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Regional Flood Frequency Analysis Using Extreme Order Statistics of the Annual Peak Record

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A regional flood frequency model is developed for estimating recurrence intervals of extreme floods. The regionalization procedure developed in this paper differs notably from the U.S. Geological Survey index flood method (Dalrymple, 1960) in that a large quantile, rather than the mean annual flood, is used as the "index flood." Based on a result from extreme value theory, exceedances of the specified quantile are modeled by a generalized Pareto distribution. The generalized Pareto distribution has two parameters: a scale parameter and a shape parameter. It is assumed that the shape parameter does not vary from basin to basin, implying that annual peak distributions for all basins have the same upper-tail thickness. The scale parameter, on the other hand, not only varies from basin to basin, but may also depend on covariate information, such as drainage area or indicator functions for basin geology. Likelihood-based inference procedures are developed for the regional flood frequency model. The model is applied to extreme floods of the Central Appalachian region of the United States.

1. INTRODUCTION

Potter [1987] notes that "decision-makers are presenting a new challenge which may help to bridge the gap between the statistics and science of floods. That challenge is the estimation of probabilities of extreme floods." In this paper development of a regional flood frequency model for estimating recurrence intervals of extreme floods is described. Model development is based on the premise that large floods reflect very different hydrologic and meteorological processes than smaller floods. This approach is supported by the *National Research Council* [1988] report in which "focusing on extreme floods ··· even to the exclusion of central characteristics" is given as one of three principles for improving flood frequency estimates.

Regional flood frequency models have received considerable attention in recent years (see, for example, *Greis and Wood* [1981], *Wallis* [1982], *Kuczera* [1982], *Stedinger* [1983], and *Lettenmaier et al.* [1987]). The starting point of most regional flood frequency procedures is the U.S. Geological Survey index flood method [*Dalrymple*, 1960]. In Dalrymple's procedure, the index flood is the mean annual flood. Annual peak values are divided by the sample mean annual flood for the site and pooled in order to estimate a dimensionless regional flood frequency curve.

The model developed in this paper differs notably from previous models in that the index flood is specified by a large quantile, rather than the mean annual flood. Based on results from extreme value theory [*Pickands*, 1975], it is assumed that exceedances of the specified quantile have a generalized Pareto distribution. The generalized Pareto distribution has two parameters: a scale parameter s and a shape parameter k. It is assumed that the shape parameter does not vary from basin to basin, implying that annual peak distributions for all basins have the same upper tail "thickness." The scale parameter not only varies from basin to basin but may also depend on covariate information such as drainage area or indicator functions for basin geology. As with previous regional procedures, an at-site estimator of the index flood is used. Our at-site estimator is a simple quantile estimator, instead of the sample mean annual flood. Exceedances of the estimated quantile, for all sites, are pooled to estimate the unknown scale and shape parameters.

Contents of the sections are as follows. The regional flood frequency model is developed in section 2. Likelihood-based inference procedures are developed in section 3. In section 4 the model is applied to extreme floods of the Central Appalachian region. A summary and conclusions are given in section 5.

2. MODEL DEVELOPMENT

We assume that the flood record consists of annual peak observations $\{Y_{ij}; j = 1, \cdots, n; i = 1, \cdots, m\}$ for m gaging stations each with n years of overlapping data. The random variable Y_{ij} denotes the instantaneous annual peak discharge for year j at station i. For each gaging station i we also have a vector of covariate information, $X_i = (X_{i1}, \cdots, X_{iq})$, with X_{ij} denoting the jth covariate value for site i. Covariate data may include drainage area, indicator functions for basin geology, etc. Detailed discussions of the importance of covariate information for flood frequency analysis can be found in the works by *Benson* [1962] and *Potter* [1987].

It is assumed that for each station i, annual peak discharges are independent and identically distributed with distribution

$$F_i(y) = P\{Y_{ij} \le y\} \qquad y > 0 \tag{1}$$

We are primarily concerned with estimating quantiles of F_i,

$$Q_i(p) = F_i^{-1}(p) \tag{2}$$

where p is typically very close to 1. The value of the 100-year flood at site i, for example, is given by $Q_i(0.99)$.

It is also assumed that annual flood peaks are independent from site to site. This assumption is standard in regional flood frequency models (although several authors, including *Stedinger* [1983] and *Lettenmaier et al.* [1987] have discussed problems created by this assumption). At the conclusion of section 3 it is noted that site to site independence is not strictly necessary for our statistical inference procedures; all results carry through if a particular "conditional independence" property holds.

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An important attribute of the upper tail of the annual peak distribution for site i is its upper bound

$$v_i = \sup \{x : F_i(x) < 1\} \tag{3}$$

with interest focusing on whether v_i is finite or not. The conditional exceedance distribution of F_i is defined for $u < v_i$ by

$$F_i(y \mid u) = P\{Y_{ij} \leq u + y \mid Y_{ij} > u\}$$

$$= \frac{F_i(u + y) - F_i(u)}{1 - F_i(u)} \tag{4}$$

For a fixed threshold u the conditional exceedance distribution $F_i(y \mid u)$ is the conditional probability that Y_{ij} is less than or equal to $u + y$ given that it is larger than u.

