, in sec; A is the channel wetted cross-sectional area, in ft^2 ; B is the width of the channel at the water surface, in ft; h is the water surface elevation above a datum plane, in ft; y is the depth of flow, in ft; S is the energy slope, in ft/ft; V is the mean velocity of flow, in ft/sec; S_0 is the effective channel bottom slope, in ft/ft; Q is the discharge, in ft^3/sec ; R is the hydraulic radius, in ft; n is the Manning roughness coefficient, in $\text{sec}/\text{ft}^{1/3}$; and g is the acceleration due to gravity, in ft/sec^2 .

In the development that follows, the following assumptions are made for a short section of channel containing the gaging station or forecast point:

- 1) lateral inflow or outflow is negligible;
- 2) the channel width is essentially constant; i.e., $\partial B/\partial x \simeq 0$;
- 3) energy losses from channel friction and turbulence are described by the Manning equation;
- 4) the geometry of the section is essentially permanent; i.e., any scour or fill is negligible;
- 5) The bulk of the flood wave is moving approximately as a kinematic wave which implies that the energy slope is approximately equal to the channel bottom slope; and
- 6) the flow at the section is controlled by the channel geometry, friction, and bottom slope and by the shape of the flood wave.

An expression for the energy slope is obtained by rearranging Eq (2) in the following form:

$$S = S_0 - \frac{\partial y}{\partial x} - \frac{V}{g} \frac{\partial V}{\partial x} - \frac{1}{g} \frac{\partial V}{\partial t} \quad (4)$$

The four terms on the right side of Eq (4) represent the component slopes which produce the variable energy slope S due to changing discharge. From left to right, respectively, the four slopes are attributed to: gravity force, pressure force, convective (spatial) acceleration, and local (temporal) acceleration.

Using assumption (2), Eq (1) may be expressed as:

$$\frac{\partial V}{\partial x} = - \frac{VB}{A} \frac{\partial y}{\partial x} - \frac{B}{A} \frac{\partial h}{\partial t} \quad (5)$$

Upon substituting Eq (5) and $V = Q/A$ in Eq (4), the following expression for the variable energy slope S is obtained:

$$S = S_o + \left(\frac{BQ^2}{gA^3} - 1 \right) \frac{\partial y}{\partial x} + \frac{BQ}{gA^2} \frac{\partial h}{\partial t} - \frac{1}{g} \frac{\partial(Q/A)}{\partial t} \quad (6)$$

Information concerning the characteristics of either the stage or discharge hydrograph generally is available only at the location for which the rating curve is required. Such lack of spatial resolution of the hydrograph requires that the derivative terms with respect to x in Eqs (4) and (5) be replaced by equivalent expressions which can be evaluated from the available information. Henderson [1966] shows that, if the bulk of the flood wave is moving approximately as a kinematic wave, the following expression may be used to eliminate the need for spatial resolution of the specified hydrograph:

$$\frac{\partial y}{\partial x} = \frac{-1}{c} \frac{\partial h}{\partial t} - \frac{2 S_o}{3 r^2} \quad (7)$$

in which c is the kinematic wave velocity and r is the ratio of the channel bottom slope to an average wave slope.

The kinematic wave velocity c may be determined from observations of the time interval between equal rises in stage, h , at gaging stations along the channel. Also, c may be computed from a relationship given by Henderson [1966] and Chow [1959]:

$$c = \frac{1}{B} \frac{dQ}{dh} \quad (8)$$

where dQ/dh is the slope of the single-value rating curve. If the channel is assumed to be prismatic, the kinematic wave velocity can be computed directly by substituting Eq (3) in Eq (8). After differentiation, the following is obtained:

$$c = KV = K Q/A \quad (9)$$

where

$$K = \frac{5}{3} - \frac{2A}{3B^2} \frac{dB}{dh} \quad (10)$$

and the hydraulic radius R is approximated by the hydraulic depth D ; i.e.,

$$R \simeq D = A/B \quad (11)$$

This is a good approximation of the hydraulic radius for large channels and will be used herein. If the hydraulic radius is used in lieu of the hydraulic depth, the term dB/dh in Eq (10) would be replaced by dP/dh , where P is the wetted perimeter of the channel cross-section; and, the term B^2 would be replaced by the product PB . From an inspection of Eq (10) it is evident that K has an upper limit of about 1.7 when dB/dh is negligible

and a lower limit of about 1.3 for a triangular-shaped channel. It has been observed that K can be approximated as 1.3 for many natural channels [Corbett, 1943; Linsley, *et al.*, 1949]. Although there are a number of methods for determining the kinematic wave velocity, Eq (9) is used in this study.

The value of r in Eq (7) may be taken as a constant for a particular channel. Typical values of r range from 10 to 100. It is used in Eq (7) as part of a small correction which accounts for the fact that a typical flood wave is not exactly a kinematic wave. To arrive at a value for r , the wave slope is approximated from the characteristics of a typical flood event for a particular channel location. The wave slope is determined by dividing the height of the wave by its half-length, the latter obtained by assuming that the wave travels as a kinematic wave during the interval of time from the initiation of the wave to the occurrence of the wave peak at the location of concern. The half-length is determined from the product of the average kinematic wave velocity and the time to peak stage. Eq (9) is used to determine the average kinematic velocity; with Q and A taken as the average values during the flood event and K assumed equal to 1.3. Hence, the following expression is obtained for evaluating r :

$$r = \frac{56200 (Q_p + Q_o)}{(h_p - h_o) A} \tau S_o \quad (12)$$

in which Q is the discharge prior to the start of the typical flood, in ft^3/sec ; Q_p is the peak discharge of the typical flood, in ft^3/sec ; h_o is the stage prior to the start of the typical flood, in ft; h_p is the peak stage of the typical flood, in ft; \bar{A} is the wetted cross-sectional area associated with the average stage, $(h_p + h_o)/2$, in ft^2 ; and τ is the interval of elapsed time from the beginning of a rise in stage until the occurrence of the peak stage, in days.

