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Cokriging Radar-Rainfall and Rain Gage Data

WITOLD F. KRAJEWSKI

Hydrologic Research Laboratory, National Weather Service, National Oceanic and Atmospheric Administration, Silver Spring, Maryland

An ordinary cokriging procedure has been developed to optimally merge rainfall data from radars and standard rain gages. The radar-rainfall data are given in digitized form. The covariance matrices required to perform cokriging are computed from single realization data, using the ergodicity assumption. Since the ground truth and the error structure of the radar data are unknown, parameterization of the covariance between radar data and the true rainfall is required. The sensitivity of the procedure to that parameterization is analyzed within a controlled simulation experiment. The experiment is based on a hypothesized error structure for the rainfall measurements. The effect of measurement noise and network density is examined. The usefulness of the procedure to remove the bias in radar is tested. Daily data are used.

Introduction

Recent progress in quantitative hydrology brings out in strong relief the need for accurate real-time analysis of precipitation, probably the single most important hydrometeorological input to streamflow prediction models. Because of its large variability in space and time, precipitation is difficult to measure accurately with a network of rain gages. For real-time hydrologic applications of rain gage data, automated gages should be used. Large numbers of automated rain gages are both expensive and difficult to maintain and operate, even with today's sophisticated communication networks. An alternative device which is potentially useful for precipitation measurement is meteorological radar [*Kessler and Wilk*, 1968; *Hudlow*, 1973; *Austin and Austin*, 1974; *Anderl et al.*, 1976].

Land-based weather radar provides capability to measure precipitation continuously in time and space, typically within a radius of up to 200 km. Radar measurement of precipitation is indirect, and raw reflectivity data have to be converted into rainfall units, using a "*Z-R* relationship" [*Battan*, 1973]. In order to estimate the coefficients of a *Z-R* relationship, rain gage data are used. Many radar systems are equipped with digital processors which allow them not only to convert the raw data into rainfall, but also to integrate them into a desired time and space scale.

Unfortunately, radar data, as well as data from other remote sensors, are characteristically in error because of equipment and meteorological variabilities. *Austin* [1964], *Harrold et al.* [1973], and *Wilson and Brandes* [1979], among others, discuss the various causes of these errors. The errors exhibit both systematic and random behavior and quite often can exceed 100% on a relative scale. It is impossible to eliminate these errors directly by using rain gage data to calibrate the radar, because of the generally low density of rain gage networks and the different sampling characteristics of the two sensors. Rain gages measure point precipitation on the ground level, while radar-based precipitation represents a volumetric (or areal) average above the ground at a level depending on the distance from the radar site.

In this paper an optimal estimation approach to the problem of measuring precipitation using both radar and rain gage rainfall data will be described. This represents a philosophy similar to that of *Eddy* [1979] and *Crawford* [1979], who studied the problem of radar and rain gage data merging in a multivariate analysis framework. Here a well-known geostatistical interpolation technique called kriging is examined. The use of kriging for merging radar and rain gage data was also studied by *Lebel* [1986] and *Creutin and Delrieu* [1986]. In the study reported here, a numerical simulation experiment has been designed and carried out for the purpose of testing this technique.

The study was part of the design and implementation of a precipitation-processing system being developed for hydrologic use. The system is designed to be used with the NEXRAD (Next Generation Weather Radar) radar systems and will also include satellite data. The usage of satellite data is not addressed in this paper. Ultimately, the system will work in real time, providing hourly rainfall data for input into hydrologic models. Operational constraints dictated the choice of an ordinary cokriging algorithm instead of more sophisticated methods of universal cokriging [*Myers*, 1982] or disjunctive cokriging [*Yates*, 1986]. While these latter methods are perhaps more accurate and are theoretically justified for rainfall estimation, their computational requirements currently prohibit real-time applications. For more details on the future precipitation-processing systems of the National Weather Service, refer to papers by *Hudlow et al.* [1983, 1984] and *Ahnert et al.* [1983]. In the following sections a multisensor rainfall estimation problem will be formulated and a methodology to solve this problem described. Also, a test experiment design will be discussed, along with the results.

Formulation of the Problem

Let us assume that our precipitation measurements network covering space Ω consists of two sensors: a weather radar and a set of N rain gages. Let us further assume that the radar is equipped with a digital processor, which produces accumulated rainfall estimates on a rectangular grid over time period ΔT and space Ω. Similarly, rain gage data represent point measurements for the same time period ΔT. Both data sets are schematically depicted in Figure 1.

The motivation to use both data sets to estimate rainfall stems from the error characteristics of the two sensors. Rain gage data is typically considered to provide good point accuracy, but it offers little information on the spatial distribution of rain storms, especially in convective type situations. Radar, on the other hand, is capable of accurately delineating rainfall boundaries but, because of various meteorological, equipment, and methodological factors, its estimates of rainfall are burdened with errors that are very often quite significant. Thus it is

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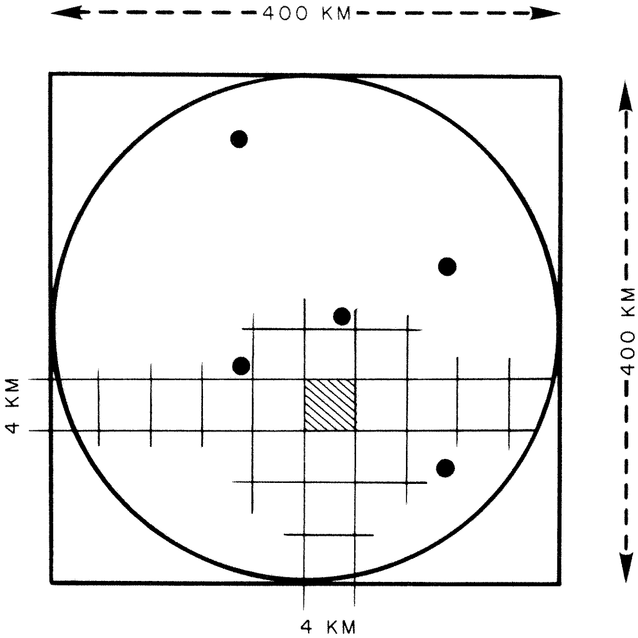

Fig. 1. Schematic representation of the radar (squares) and rain gage (dots) data in the domain Ω.

hoped that combining the data from these two sensors will result in rainfall estimates which will have both high spatial and point accuracy.

The previously mentioned facts were recognized a long time ago and contributed to slow progress in hydrologic applications of radar. Other methods of combining radar and rain gage data, such as those presented by *Brandes* [1975], *Smith and Cain* [1983], and *Hildebrand et al.* [1979] fail to account explicitly for the different sampling characteristics of the two sensors and the existence of various processes, such as evaporation and advection, taking place between the radar sampling volume and the ground. Papers by *Eddy* [1979] and *Crawford* [1979] essentially describe the same method, based on spatial correlation, which also does not account for the sampling differences. However, other differences between the two data sets due to the environmental processes of evaporation and advection are described statistically by modeling the spatial cross-correlation function. The technique described here is conceptually very similar, although its realization is quite different.

Mathematically, if we consider rainfall accumulated over the period ΔT as a two-dimensional random process $Z(\mathbf{u})$, $\mathbf{u} \in R^2$, our data are represented by (1) the radar data:

$$R_{ij} = \frac{1}{|A_R|} \int_{A_R} Z(\mathbf{u}_{ij})\, d\mathbf{u} + \varepsilon_{R_{ij}} \tag{1}$$

$$i = 1, 2, \cdots, N_x \qquad j = 1, 2, \cdots, N_y \qquad A_R \in \Omega$$

and (2) the rain gage data:

$$G_k = Z(\mathbf{u}_k) + \varepsilon_{G_k} \qquad k = 1, 2, \cdots, N \tag{2}$$

where A_R is the integration area of a single radar measurement, i, j are coordinates of the corresponding location, $\varepsilon_{R_{ij}}$ is the error associated with ijth radar observation, and ε_{G_k} is the error associated with kth gage observation. Notation $R_k(\mathbf{u}_k)$ will also be used for the radar data.