Closely associated with conditional exceedance distributions is the generalized Pareto distribution

$$G(y \mid s, k) = 1 - (1 - ks^{-1}y)^{k^{-1}} \tag{5}$$

where the scale parameter s is positive and the shape parameter k is real-valued. The range of the generalized Pareto distribution is $(0, k^{-1}s]$ if k is positive; for k less than or equal to 0, the range is the positive half line. *Pickands* [1975] shows that if the underlying annual peak distribution $F_i(y)$ has an extreme value domain of attraction [see *Leadbetter et al.*, 1983] then the conditional exceedance distribution $F_i(y \mid u)$ can be closely approximated by a generalized Pareto distribution as u becomes large. Importance of Pickand's result follows from the fact that most continuous "textbook" distributions have an extreme value domain of attraction and thus have generalized Pareto upper tails.

Our regional flood frequency model is specified in terms of the conditional exceedance distribution. Heuristically, the model can be described as follows. For each site i, the flood threshold u_i has exceedance probability $(1 - p_0)$. The amount that a flood exceeds its flood threshold, given that it does in fact exceed it, has a generalized Pareto distribution. The shape parameter for the regional generalized Pareto model is the same for all sites. The scale parameter is a function of the covariate information X_i for site i. More formally, it is assumed that for sufficiently large p_0 in $(0, 1)$,

$$F_i(y \mid u_i) = G(y \mid S_i, k) \tag{6}$$

where

$$u_i = F_i^{-1}(p_0) \tag{7}$$

$$S_i = \exp \{c + b_1 X_{i1} + \cdots + b_q X_{iq}\} \tag{8}$$

The parameter u_i is the index flood for site i. The model contains $q + 2$ regional parameters $(k, c, b_1, \cdots, b_q)$.

The generalized Pareto density is given by

$$\begin{aligned} g(y \mid s, k) &= s^{-1}(1 - ks^{-1}y)^{(k^{-1} - 1)} \qquad k \neq 0 \\ g(y \mid s, k) &= s^{-1} \exp \{-s^{-1}y\} \qquad k = 0 \end{aligned} \tag{9}$$

If k is positive, the distribution of flood peaks, for all sites, is bounded above; the upper bound for site i is

$$v_i = u_i + k^{-1}S_i \tag{10}$$

If k equals 0, the upper tail of F_i is exponential with scale parameter S_i. If k is negative, the distribution is unbounded and "thick tailed" (see *Schuster* [1984] for a discussion of upper tail thickness). By a straightforward application of Bayes' theorem, using (4)–(6), it follows that the quantile function of F_i, for p greater than p_0, is given by

$$Q_i(p) = u_i + S_i k^{-1}[1 - ((1 - p)/(1 - p_0))^k] \qquad k \neq 0 \tag{11a}$$

$$Q_i(p) = u_i - S_i \log [(1 - p)/(1 - p_0)] \qquad k = 0 \tag{11b}$$

3. Statistical Inference

Inference procedures are likelihood-based and rely on exceedances of u_i, the flood quantile with recurrence interval $(1 - p_0)^{-1}$ for site i. We denote the number of years, out of n, for which annual peaks at site i exceed u_i by N_i. The total number of floods at all sites that exceed thresholds u_i is denoted N, that is, N is the sum of the m values of N_i. We denote the exceedances by $\{Z_{ij}; i = 1, \cdots, m; j = 1, \cdots, N_i\}$; the "normalized" flood value Z_{ij} equals $Y_{ik} - u_i$ if year k contains the jth annual peak at site i that exceeds u_i.

It follows from (5)–(9) and the site to site independence assumption that the log likelihood function for our model is given by

$$L_n(a) = \sum_{i=1}^{m} \sum_{j=1}^{N_i} \log (g(Z_{ij} \mid S_i, k)) + C \tag{12}$$

where $a = (k, c, b_1, \cdots, b_q)$ is the vector of unknown parameters. The second term, C, in (12) is a function of annual peaks that are smaller than the thresholds u_i, but not of the parameters a.

The score functions are partial derivatives of the log likelihood function with respect to the parameters a,

$$U_n(a)_j = \frac{\partial L_n(a)}{\partial a_j} \qquad j = 1, \cdots, q+2 \tag{13}$$

Maximum likelihood estimators $\hat{a}$ are solutions to the system of equations

$$U_n(a)_j = 0 \qquad j = 1, \cdots, q+2 \tag{14}$$

It is straightforward to show that standard properties of maximum likelihood estimators hold (see, for example, *Bickel and Doksum* [1977]). In particular, our estimators are consistent, implying that if our data set is large and if the model is correct for the data, estimated parameters will be close to their true values with high probability. The invariance property of maximum likelihood estimators implies that the maximum likelihood estimator of $Q_i(p)$, for $p > p_0$, is obtained by replacing the parameters $a = (k, c, b)$ in (11) by the maximum likelihood estimators $\hat{a}$. The resulting estimator, $\hat{Q}_i(p)$, is a consistent estimator of $Q_i(p)$ for $p > p_0$. The estimators $\hat{a}$ and $\hat{Q}_i(p)$ are also asymptotically normal. This result can be used to estimate standard errors of the estimators $\hat{a}$ and $\hat{Q}_i(p)$ [see *Smith*, 1987]. A useful property of maximum likelihood estimators for hypothesis testing problems (see section 4) is that likelihood ratios have a limiting chi-squared distribution.