Since c and r are defined by Eqs (9) and (12), Eq (7) can be substituted in Eq (6), with the partial derivatives in the latter replaced by a finite difference expression. After some rearrangement, the following equation is obtained:

$$S = S_o + \left[\frac{A}{KQ} + \left(1 - \frac{1}{K}\right) \frac{BQ}{gA^2} \right] \delta h_s + \frac{Q'/A' - Q/A}{g\Delta t} + \frac{2S_o}{3r^2} \left(1 - \frac{BQ^2}{gA^3}\right) \quad (13)$$

where K is given by Eq (10); Δt is the computational time step, in sec; Q' is the discharge at time $t-\Delta t$, in ft^3/sec ; A' is the cross-sectional area at time $t-\Delta t$, in ft^2 ; and δh_s is the change in water surface elevation during the Δt time interval, in ft/sec, and $\delta h_s = (h-h')/\Delta t$, where h' is the stage at time $t-\Delta t$.

Eq (13) is the expression for the variable energy slope S which is caused by varying discharge. All of the terms on the right side of the equation except S_o account for the effect of the dynamic characteristics of the flow. If the flow is steady (unchanging with time) the energy slope is constant and equivalent to the bottom slope, S_o . This is evident from Eq (13) since all terms on the right side of the equation except the first term vanish when the flow is steady; i.e., $Q' = Q$, $\delta h_s = 0$, and r is infinitely large since the wave slope vanishes for steady uniform flow.

The dynamic relationship between stage and discharge can be determined from the Manning equation wherein the energy slope is evaluated using Eq (13). In this paper, the

hydraulic radius is approximated by the hydraulic depth. Upon substituting Eq (13) and Eq (11) into Eq (3), the following is obtained:

$$Q - 1.486 \frac{AD^{2/3}}{n} \left\{ S_0 + \left[\frac{A}{KQ} + \left(1 - \frac{1}{K}\right) \frac{BQ}{gA^2} \right] \delta h_s + \frac{Q'/A' - Q/A}{g\Delta t} \right. \\ \left. + \frac{2S_0}{3r^2} \left(1 - \frac{BQ^2}{gA^3}\right) \right\}^{1/2} = 0 \quad (14)$$

Eq (14) can be used to determine either discharge when the rate of change of stage is known (as in stream gaging) or stage when the rate of change of discharge is known (as in stream forecasting).

If Eq (14) is used to determine the discharge when the stage is specified (observed or predicted), the known quantities consist of constants (S_0 , r , g , Δt); known functions (A , B , n , K) of the specified stage h , and known quantities (Q' , A' , h') associated with the time $t-\Delta t$. The only unknown is Q .

If Eq (14) is used to determine the stage when the discharge is specified, the known quantities consist of constants (S_0 , r , g , Δt), the specified discharge Q , and the quantities (Q' , A' , h') associated with the time $t-\Delta t$. The terms (A , B , n , K) are functions of the unknown stage.

In each of the above cases, the equation is nonlinear with respect to the unknown variable. Thus, an explicit solution is not possible; therefore, an iterative solution via a digital computer is required. A very efficient solution can be obtained via Newton Iteration [Isaacson and Keller, 1966]. Details of the application of the Newton Iteration Technique are presented by Fread [1973]. A Fortran IV computer program which provides either the computation of discharge from a specified stage or vice versa is also given by Fread [1973].

Eq (14) differs from other equations given in the literature, e.g. Linsley *et al.* [1949], Corbett, [1943], which use only gravity and pressure forces in the computation of the variable energy slope, i.e. only the first two terms of Eq (4). In Eq (14), the energy slope is evaluated using gravity, pressure, and inertia forces. This feature enables Eq (14) to be more accurate for the computation of stage-discharge relationships associated with the normal flood flows and to be capable of treating those flows in which the inertia terms are dominant such as hurricane surges, reservoir releases, and dam failures. Also, the other equations presented in the literature provide only for the computation of discharge. In this paper, Eq (14) is used to compute either stage or discharge via the Newton Iteration Technique.

APPLICATIONS

In either application of Eq (14) to determine the dynamic relation of stage and discharge, the following information is required:

- 1) the effective bottom slope S_0 determined from a low flow water surface profile; if the low flow profile is noticeably affected by the irregularity of the channel bottom, the high flow profile should be used;
- 2) the cross-sectional area A and the surface width B as functions of the stage; usually about three to ten values of A , B , and h will adequately describe the variation of

area and width with stage, and linear interpolation will provide any intermediate values required during the computation of the stage-discharge relation.