In (1) it is assumed that three-dimensional sampling volume of radar measurements is projected on two-dimensional space R^2. The two-dimensional sampling space of radar data, i.e., the grid boxes of Figure 1 will be called "radar bins."

The problem of rainfall estimation, using the two sensors, can then be formulated as follows: Find the best estimate $V^*(\mathbf{u}_0)$ of $V(\mathbf{u}_0)$, defined as

$$V(\mathbf{u}_0) = \frac{1}{|A|} \int_A Z(\mathbf{u}_0)\, d\mathbf{u} \qquad |A| \equiv |A_R| \tag{3}$$

Thus $V^*(\mathbf{u}_0)$ is an estimate of the mean areal precipitation on the ground level over the same area as sampled by radar.

Model Description

As a solution to the problem formulated above, a linear model is proposed:

$$V^*(\mathbf{u}_0) = \sum_{i=1}^{N_G} \lambda_{G_i} G_i(\mathbf{u}_i) + \sum_{i=1}^{N_R} \lambda_{R_i} R_i(\mathbf{u}_i) \tag{4}$$

where $N_G \le N$ is the number of gages in the local vicinity of location $\mathbf{u}_0$, N_R is the number of radar bins surrounding the location $\mathbf{u}_0$, and λ_{G_i} and λ_{R_i} are corresponding coefficients (weights) that need to be estimated.

It is assumed in the model that the rainfall field $Z(\mathbf{u})$ is second-order stationary and ergodic over the space Ω. It is also assumed that rain gage observation errors are random with zero mean, and variance $\sigma_{\varepsilon_G}{}^2$ and uncorrelated in space. The radar observation errors are random with mean m_{ε_R} and spatial covariance $\mathrm{cov}_{\varepsilon_R}(\mathbf{u})$. Both assumptions have bases in various experiments with real world data; however, the spatial error structure of radar is, to the best of the author's knowledge, unknown at this point.

The weights λ_{G_i} and λ_{R_i} can be obtained minimizing the estimation variance:

$$\begin{aligned}
\sigma_v{}^2 = E\{[V - V^*]^2\} = \frac{1}{|A|^2} \iint_A \mathrm{Cov}_V(\mathbf{u} - \mathbf{v})\, d\mathbf{u}\, d\mathbf{v} \\
-2 \sum_{i=1}^{N_G} \lambda_{G_i} \frac{1}{|A|} \int_A \mathrm{Cov}_{GV}(\mathbf{u} - \mathbf{u}_i)\, d\mathbf{u} \\
-2 \sum_{j=1}^{N_R} \lambda_{R_j} \mathrm{Cov}_{RV}(\mathbf{u}_0 - \mathbf{u}_j) \\
+ \sum_{i=1}^{N_G} \sum_{j=1}^{N_R} \lambda_{G_i} \lambda_{R_j} \mathrm{Cov}_{GR}(\mathbf{u}_i - \mathbf{u}_j) \\
+ \sum_{i=1}^{N_G} \sum_{j=1}^{N_G} \lambda_{G_i} \lambda_{G_j} \mathrm{Cov}_{GG}(\mathbf{u}_i - \mathbf{u}_j) \\
+2 \sum_{i=1}^{N_R} \sum_{j=1}^{N_R} \lambda_{R_i} \lambda_{R_j} \mathrm{Cov}_{RR}(\mathbf{u}_i - \mathbf{u}_j)
\end{aligned} \tag{5}$$

where $E\{\ \}$ is the expectation operator, $\mathbf{u}_0$ is the middle point in block of area A at the location of interest, and $\mathbf{u}$ and $\mathbf{v}$ are other points within the same block A. The covariance of the true area average process is denoted as $\mathrm{Cov}_V(\ )$; $\mathrm{Cov}_{GV}(\ )$ is an unknown covariance between the integrated process Z and the rain gage observations; $\mathrm{Cov}_{RV}(\ )$ is an unknown covariance between the integrated process Z and the radar observations; $\mathrm{Cov}_{GG}(\ )$ is the covariance of the rain gage data; $\mathrm{Cov}_{GR}(\ )$ is the cross covariance between gage and radar data; and $\mathrm{Cov}_{RR}(\ )$ is the covariance of the radar data.

For the estimate V^* to be unbiased, it has to satisfy

$$E\{V^*\} = E\{V\} \tag{6}$$

Under our assumptions about the stationarity and error structure, the following conditions apply:

$$\sum_{i=1}^{N_G} \lambda_{G_i} = 1 \tag{7}$$

$$\sum_{j=1}^{N_R} \lambda_{R_j} = 0 \tag{8}$$

It should be noted, however, that if $m_{\varepsilon_R} = 0$, i.e., the radar-rainfall field is unbiased, then (7) and (8) reduce to

$$\sum_{i=1}^{N_G} \lambda_{G_i} + \sum_{j=1}^{N_R} \lambda_{R_j} = 1 \tag{9}$$

The problem of σ_v^2 minimization under unbiased conditions can be solved using the Lagrange multiplier technique. Minimization of the Lagrangian function leads to a set of simultaneous linear equations that can be written in matrix form as:

$$\begin{vmatrix} \mathbf{Cov}_{RR} & \mathbf{Cov}_{RG} & 1 & 0 \\ \mathbf{Cov}_{GR} & \mathbf{Cov}_{GG} & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{vmatrix} \times \begin{vmatrix} \boldsymbol{\lambda}_R \\ \boldsymbol{\lambda}_G \\ \mu_R \\ \mu_G \end{vmatrix} = \begin{vmatrix} \mathrm{Cov}_{VG} \\ \mathrm{Cov}_{VR} \\ 0 \\ 1 \end{vmatrix} \tag{10}$$

where

$$\mathbf{Cov}_{RR} = \begin{vmatrix} \mathrm{Cov}_{RR}(\mathbf{u}_{R1}, \mathbf{u}_{R1}) & \cdots & \mathrm{Cov}_{RR}(\mathbf{u}_{RN_R}, \mathbf{u}_{R1}) \\ \vdots & & \vdots \\ \mathrm{Cov}_{RR}(\mathbf{u}_{RN_R}, \mathbf{u}_{R1}) & \cdots & \mathrm{Cov}_{RR}(\mathbf{u}_{RN_R}, \mathbf{u}_{RN_R}) \end{vmatrix}$$

$$\mathbf{Cov}_{RG} = \begin{vmatrix} \mathrm{Cov}_{RG}(\mathbf{u}_{R1}, \mathbf{u}_{G1}) & \cdots & \mathrm{Cov}_{RG}(\mathbf{u}_{RN_R}, \mathbf{u}_{G1}) \\ \vdots & & \vdots \\ \mathrm{Cov}_{RG}(\mathbf{u}_{RN_R}, \mathbf{u}_{G1}) & \cdots & \mathrm{Cov}_{RG}(\mathbf{u}_{RN_R}, \mathbf{u}_{GN_G}) \end{vmatrix}$$