- 3) the Manning's coefficient n can be computed via Eq (3) from observed stages and discharges where Q is the discharge associated with a mean single-value rating curve and S_o is assumed for the energy slope S ; n is expressed as a linear function of stage, i.e.,

$$n = n_{L0} + \frac{(n_{L1} - n_{L0})}{(h_{L1} - h_{L0})} (h - h_{L0}) \quad (15)$$

where n_{L0} is the n value associated with the stage h_{L0} and n_{L1} is the value associated with the stage h_{L1} . The n - h relation may vary sufficiently to require two linear relations, i.e. Eq (15) for a lower range of stage $h_{L0} < h < h_{L1}$; and a similar equation for an upper range in stage $h_{U0} < h < h_{U1}$; and

- 4) the specified (observed or predicted) stage or discharge hydrograph; the predicted discharge hydrograph may be the product of some hydrologic flow routing technique such as the Muskingum method.

Eq (14) was tested on several stage-discharge relationships at several locations where the stage-discharge relation is significantly affected by unsteady flow. The floods selected for testing Eq (14) caused minimal scour and fill at the selected locations.

TABLE 1. Cross-Section and Hydraulic Data for Locations on the Lower Mississippi, Red, and Atchafalaya Rivers.

Location	S_o	Year	n_{L0}	n_{L1}	h_{L0}	h_{L1}	n_{U0}	n_{U1}	h_{U0}	h_{U1}	h	A	B
Mississippi River											16.0	72500.	3000.
Tarbert Landing, Louisiana	0.0000143	1967	0.0156	0.01542	5.	50.					34.0	134000.	3540.
											41.2	164000.	3630.
											48.0	200000.	3690.
Mississippi River											15.0	55300.	3020.
Red River Landing, Louisiana	0.0000143	1963	0.0108	0.0113	15.	50.					16.0	58000.	3060.
											18.0	64400.	3160.
											19.0	68000.	3360.
											30.0	104000.	3420.
											37.0	133500.	4310.
											44.0	163300.	4400.
Red River											45.0	4800.	463.
Alexandria, Louisiana	0.0000233	1966	0.031	0.018	45.	55.	0.0180	0.0130	55.	80.	52.0	8300.	528.
											60.0	13000.	576.
											80.0	26600.	696.
Atchafalaya River											1.0	20000.	960.
Simmesport, Louisiana	0.0000211	1964	0.0395	0.0196	5.	16.1	0.0196	0.0143	16.1	40.	20.0	42100.	1250.
											40.0	66000.	1425.

Using the information shown in Table 1 and observed stage-hydrographs, stage-discharge rating curves and discharge hydrographs were computed for selected floods at selected locations on the Lower Mississippi, Red and Atchafalaya Rivers. The dynamic

stage-discharge relationship for Tarbert Landing (river mile 306.3) on the Lower Mississippi River is shown in Fig. 2. The solid line representing the computed rating curve for the flood of 1967 was obtained by: 1) using Eq (14) to compute the discharge hydrograph shown in Fig. 3 from the specified multiple-peaked stage hydrograph shown

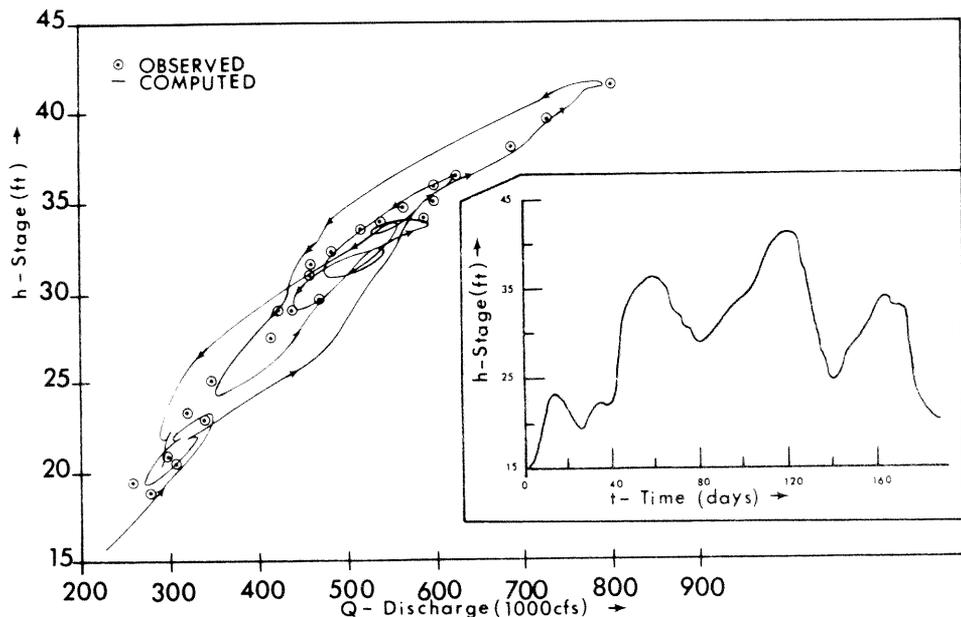


Figure 2. Stage-Discharge Relation for Mississippi River, Tarbert Landing, Louisiana, (2/1/67 - 8/3/67).

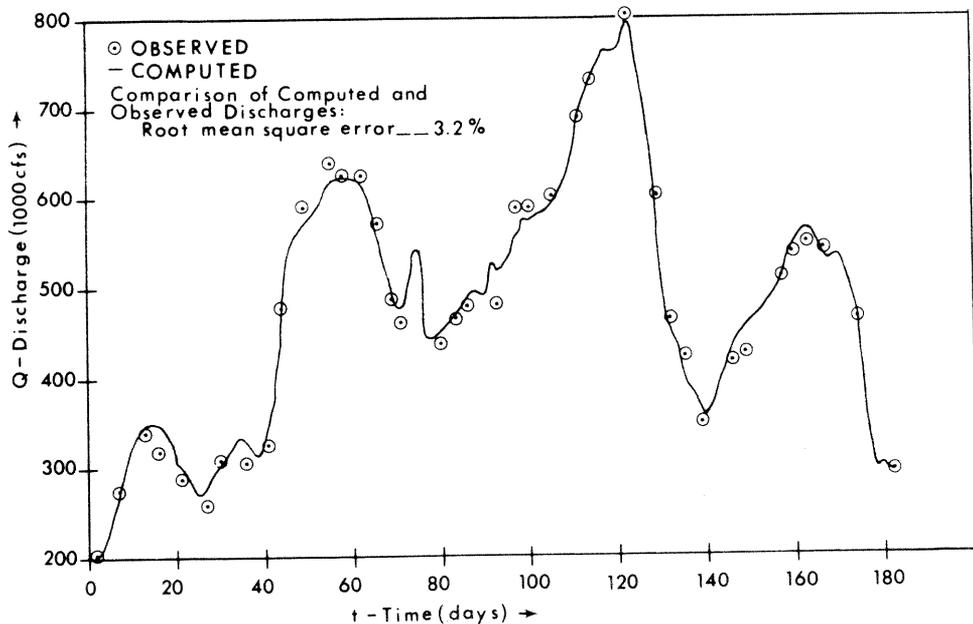


Figure 3. Discharge Hydrograph for Mississippi River, Tarbert Landing, Louisiana, (2/1/67 - 8/3/67).

in the insert of Fig. 2 and 2) plotting the observed stage against the computed discharge. Measured values of stage and discharge are shown in Fig. 2 also. The computed rating curve has a substantial dynamic loop, e.g. the discharge value of 450,000 cfs has two associated stages, one on the rising limb and one on the recession limb of the specified stage hydrograph, which differ by approximately 5 ft. The loop rating curve has irregularities which reflect each variation in the rate of change of the stage hydrograph. Each peak of the specified hydrograph produces a different loop in the rating curve. A comparison of the computed and observed discharges is shown in Fig. 3 where the root mean square (rms) error is approximately 3 percent.

At Red River Landing (river mile 302.4) on the Lower Mississippi River, the 1963 observed stage hydrograph, shown in the insert of Fig. 4, yields the computed stage-discharge rating curve of Fig. 4. The dynamic loop is rather significant; for a discharge of

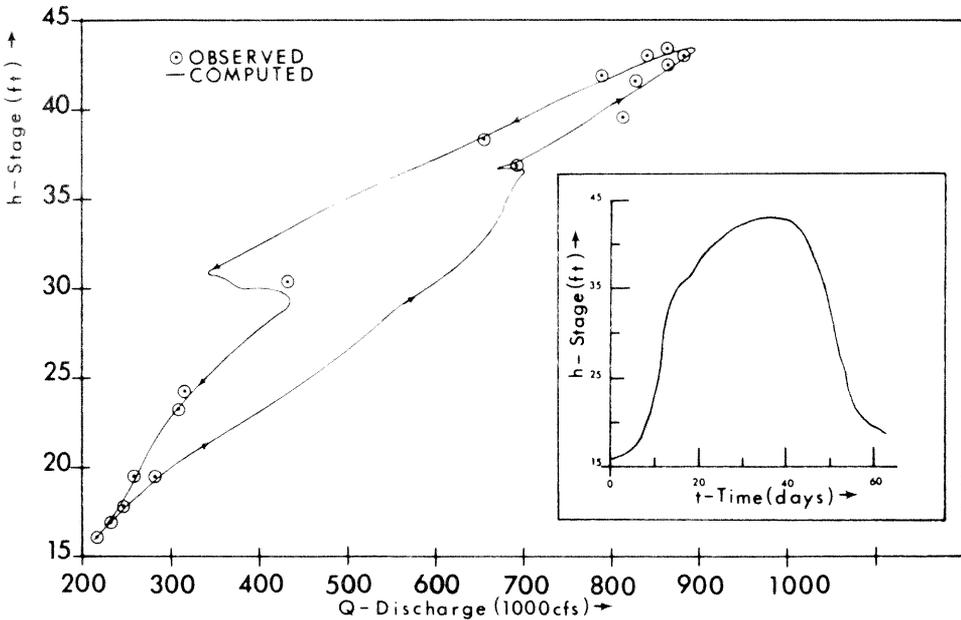


Figure 4. Stage-Discharge Relation for Mississippi River, Red River Landing, Louisiana, (3/11/63 - 5/13/63).

450,000 cfs, the maximum difference between stages of the rising and falling limbs of the hydrograph is about 9 ft. This is a result of the rather severe rate of change of stage which exists for both the rising and the recession limbs of the stage hydrograph. Also, noticeable variations in the rate of change of the stage hydrograph are reflected as irregularities superimposed on an otherwise smooth single-looped rating curve. The computed discharge hydrograph, shown in Fig. 5, compares with measured discharges quite well, the rms error being 1.6 percent.