$$\mathbf{Cov}_{GG} = \begin{vmatrix} \mathrm{Cov}_{GG}(\mathbf{u}_{G1}, \mathbf{u}_{G1}) & \cdots & \mathrm{Cov}_{GG}(\mathbf{u}_{GN_G}, \mathbf{u}_{G1}) \\ \vdots & & \vdots \\ \mathrm{Cov}_{GG}(\mathbf{u}_{GN_G}, \mathbf{u}_{G1}) & \cdots & \mathrm{Cov}_{GG}(\mathbf{u}_{GN_G}, \mathbf{u}_{GN_G}) \end{vmatrix}$$

$$\mathbf{Cov}_{GR} = [\mathbf{Cov}_{RG}]^T$$

$$\mathbf{Cov}_{VR} = \langle \mathrm{Cov}_{VR}(\mathbf{u}_0, \mathbf{u}_{R1}), \cdots, \mathrm{Cov}_{VR}(\mathbf{u}_0, \mathbf{u}_{RN_R}) \rangle^T$$

$$\mathbf{Cov}_{VG} = \left\langle \frac{1}{|A|} \int_A \mathrm{Cov}_{GG}(\mathbf{u}, \mathbf{u}_{G1})\, d\mathbf{u}, \cdots, \frac{1}{|A|} \int_A \mathrm{Cov}_{GG}(\mathbf{u}, \mathbf{u}_{GN_G})\, d\mathbf{u} \right\rangle^T$$

$$\boldsymbol{\lambda}_R = \langle \lambda_{R_1}, \cdots, \lambda_{RN_R} \rangle^T$$

$$\boldsymbol{\lambda}_G = \langle \lambda_{G_1}, \cdots, \lambda_{GN_G} \rangle^T$$

and μ_G and μ_R are scalar Lagrangian multipliers. The superscript T denotes matrix or vector transpose operator.

The system (equation (10)) yields a unique solution if the covariance matrix is positive definite. Because of the irregular pattern of the network of rain gages the system needs to be solved for each location, and it is necessary to approximate the matrices $\mathbf{Cov}_{GG}$ and $\mathbf{Cov}_{RG}$ with a function model. Although the relative configuration of radar data does not change from location to location and the number of data values used in the $\mathbf{Cov}_{RR}$ computation is typically large, the matrix $\mathbf{Cov}_{RR}$ also needs to be approximated to ensure positive definiteness.

To avoid solving the system (Equation (10)) at each location in Ω, a modified algorithm is proposed. First, the rain gage data are interpolated onto the same grid blocks as those for which the radar data are given. This is done using block kriging estimation [see *Journel and Huijbregts*, 1978]. Then the above described algorithm is used to cokrige the two fields: the radar data field and the field obtained by block kriging the rain gage data. Now the relative geometry in either field and between the two fields is constant throughout Ω (except on the edges), the system (10) needs to be solved only once, and the same weights can be applied at each location in the field. The matrices $\mathbf{Cov}_{GG}$, $\mathbf{Cov}_{RR}$, and $\mathbf{Cov}_{RG}$ are approximated by exponential models.

The computational algorithm can be summarized as follows:

1. Estimate the rain gage data covariance function, using the exponential isotropic model. This is done by a least squares fit.
2. Block krige the rain gage data to estimate

$$V_G^*(i, j) = \frac{1}{|A_R|} \int Z(\mathbf{u}_{ij})\, d\mathbf{u} \qquad \mathbf{u}_{ij} \in \Omega$$

3. Estimate $\mathbf{Cov}_{GG}$, $\mathbf{Cov}_{RR}$, and $\mathbf{Cov}_{RG}$ from the radar and the new kriged rain gage fields. Now the elements of $\mathbf{Cov}_{RG}$ represent estimates of the cross covariance between two areal average observation fields. The estimation of $\mathbf{Cov}_{VR}$ and $\mathbf{Cov}_{VG}$ will be explained later.
4. Model $\mathbf{Cov}_{GG}$, $\mathbf{Cov}_{RR}$, and $\mathbf{Cov}_{RG}$, using exponential isotropic models. This is done by least squares fit, also.
5. Compute $\boldsymbol{\lambda}_R$ and $\boldsymbol{\lambda}_G$.
6. Cokrige the two fields.

The choice of exponential model was made for two main reasons: first, the behavior of the model near the origin, which seems appropriate for rainfall estimation [*Rodriguez-Iturbe and Mejia*, 1974*b*], and second, its computational efficiency. Computational efficiency was also the main reason for using least squares as the method of covariance estimation. The proposed algorithm is much faster than using direct rain gage observations and varying network configuration from location to location, and at the same time it results in minimal degradation of accuracy.

A few additional remarks are in order. First is the problem of a changing pattern along the edges of the domain Ω. In the work presented here, no special accommodation has been made for this problem. The same weights are applied along the edges as in the middle of Ω, but the bins located outside of Ω are treated as missing values. Such a procedure results in local bias, but it did not affect the results, since they were based only on the points located inside Ω, separated by a few bins from the edges. The second important problem is that of the small sample size of the rain gage data. In a real-world application of the method presented here, where there is not enough data to compute reliable covariance, computed covariances can be substituted with climatological covariances computed from historical data. However, for reasons explained in the next section, this problem is not of concern in this paper.

To solve the system (10), one needs to estimate the vectors $\mathbf{Cov}_{VR}$ and $\mathbf{Cov}_{VG}$. Elements of these vectors are covariances between radar and rain gage data, respectively, and the true precipitation V. Since V is unknown, $\mathbf{Cov}_{VR}$ and $\mathbf{Cov}_{VG}$ have to be approximated. The approximation used has the form:

$$\mathbf{Cov}_{VR} = \beta_R\, \mathbf{Cov}_{RR} \qquad \beta_R \in (0, 1) \tag{11}$$

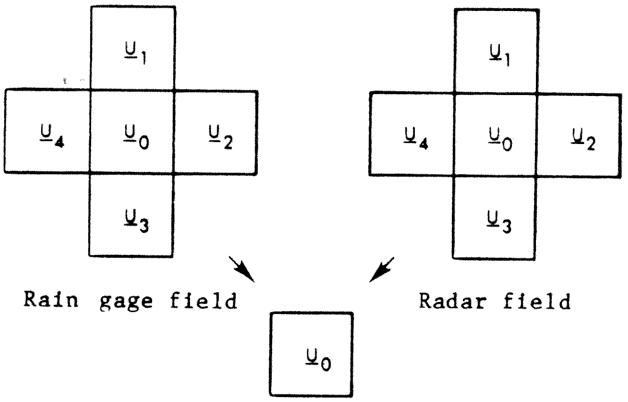

Fig. 2. Schematic representation of the configuration of the data included in the cokriging system from each field.

and

$$\mathbf{Cov}_{VG} = \beta_G \, \mathbf{Cov}_{GG} \qquad \beta_G \in (0, 1) \tag{12}$$

where the elements of $\mathbf{Cov}_{GG}$ have the meaning described in point 3 of the algorithm. The values of β_R and β_G are unknown scalars and, in general, are extremely difficult (if not impossible) to estimate from the data. However, since they represent a measure of the accuracy of radar and gage measurement in relation to V, they can be estimated based on experience or specially designed experiments. The sensitivity analysis of the previously presented method with respect to these two parameters will follow, and some recommended values will be given.

It should be pointed out that $\mathbf{Cov}_{VR}$ and $\mathbf{Cov}_{VG}$ can be expressed in terms of data error characteristics. However, because of the generally unknown statistical error structure of radar data, it was decided to investigate the previously described approach first.

Before the weights $\boldsymbol{\lambda}_R$ and $\boldsymbol{\lambda}_G$ are computed, one needs to decide how many data points should be used for estimation at each location. The configuration of the data used in the present study was constant and is schematically presented in Figure 2. The relatively small number of data points (five from each field) was dictated again by the computational efficiency.