At Alexandria (river mile 104.4) on the Red River, the single-peak and uniform variation of the observed stage hydrograph of 1966, shown in the insert of Fig. 6, yields the smooth single-loop rating curve shown in Fig. 6 and the computed discharge hydrograph of Fig. 7. The rms error of the computed versus observed discharges is 4.6 percent.

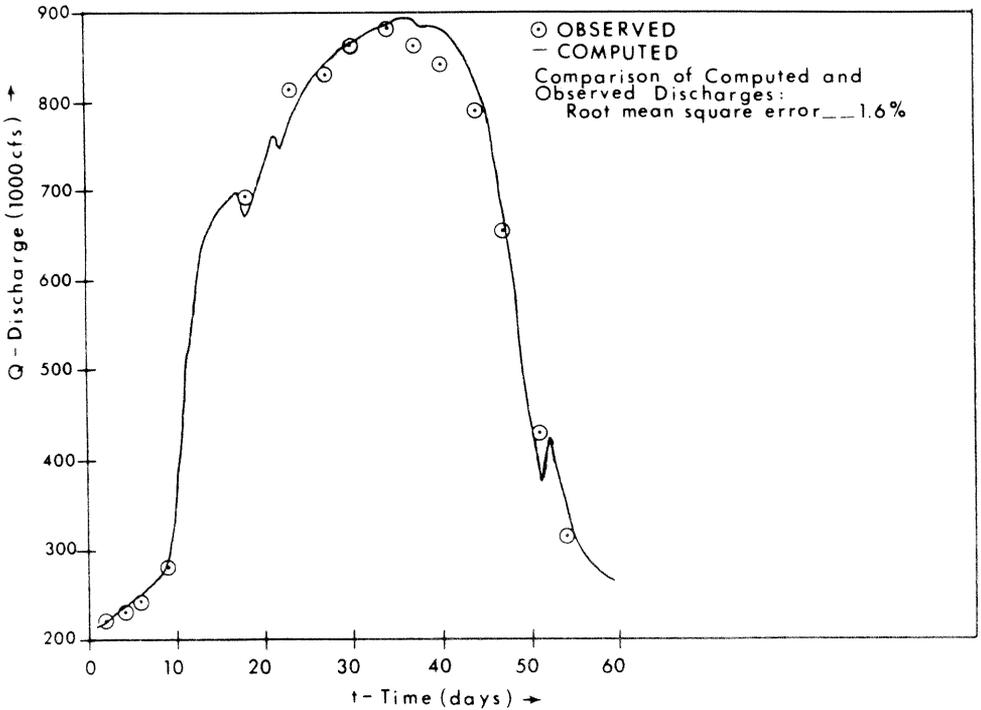


Figure 5. Discharge Hydrograph for Mississippi River, Red River Landing, Louisiana, (3/11/63 - 5/13/63).

The 1964 observed stage hydrograph at Simmesport on the Atchafalaya River, shown in the insert of Fig. 8, results in the computed rating curve of Fig. 8. The irregularity of the stage hydrograph causes the computed rating curve to contain several small loops and other irregularities. The dynamic loop of the rating curve has a maximum difference of 8 ft in rising and falling stages corresponding to the single discharge value of 200,000 cfs. The irregularities observed in the stage hydrograph are reflected in the computed discharge hydrograph of Fig. 9. The computed and observed discharges have an rms error of 7.5 percent.

In each of the above applications, the dynamic loop of the computed stage-discharge rating curve is quite significant. This occurs, even though the maximum rate of change of stage is of the order of a few feet per day, because the effective bottom slope at each location is very mild. The importance of the bottom slope and the rate of change of stage will be discussed later. Equation (14) provides computed discharges which agree closely with the measured values, the average rms error being approximately 4 percent.

ESTIMATED MAGNITUDE DYNAMIC LOOP

In this section, a simple graphical procedure is developed to estimate the magnitude of the dynamic loop. This magnitude is denoted by Δh as shown in Fig. 1; Δh is the difference between the stage associated with the single-value rating curve and the stage (at the same discharge) associated with the rising or recession limb of the hydrograph. The graphical procedure is based on simplifying assumptions concerning the dynamic relation-