The presented algorithm accounts for the sampling differences of the two sensors. It also uses the spatial cross-correlation function to account for differences between the error fields involved.

Once the coefficients λ_R and λ_G are determined, one can calculate the estimation variance as

$$\hat{\sigma}_v^{\,2} = \mathrm{Cov}_{GG}(\mathbf{u}_0, \mathbf{u}_0) - \mu_G - \sum_{i=1}^{N_R} \lambda_{R_i} \mathrm{Cov}_{VR}(\mathbf{u}_0, u_i) - \sum_{i=1}^{N_G} \lambda_{G_i} \mathrm{Cov}_{VG}(\mathbf{u}_0, \mathbf{u}_i) \tag{13}$$

In this expression Cov_{GG} was substituted for Cov_V.

Test of the Method

Probably the most natural way to test any method of estimating mean areal precipitation would be to compare the results with data from a very dense network which would constitute a "ground truth." In the present case, however, such an approach seems to be infeasible. First, there are not many (if any) rain gage networks with high enough density and large enough coverage. Second, it may be impossible to find corresponding radar data, since the systematic archiving of the high-resolution RADAP II data by the National Weather Service (NWS) started in 1985. Also, a significant data management effort is required to handle enough data to give the experiment statistically meaningful results.

In order to avoid these problems, a numerical experiment has been designed following the ideas of *Greene et al.* [1980]. In the experiment the sampling and measurement properties of radar and rain gages are mathematically simulated by generating radar and rain gage data from an original rainfall field which constitutes the "ground truth." Such an experiment has many advantages: (1) full control of the experiment with minimum effort; (2) knowledge of the true field; (3) control of the measurement errors; (4) control of the measurement network configuration (sampling scheme, network density); (5) feasibility of performing sensitivity analysis with respect to measurement errors and network density; (6) statistically valid conclusions.

The experimental system which is schematically depicted in Figure 3 consists of four elements. These are described in the following sections.

Original Field

The original field could be generated by a space-time rainfall model, such as developed by *Waymire et al.* [1984], or it could be a high-quality radar field. The latter was selected, and the original fields are the radar-rainfall fields from the GARP Atlantic Tropical Experiment (GATE) conducted in 1974. The GATE data represent convective systems and there-

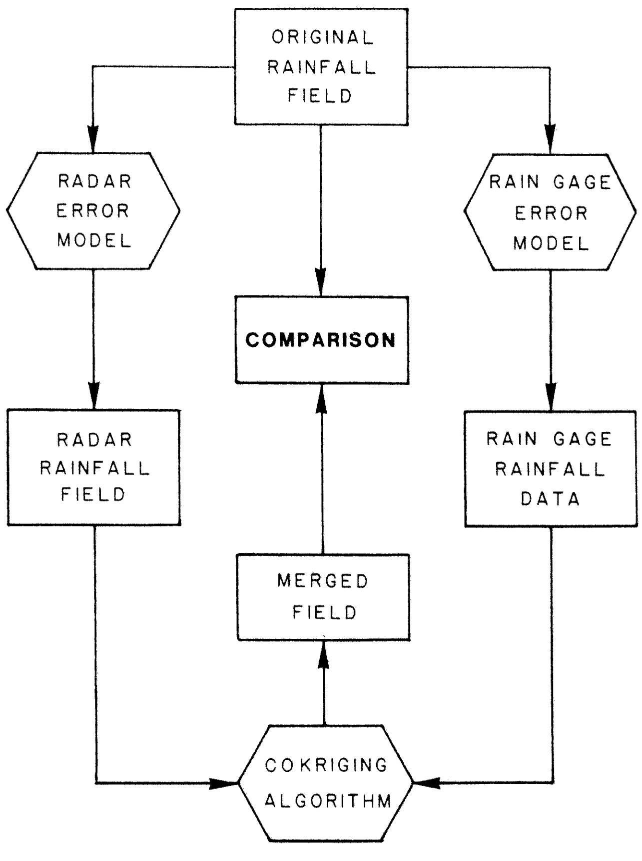

Fig. 3. Schematic of the numerical simulation experiment used to test the cokriging algorithm.

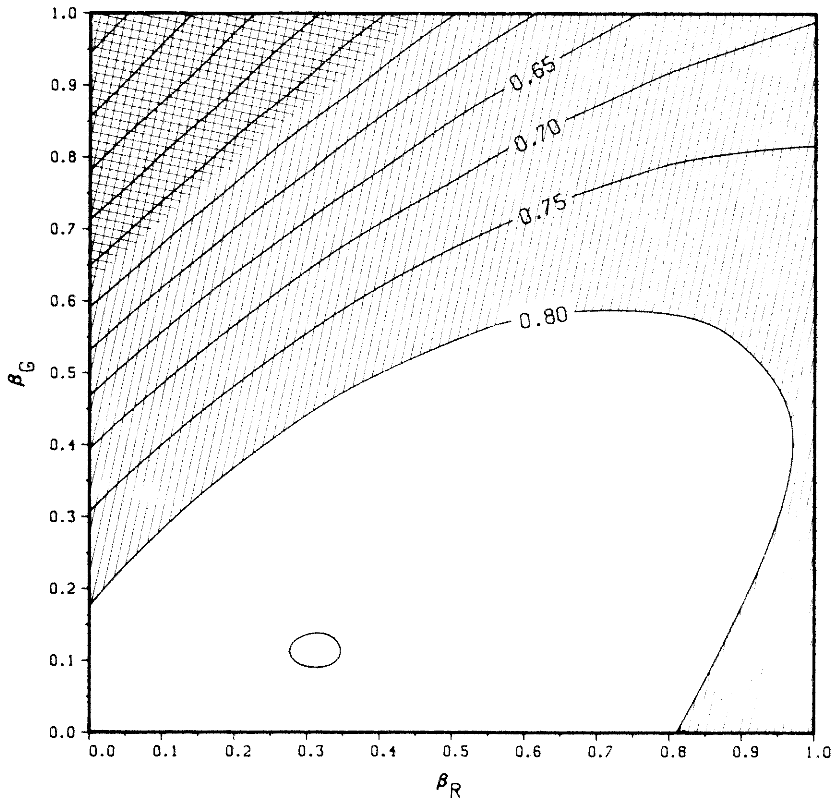

Fig. 4. Contours of the correlation coefficient. Parameters are $L_R^* = 0.03$, $\rho_\varepsilon = 8$ km, no bias, and 50 gages.

fore provide a stringent test for the present estimation procedure. The data are described by *Hudlow and Patterson* [1979]. The GATE data underwent substantial analysis prior to their release. They are considered to be of high quality in that anomalous propagation and other outliers are removed, and all the major features of real storm events are preserved. The GATE radar data defines the Ω region as being a circle inscribed in a 400 × 400 km square. Data points are given on a rectangular 4 × 4 km grid.

Radar Generator

The radar generator used in this study is described by *Krajewski and Georgakakos* [1985]. The radar field is generated as

$$R(i, j) = \theta(i, j)10^{\varepsilon(i,j)S(i,j)} \tag{14}$$

where $R(i, j)$ is the radar field at the location (i, j); $\theta(i, j)$ is the original field at the same location; $\varepsilon(i, j)$ is the random component of the noise field and is a stationary, isotropic random field, with the mean μ_ε, variance σ_ε^2, and spatial correlation function $\rho(\tau)$; and $S(i, j)$ is the deterministic component of the noise field.

The deterministic component $S(i, j)$ accounts for the measurement behavior of the radar as a sensor. It usually exhibits higher errors in high rainfall intensity and high gradient areas. The form of the function $S(i, j)$ was borrowed from *Greene et al.* [1980]:

$$S(i, j) = 0.5\{\langle|\nabla\theta(i, j)|\rangle[|\nabla\theta(i, j)|_m]^{-1} + \theta(i, j)\theta_m(i, j)^{-1}\} \tag{15}$$

where $\langle|\nabla\theta(i, j)|\rangle$ is the average absolute value of the gradient computed in four directions around the point (i, j) in the original field; $|\nabla\theta(i, j)|_m$ is the maximum absolute gradient in the original field; and $\theta_m(i, j)$ is the maximum value in the original field. The parameters of the random component ε are, in general, unknown but can be estimated for the purpose of the generation by setting requirements on the resultant radar field R. In the present study these requirements were (1) The mean of the radar field is required to be M_R^*. Thus

$$M_R^* = \frac{1}{|\Omega|}\int_\Omega E\{R\}\, d\Omega \tag{16}$$

(2) The logarithmic variance of the ratio R/θ is required to be L_R^*:

$$L_R^* = \frac{1}{|\Omega|}\int_\Omega E\{\log_{10}{}^2(R/\theta)\}\, d\Omega - \frac{1}{|\Omega|}\int_\Omega E^2\{\log_{10}(R/\theta)\}\, d\Omega \tag{17}$$

Solving the system of (16) and (17), one can obtain μ_ε and σ_ε^2. The correlation function $\rho(\tau)$ of the ε is assumed to be isotropic and exponential. Its parameter was specified directly in the present experiment and was a subject of the sensitivity analysis. Once the parameters of ε are specified, ε is generated using, for example, the Turning Bands method [*Montoglou and Wilson*, 1982]. After ε is generated and the values of the function S are computed for each location in Ω, the radar field can be generated using (14).

Rain Gage Data Generator

Rain gage data are generated in two steps. First, the locations of the gages are selected based on a uniform random distribution in Ω. Second, a rainfall value is assigned to each location, based on the original field values. The original field of choice (GATE radar data) represents areal averages, but we want to generate the corresponding point process values. To do that, the relationship between the variances of the point

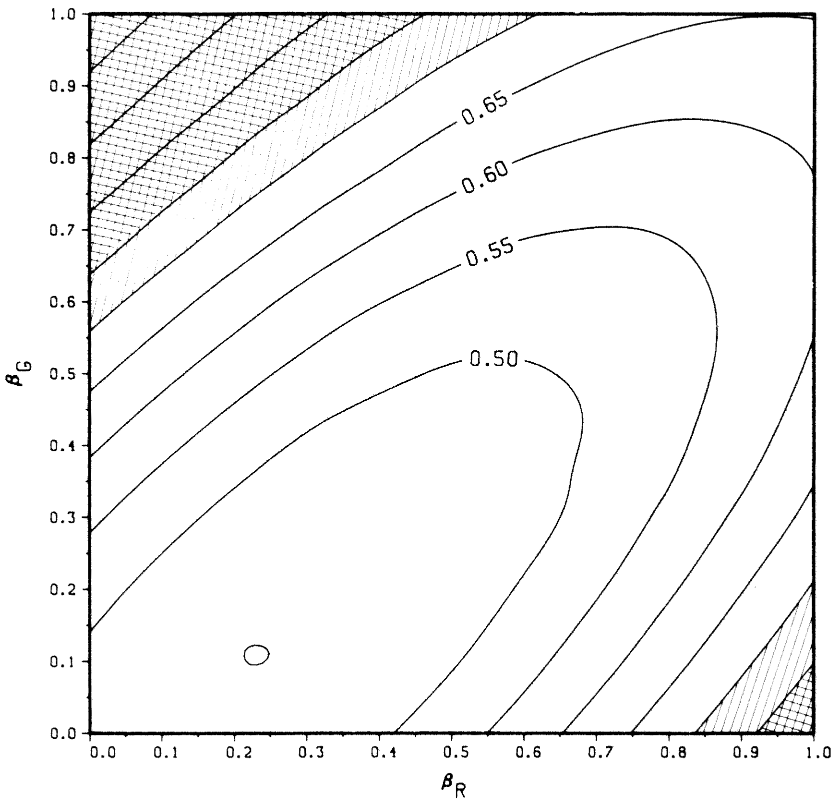

Fig. 5. Contours of estimation variance. Parameters are $L_R{}^* = 0.03$, $\rho_\varepsilon = 8$ km, no bias, and 50 gages.

process and the areal average process given by *Rodriguez-Iturbe and Mejia* [1974*a*] was used:

$$\sigma_p{}^2 = \sigma_A{}^2\left[\int_0^d r(v)G(v)\,dv\right]^{-1} \tag{18}$$

where $\sigma_p{}^2$ and $\sigma_A{}^2$ are the point and areal average process variances, respectively; v is a distance between two points in the region A; $r(\ )$ is the correlation function of the point process; $G(\ )$ is the probability density function for random distribution between two points in the averaging area (a square in the present case); and d is the largest distance in that area. The distribution $G(\ )$ for a rectangular area is given by *Gosh* [1951].

It was assumed that the rainfall data are lognormally distributed; consequently, the gage values $G_k(i, j)$ are generated as

$$G_k(i, j) = LN\{\theta(i, j), \sigma_p{}^2\} + LN\{0, 0.0001m^2\theta^2(i, j)\} \tag{19}$$

$$k = 1, \cdots, N$$

where m is measurement error expressed as a percentage of the mean, and $LN\{a, b\}$ denotes lognormal distribution, with the mean a and variance b.

The correlation function $r(\ )$ was assumed to be exponential and was estimated for the local vicinity of $G_k(i, j)$. The correlation parameter h was obtained from the equation

$$r_A(\tau_1) = \iint_A r(\mathbf{u}, h)\,d\mathbf{u} \tag{20}$$

where $r_A(\tau_1)$ is the lag one correlation of the areal average process, as computed from the data.

Performance Criteria

The evaluation of the cokriging algorithm to merge radar and rain gage rainfall data is based on the comparison of the cokriged field and the original field. The comparison can be done by visual inspection of the resultant maps or by a set of statistics. The latter approach is more appropriate if the objective of the experiment is sensitivity analysis. The statistics selected for comparison include the mean and variance of the fields, the correlation coefficient with the original field, the estimation variance (both computed and estimated), the mean square error for averaging areas ranging from 16 km^2 to 1000 km^2, and the maximum mean square error in the field for the same areas. Inspection and proper interpretation of all of these statistics allows us to evaluate the proposed methodology in a fair way. For the sake of clarity, the expressions used to compute the mean square error and estimation variance are given:

Mean square error

$$\frac{1}{N_A}\sum_{i=1}^{N_A}(\theta_i - M_i)^2 \tag{21}$$

Estimation variance

$$\frac{1}{N_A}\sum(\theta_i - M_i)^2 - \left[\frac{1}{N_A}\sum(\theta_i - M_i)\right]^2 \tag{22}$$

where N_A is the number of points used in comparison and θ_i and M_i are the original and merged field values, respectively.