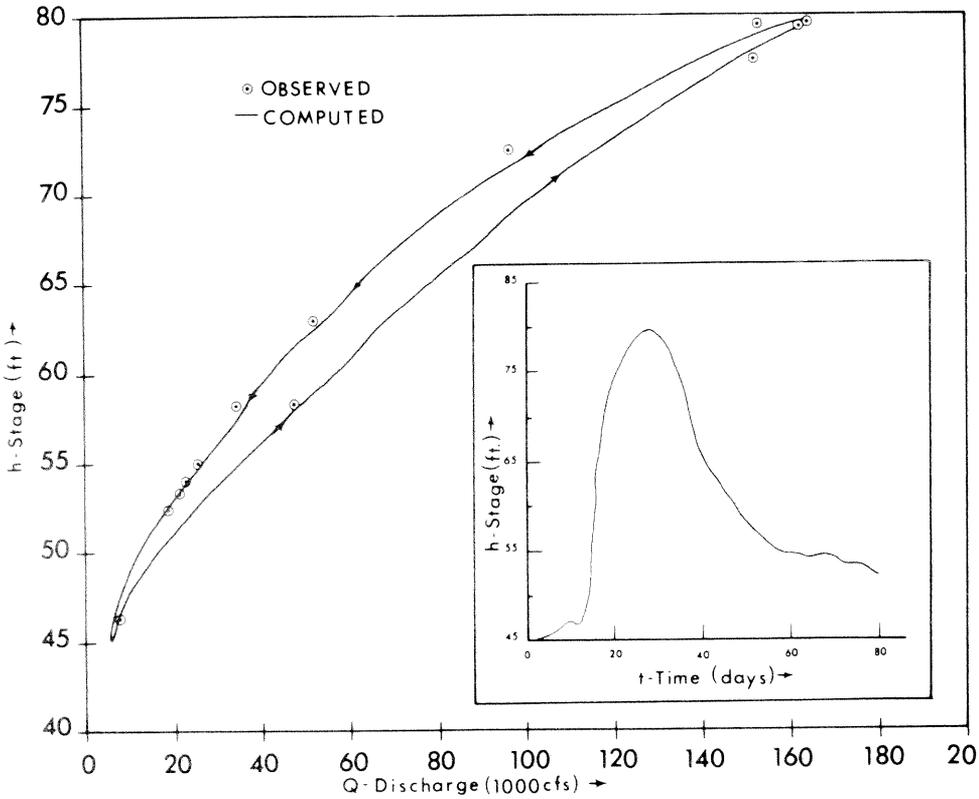


Figure 6. Stage-Discharge Relation for Red River, Alexandria, Louisiana, (4/13/66 - 7/1/66).

ship between stage and discharge, however, it is intended to provide an approximate value for Δh so that it may be determined if Eq (14) need be used for a particular river station and flood event.

If the hydraulic depth D is substituted for the hydraulic radius in Eq (3), the following expression for D is obtained:

$$D = \left[\frac{Q n}{1.486 B} \right]^{0.6} S^{-0.3} \tag{16}$$

Then, by assuming a small change in hydraulic depth is equivalent to a small change in stage (i.e. $\Delta h = \Delta D$) and evaluating Q in Eq (16) via Eq (3) in which R is replaced by D and S by S_0 , the following approximation for Δh can be obtained:

$$\Delta h \simeq D \left[1 - \left(\frac{S_0}{S} \right)^{0.3} \right] \tag{17}$$

in which the energy slope S is approximated by:

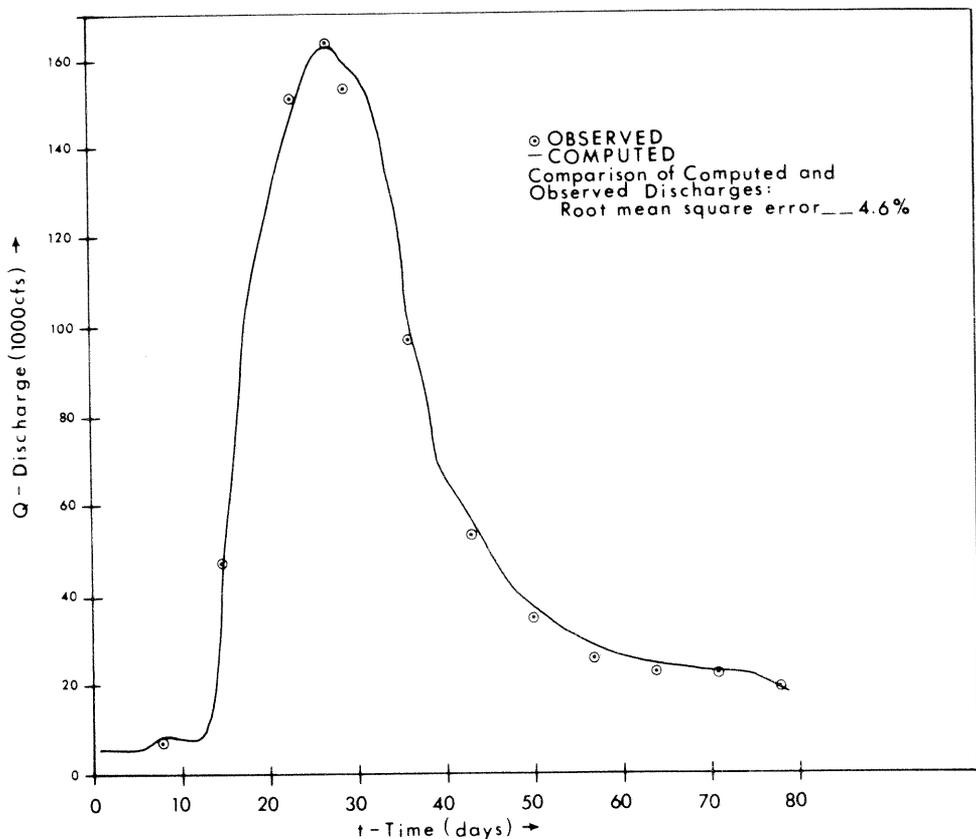


Figure 7. Discharge Hydrograph for Red River, Alexandria, Louisiana, (4/13/66 - 7/1/66).

$$S \simeq S_0 + \delta h_s \left[\frac{0.52}{S_0^{1/2} D^{2/3} n} + \frac{0.01 S_0^{1/2} D^{2/3}}{D n} \right] \quad (18)$$

Eq (18) is a simplification of Eq (13) obtained by assuming: 1) $K = 1.3$, 2) Q is evaluated via Eq (3) in which S is replaced by S_0 , and 3) all terms on the right side of Eq (13) except the first two may be neglected.