It should be realized that the general validity of such a numerical experiment is limited by the validity of the error models of radar and rain gage data. There have been several

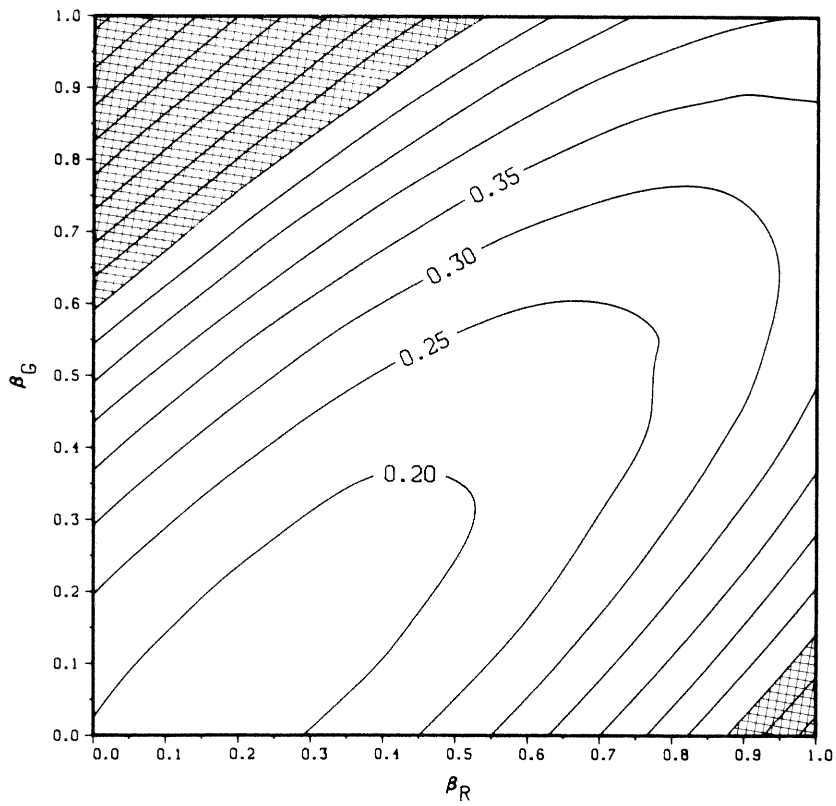

Fig. 6. Contours of the mean square error. Parameters are $L_R{}^* = 0.03$, $\rho_\varepsilon = 8$ km, no bias, and 50 gages.

studies done on the error characteristics of rain gage data (see, for instance, *Larson and Peck* [1974] and *Sevruk* [1982]). In the light of these studies the proposed model seems to be adequate. The situation is quite different as far as radar data are concerned. There were many studies done in the past on the comparison of radar and rain gage data, but the question of what the statistical structure of radar rainfall errors is in space remains unanswered. Such a question, however, is critical in the design of an experiment like the one described here. The radar error model presented has certain qualitative features identified by previous studies:

1. The model delineates rainfall patterns correctly, i.e., anomalous propagation (AP) is not modeled in clear air. It was assumed that in the operational environment the radar data would undergo quality control steps that would, perhaps with the aid of satellite data, eliminate AP.
2. Errors are higher in high-rainfall gradient areas.
3. Errors are higher in high-rainfall intensity areas.
4. Errors are correlated in space.

Among the effects that are not modeled are (1) Range effect due to attenuation of electromagnetic wave (it was assumed that for the radars with parameters corresponding to those planned for the NEXRAD, this effect is negligible [*Hudlow et al.*, 1984]); (2) Beam-filling effect (however, the effect of this problem also will be significantly reduced using NEXRAD equipment and processing); and (3) Complete evaporation of rain before hitting the ground (thus application of such an error model in some situations may be inappropriate). Summarizing, one could say that if the error models are valid, then the results of this study are valid also.

RESULTS OF THE SENSITIVITY ANALYSIS

The sensitivity analysis of the merging method described previously was performed using daily data from the GATE experiment. It is very expensive in terms of computer central processing unit (CPU) time to perform a truly comprehensive experiment. It was estimated that such an experiment would take over 5 years of CPU time on the PR1ME 750 computer. Therefore a limited experiment was performed instead. The optimal combination of the parameters β_G and β_R was sought as a function of various noise parameters and rain gage densities. The noise parameters selected here for investigation were the bias of the radar field, the variance $L_R{}^*$ (see equation (17)), and the correlation distance h_ε in the ε field. The effect of measurement error in the gage observations was not studied. An error of 10% was specified for all the runs.

Since the framework of the methodology described here is the estimation of random fields, the proper way of conducting this experiment would be to repeat the analysis for a number of realizations (at least 30) for each field, keeping the same noise parameters, and then to average the results across the ensemble of realizations. Such a procedure, however, is very costly and prohibits even a limited experiment. To evaluate the variability of the results across realizations, a few (five) realizations were used for a selected set of radar noise parameters and a network of 50 gages. The data for GATE day 245 (September 2, 1974) were used. The radar noise parameters were (1) no bias in the radar field, (2) high noise ($L_R{}^* = 0.03$, which is in the range given by *Greene et al.* [1980] and was also found to generate outliers [*Krajewski and Georgakakos*, 1985]), and (3) correlation distances in the ε field of 8 and 20

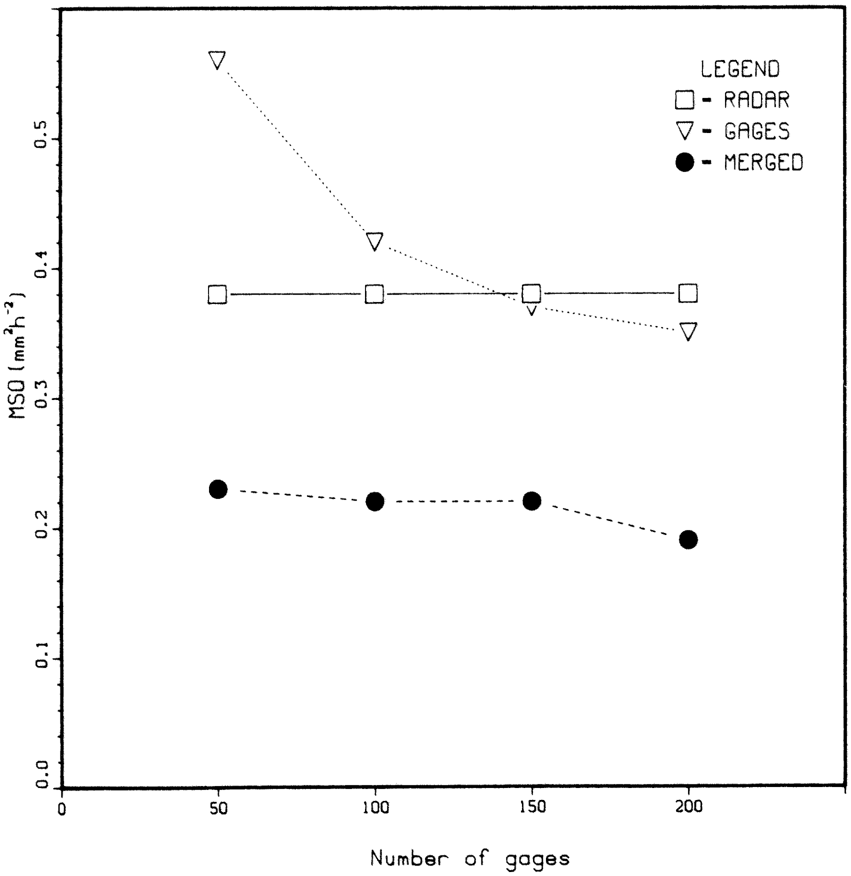

Fig. 7. Mean square errors as a function of rain gage network density. Parameters are $L_R{}^* = 0.03$, $\rho_\varepsilon = 20$ km, no bias, $\beta_R = 0.45$, and $\beta_G = 0.35$.

km. It was found that the variability of the location of the optimal set of β_R and β_G was negligible compared to the effects of other noise parameters, and therefore only one realization was used for all other runs.

Two basic situations were investigated: the unbiased radar field case and the biased radar field case. It is important to distinguish between these two cases, since in the operational environment of NWS, where the previously described method is to be implemented [*Ahnert et al.*, 1983; *Hudlow et al.*, 1983], there will be a bias removal procedure, based on the Kalman filter concept [*Ahnert et al.*, 1986]. The procedure will attempt to remove overall bias often present in the radar data, based on limited number of rain gage observations. If such a procedure does not precede the merging step, or does not work properly because of a lack of adequate information, the bias has to be removed by the cokriging algorithm.

First, let us consider the no-bias case. Two levels of $L_R{}^*$ were considered: $L_R{}^* = 0.03$, a rather high noise, and $L_R{}^* = 0.01$, a medium-to-low noise. The correlation distance ρ_ε of the ε field was 8, 20, and 40 km. The number of rain gages ranged from 50 to 200. The 50-gage case corresponds to approximately 1 gage per 2500 km^2 and the 200-gage case to 1 gage per 600 km^2. In order to study the selected statistics in the β_R, β_G parameter space, 100 runs were made for each noise parameter combination. The statistics are correlation coefficient with the original field, the variance of residuals, and the mean square error for various averaging areas. Figures (4–6) are examples of the plots of the correlation coefficient, the variance of residuals, and the mean square error surfaces, respectively. The shaded area corresponds to a region where radar alone did better than the merging procedure. For case presented here, rain gage analysis based on kriging 50 gages was worse than the analysis based on radar data. The cross-hatched areas correspond to those combinations of β_R and β_G which result in a worse performance than the gage data analysis. The shape of the surfaces is very regular with a flat optimum vicinity, which means that very precise location of the optimum combination of β_G and β_R is not needed. Also, note that the optima for all three criteria have approximately the same location. This is probably due to the quadratic character of all the selected statistics. The method seems to be more sensitive to proper specification of β_G than β_R. The location of optimum moves to higher values of β_R with noise $L_R{}^*$ decreased and moves to higher values of β_G with increased density of the rain gage network. This is obviously an expected behavior. Figure 7 shows the mean square error plot as a function of the number of rain gages for $L_R{}^* = 0.03$ and $\rho_\varepsilon = 20$ km.

In the case of the biased radar field one would like to distinguish between overestimation and underestimation of the rainfall field by radar. An underestimated radar-rainfall field was generated by requiring the mean of the radar field to be half of the original field mean. It was found that in such a case, for all the combinations of other parameters specified, the model was practically insensitive to the choice of β_R and β_G (in terms of selected criteria). The bias was effectively removed for any number of rain gages in the range 50–200, but the improvement offered by the merging procedure over rain

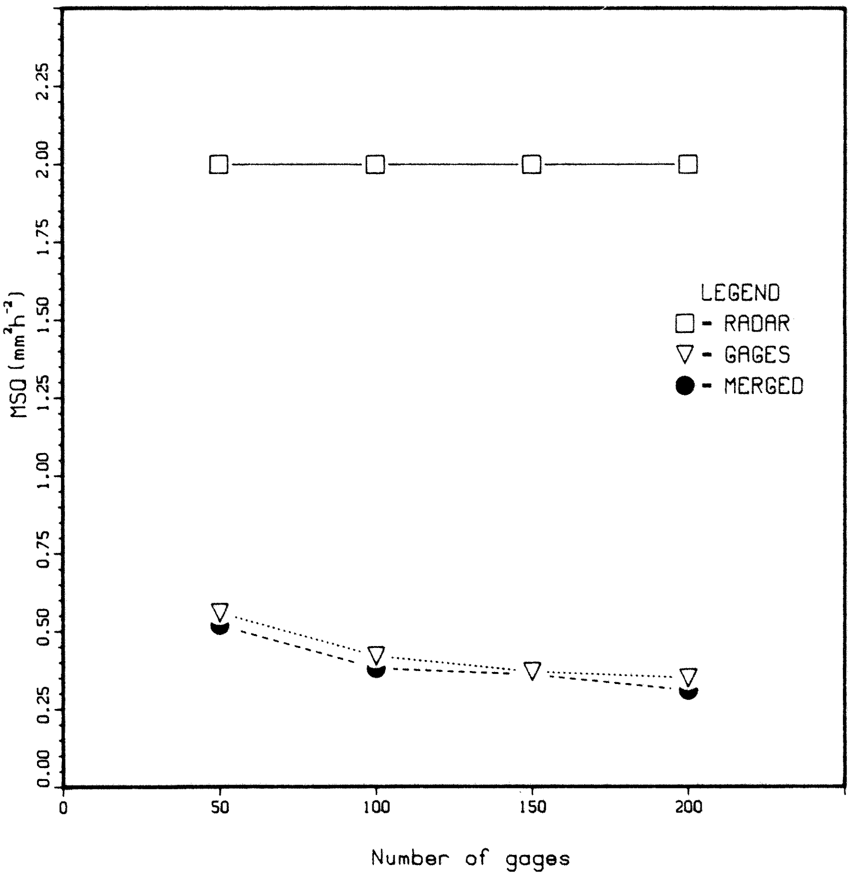

Fig. 8. Mean square errors as a function of rain gage network density. Parameters are $L_R^* = 0.03$, $\rho_\varepsilon = 20$ km, bias = 1.5, $\beta_R = 0.15$, and $\beta_G = 0.35$.

gage data only was marginal ($\leq 5\%$). Whether this is a consistent result remains to be seen until more data fields are investigated.

The second situation, with bias present, was the case of the overestimated field, i.e., the radar field mean was generated as 150% of the mean of the original field. Again, the bias was effectively removed even by 50 gages (error in the mean was less than 5%). This time, however, there was a well-defined optimum in the β_R, β_G parameter space corresponding to approximately $\beta_R = 0.2$ and $\beta_G = 0.3$. The model is more sensitive to the β_R parameter than to β_G. It is interesting that although even 50 gages merged with radar improved the mean square error (see Figure 8) and the estimation variance, 100 gages were required to produce a merged field with a correlation coefficient statistic equal to that of the radar field, and 150 gages were required to produce a merged field with a slightly improved correlation coefficient. The effect of the correlation distance of the radar error field was found to be minimal in the investigated range of 8–20 km. Less-correlated noise leads to some ($\leq 3\%$) degradation of performance statistics and slight increase of β_R values.

Two general results were evident. First, the maximum square error in the radar field (which was in all cases the more noisy field), was always reduced to a level limited by the accuracy of the rain gage data. Second, estimates for larger areas showed reduced average error characteristics. This reduction of errors was again limited by the accuracy of the rain gage field.

It should also be pointed out that although the noise (L_R^*) was set to the same value of 0.03 for both bias and no-bias cases, the "amount" of random noise was really higher in the no-bias case (in which all the noise expressed in terms of error characteristics can be attributed to L_R^*), while in the bias cases (especially in the case of overestimation) the bias is responsible for high mean square values, but the correlation of the radar and original fields is still good. Thus the improvement of the mean square error is the most that should be expected from the merging procedure.

CONCLUSIONS

A method for merging radar-rainfall and rain gage data was presented. The method was tested via a numerical experiment, with error fields of both sensors being modeled. The results presented represent a preliminary testing phase and are limited to daily rainfall data. Ongoing testing of the described technique, prior to its operational implementation by the National Weather Service, proceeds along two paths. One is based on the methodology described here and will be followed by similar analyses for more daily fields and also for 6-hourly and 1-hourly data fields. The second (semioperational test) is based on real-time data from the Oklahoma City radar and Tulsa River Forecast Center rain gage data. The comparison is based on hydrographs resulting from mean areal precipitation estimated from rain gages only (as is currently being done in the operational environment) and via the merging procedure for selected basins. For more details on this approach and some preliminary results, see *Krajewski and Ahnert* [1986]. As the results presented in this paper suggest, the best configuration of a precipitation-processing system, using data from radar and a network of rain gages, includes a separate bias removal procedure, so that an unbiased radar-rainfall field enters the merging step. Then, if the bias is effectively removed and the noise in the radar field is low, the merging will not substantially alter the rainfall field. If, however, the noise is high, then a substantial improvement can be expected.