An inspection of Eqs (17) and (18) indicates that the independent parameters necessary to approximate the magnitude of Δh are: S_0 , δh_s , D , and n . Thus, by allowing these parameters to assume values which encompass the practical range of each, Eqs (17) and (18) may be used to determine the Δh associated with various parameter values. The results of these computations are summarized by the family of graphical relationships shown in Fig. 10. The curves of Fig. 10 can be used to determine Δh for any combination of the relevant parameters (S_0 , δh_s , D , and n). The following example illustrates the use of Fig. 10 to estimate the magnitude of the dynamic loop:

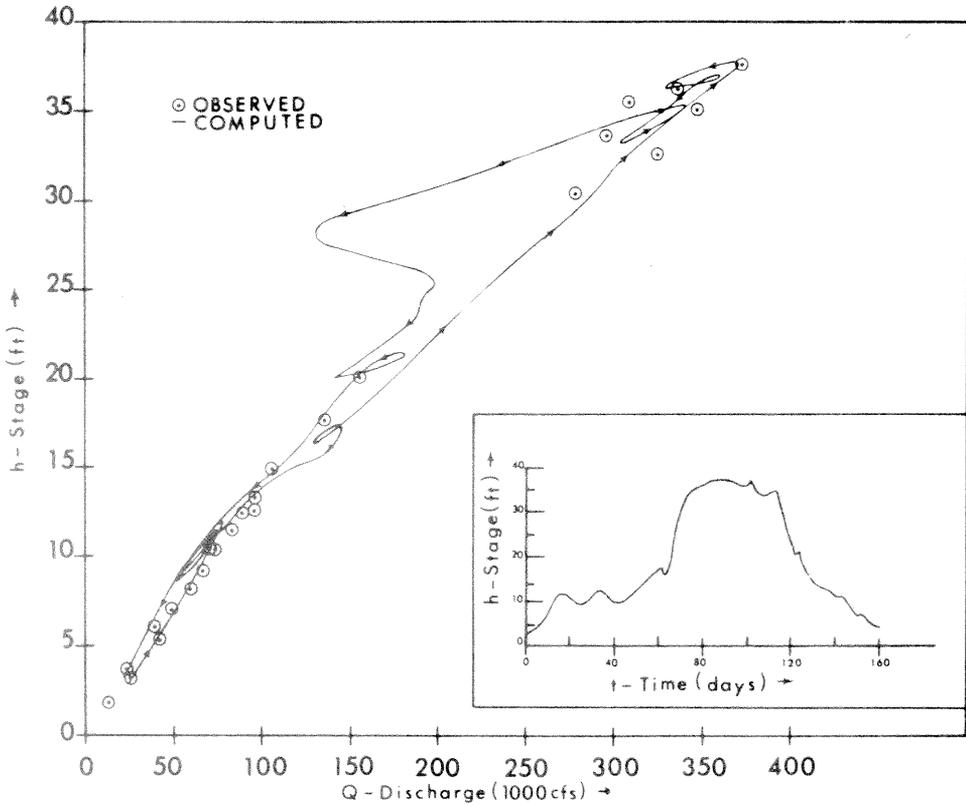


Figure 8. Stage-Discharge Relation for Atchafalaya River, Simmesport, Louisiana, (1/7/64 - 6/16/64).

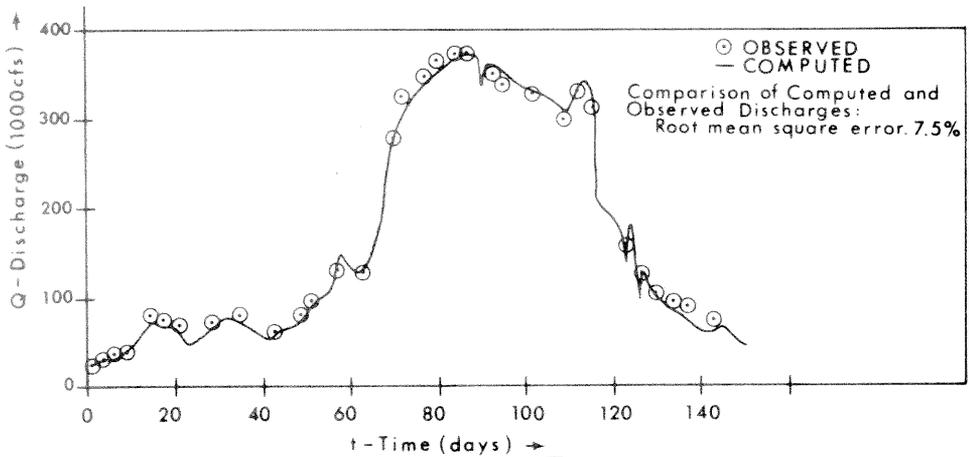


Figure 9. Discharge Hydrograph for Atchafalaya River, Simmesport, Louisiana, (1/7/64 - 6/16/64).

The approximate value of Δh is to be determined when $S_o = 0.00008$, $D = 20.0$ ft, $n = 0.020$, and $\delta h = 1.0$ ft/hr. First, the parameter $\frac{D^{2/3}}{n}$ is computed to be 368. With this value, and the given value of δh , Graph A is used to obtain a value of 0.82 for K_o .