As far as the specification of the values of the parameters β_R and β_G is concerned, the preliminary results show that the robust region is somewhere in the range of 0.2–0.4 for both parameters. The robustness of these results needs to be further investigated. Also, for data collected at other than daily intervals (hourly, 6-hourly, etc.), these results may not be valid. It is expected that the performance of the presented method, relative to rain gage data analysis, should be better for hourly data.

The described approach, since geared toward an operational environment with its computational time constrains, presents some compromises between mathematical and physical appropriateness and practical efficiency. These are manifested by using ordinary kriging versus the way of the previously mentioned methods of universal and disjunctive kriging, parameter estimation, and also, now from the physical point of view, a lack of accounting for temporal correlation of the rainfall process. The practical consequences of these compromises are being investigated and will be reported.

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REFERENCES

Ahnert, P. R., M. D. Hudlow, E. R. Johnson, D. R. Greene, and M. R. Dias, Proposed "on-site" precipitation processing system for NEXRAD, *Preprints of the 21st Conference on Radar Meteorology*, pp. 378–385, American Meteorological Society, Boston Mass., 1983.

Ahnert, P. R., W. F. Krajewski, and E. R. Johnson, Kalman filter estimation of radar-rainfall field bias, in *Preprints of the 23rd Conference on Radar Meteorology Conference*, vol. 3, pp. JP33–JP37, American Meteorological Society, Boston, Mass., 1986.

Anderl, B., W. Attmannspacher, and G. A. Schultz, Accuracy of reservoir inflow forecasts based on radar rainfall measurements. *Water Resourc. Res.*, *12*(2), 217–223, 1976.

Austin, G. L., and L. B. Austin, The use of radar in urban hydrology, *J. Hydrol.*, *22*, 131–142, 1974.

Austin, P. M., Radar measurements of precipitation rate, in *Proceedings of the 11th Weather Radar Conference*, pp. 120–125, American Meteorological Society, Boston, Mass., 1964.

Batton, L. T., *Radar Observations of the Atmosphere*, University of Chicago Press, 1973.

Brandes, E. A., Optimizing rainfall estimates with the aid of radar, *J. Appl. Meteorol.*, *14*, 1339–1345, 1975.

Crawford, K. C., Considerations for the design of a hydrologic data network using multivariate sensors, *Water Resourc. Res.*, *15*(6), 1752–1762, 1979.

Creutin, J. D., and J. Delrieu, Remote sensed and ground measurement combination for rainfall field estimation, paper presented at the Chapman Conference on Modeling of Rainfall Fields, AGU, Caracas, Venezuela, 1986.

Eddy, A., Objective analysis of convective scale rainfall using gages and radar, *J. Hydrol.*, *44*, 125–134, 1979.

Gosh, B., Random distances within a rectangle and between two rectangles, *Bull. Calcutta Math. Soc.*, *43*, 17–24, 1951.

Greene, D. R., M. D. Hudlow, and E. R. Johnson, A test of some objective analysis procedures for merging radar and raingage data, in *Preprints of the 19th Conference on Radar Meteorology*, pp. 470–479, American Meteorological Society, Boston, Mass, 1980.

Harrold, T. W., E. J. English, and C. A. Nicholas, The Dee weather radar project: The measurements of area precipitation using radar, *Weather*, *28*, 332–338, 1973.

Hildebrand, P. H., N. Towery, and M. R. Snell, Measurement of convective mean rainfall over small areas using high-density raingages and radar, *J. Appl. Meteorol.*, *18*, 1316–1326, 1979.

Hudlow, M. D., Use of digital radar data in operational hydrology, paper presented at National Water Resources Engineering Meeting, Am. Soc. of Civil Eng., Washington, D. C., 1973.

Hudlow, M. D., and V. L. Patterson, GATE radar rainfall atlas, *NOAA Spec. Rep.*, Natl. Oceanic and Atmos. Admin., Washington, D. C., 1979.

Hudlow, M. D., D. R. Greene, P. R. Ahnert, W. F. Krajewski, T. R. Sivaramakrishnan, E. R. Johnson, and M. R. Dias, Proposed "off-site" precipitation processing system for NEXRAD, in *Preprints of the 21st Conference on Radar Meteorology*, pp. 394–403, American Meteorological Society, Boston, Mass., 1983.

Hudlow, M. D., R. K. Farnsworth, and P. R. Ahnert, NEXRAD technical requirements for precipitation estimation and accompanying economic benefits, *Hydro Tech. Note 4*, Off. of Hydrol., Natl. Weather Serv., NOAA, Silver Spring, Md., 1984.

Journel, A. G., and A. T. Huijbregts, *Mining Geostatistics*, Academic, Orlando, Fla., 1978.

Kessler, E., and K. E. Wilk, Radar measurements of precipitation for hydrological purposes, *WMO/IHD Rep. 5*, Secr. of the World Meteorol. Organ., Geneva, 1968.

Krajewski, W. F., and P. R. Ahnert, Near real-time tests of a multivariate analysis system, in *Preprints of the 23rd Conference on Radar Meteorology*, vol. 3, pp. JP66–JP69, American Meteorological Society, Boston, Mass., 1986.

Krajewski, W. F., and K. P. Georgakakos, Synthesis of radar rainfall data, *Water Resour. Res.*, *21*(5), 764–768, 1985.

Larson, L. W., and E. L. Peck, Accuracy of precipitation measurements for hydrologic modeling, *Water Resourc. Res.*, *10*(4), 857–863, 1974.

Lebel, T. F., A preliminary study of rainfall estimation from PROFS radar and raingage data, internal report, Environ. Res. Lab., Natl. Oceanic and Atmos. Admin., Boulder, Colo., 1986.

Mantoglou, A., and T. L. Wilson, The turning bands method for simulation of random fields using line generation by a spectral method, *Water Resourc. Res.*, *18*(5), 1379–1394, 1982.

Myers, D. E., Matrix formulation of co-kriging, *Math. Geol.*, *14*, 249–257, 1982.

Rodriguez-Iturbe, I., and T. J. Mejia, On the transformation of point rainfall to areal rainfall, *Water Resourc. Res.*, *10*(4), 729–735, 1974*a*.

Rodriguez-Iturbe, I., and T. J. Mejia, The design of rainfall networks in time and space, *Water Resourc. Res.*, *10*(4), 713–728, 1974*b*.

Sevruk, B., Methods of correction for systematic error in point precipitation measurement for operational use, *Oper. Hydrol. Rep. 21*, Secr. of the World Meteorol. Organ., Geneva, 1982.

Waymire, E., V. K. Gupta, and I. Rodriguez-Iturbe, A spectral theory of rainfall intensity at the meso-β scale, *Water Resour. Res.*, *20*(10), 1453–1465, 1984.

Wilson, J. W., and E. A. Brandes, Radar measurement of rainfall—A summary, *Bull. Am. Meteorol. Soc.*, *60*(9), 1048–1958, 1979.

Yates, S. R., Disjunctive kriging, 3, Cokriging, *Water Resourc. Res.*, *22*(10), 1371–1376, 1986.

W. F. Krajewski, Hydrologic Research Laboratory, National Weather Service, National Oceanic and Atmospheric Administration, 8060 13th Street, Room 530, Silver Spring, MD 20910.

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