Then, with the value of K_0 and the given value of S_0 , Graph B is used to obtain a value of 0.000137 for S . Finally, the ratio S_0/S is computed to be 0.584 and this value, along with the given value of D , is used in Graph C to obtain the Δh value of 2.9 ft.

The curves of Fig. 10 can be used to study the sensitivity of the parameters (Δh , S_0 , D , n) in Eqs (17) and (18). If values of D and n are selected and held constant while δh and S_0 are allowed to vary, it is found that:

- 1) Δh increases as δh increases for a constant value of S_0 ;
- 2) Δh increases as S_0 decreases for a constant value of δh ;

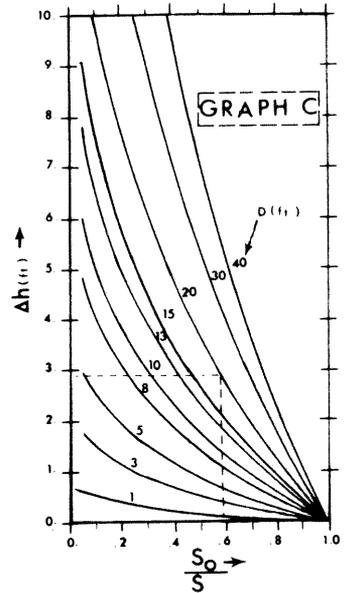
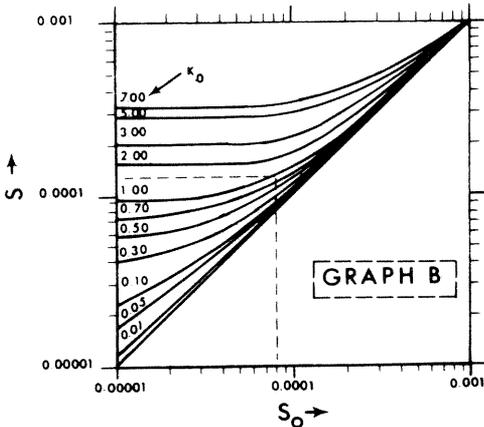
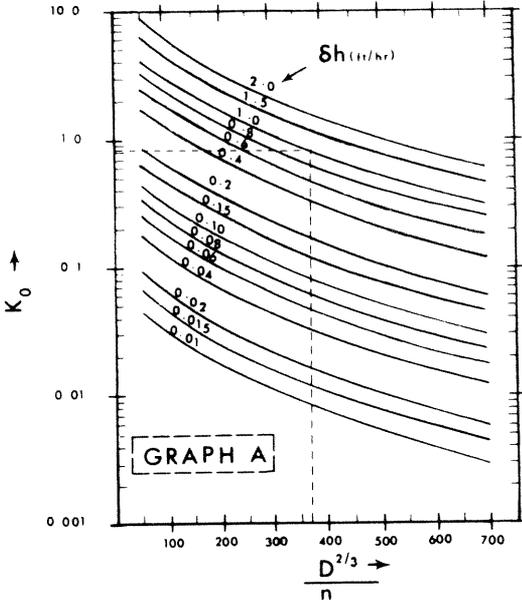


Figure 10. Generalized Graphical Estimation of Magnitude (Δh) of the Dynamic Loop.

- 3) Δh is insignificant (say less than about 0.1 ft) when $S_o > 0.001$ for most naturally occurring values of δh , say $0 < \delta h < 4$ ft/hr;
- 4) Δh may be significant when $0.0001 < S_o < 0.001$ for $0.1 < \delta h < 3$ ft/hr; and
- 5) Δh is significant when $S_o < 0.0001$ for $\delta h > 0.05$ ft/hr.

If values of S_o and δh are selected and held constant while D and n are allowed to vary, it is found that:

- 1) Δh increases as D increases for a constant value of n ; and
- 2) Δh increases as n increases for a constant value of D .

SUMMARY AND CONCLUSIONS

From the equations of unsteady flow, a mathematical model is developed which computes either stage or discharge if the other is specified along with the channel slope, cross-sectional properties and Manning's n . The model simulates the dynamic relationship which exists between stage and discharge due to the effect of a variable energy slope caused by changing discharge. This effect, which is often observed as a loop in stage-discharge rating curves, was accurately modeled in several test applications; however, caution must be exercised when applying the model to locations where significant scour, fill and/or bed form changes occur since the model is only as accurate as the specified data.

The model can be used in forecasting to convert the forecast discharge hydrograph into a stage hydrograph which properly reflects the dynamic relationship which exists between stage and discharge due to a variable energy slope. Also, the model can be used in stream gaging to convert an observed stage hydrograph into a discharge hydrograph when the effect of changing discharge is significant.

A convenient graphical procedure is given to estimate the magnitude of the changing discharge effect on stage-discharge ratings. This is useful in determining if the magnitude of the dynamic loop warrants the use of the mathematical model. The magnitude of the dynamic loop has been found to be related inversely to the channel bottom slope, and directly to the rate of change of stage, the hydraulic depth, and the Manning's n . As a general rule, the dynamic loop may be significant if the channel bottom slope is less than 0.001 ft/ft (about 5 ft/mile) and the rate of change of stage is greater than about 0.10 ft/hr.

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