In the index flood procedure of *Dalrymple* [1960] the at-site estimator of the index flood is the sample mean annual flood. The index flood for our model is the quantile u_i, instead of the mean annual flood. To implement the estimation procedure described above, the at-site estimator of u_i is

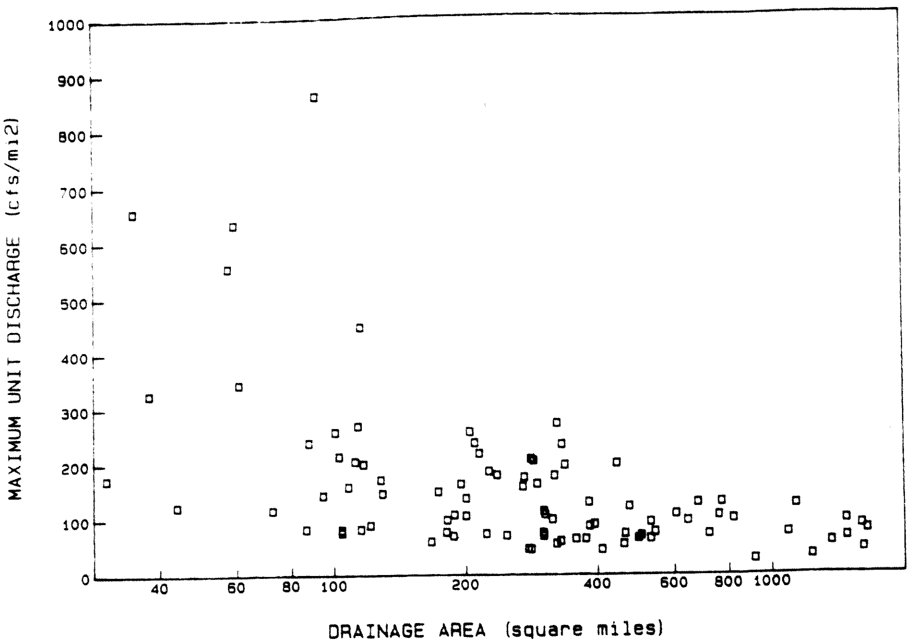

Fig. 1. Maximum unit discharge for 53 stations in the Central Appalachian Region. Maximum unit discharge is expressed in units of cubic feet per second per square mile.

$$\hat{u}_i = \inf \{y : \hat{F}_i(y) > p_0\} \qquad (15)$$

where $\hat{F}_i$ is the sample distribution of annual peaks for site i. The estimator can be represented in terms of the order statistics of the annual peaks for site i, $Y_{(i,1)} < \cdots < Y_{(i,m)}$, as follows:

$$\hat{u}_i = Y_{(i,j)} \qquad p_0 \in [(j-1)/(n+1), j/(n+1)] \qquad (16)$$

The site to site independence assumption is not strictly necessary for our statistical inference procedures. We obtain identical maximum likelihood estimators and asymptotic results under a weaker assumption. The assumption is that flood peaks are conditionally independent from site to site, given that they are larger than the appropriate thresholds $(u_1, \cdots, u_m)$. Under the conditional independence assumption the only effect on the likelihood function of (12) is that the second term C may change. Because this term is not a function of the unknown parameters a, maximum likelihood estimators, and asymptotic properties associated with the estimators are unaffected.

Loosely speaking, the conditional independence assumption protects us from being too surprised that big floods often occur at a number of sites in the same year. Given that a big flood occurs at a particular site, however, the conditional independence assumption specifies that no further information can be obtained about its magnitude from observations at other sites. This assumption has theoretical support from results in the work by *Leadbetter et al.* [1983] which assert that in a variety of settings, exceedances of thresholds of random processes are asymptotically independent as thresholds become large. The assumption has empirical support in *Buishand*'s [1984] study of extreme rainfall data in the Netherlands. From a practical standpoint the conditional independence assumption has implications for assessing the effective size of a regional flood peak sample. If the conditional independence assumption holds, the effective loss of data for the regional model of section 2, relative to procedures that use all of the annual peak data, will be less than it at first appears.

4. Extreme Floods of the Central Appalachians

Application of the regional flood frequency model is illustrated in this section. The setting for the application is the Central Appalachian region of the United States, which consists of the Piedmont, Valley and Ridge, and Appalachian Plateau provinces of Virginia, Maryland, West Virginia, and Pennsylvania. In the Central Appalachians annual peak distributions are subject to a wide variety of hydrologic and meteorological processes. A central assumption of this paper is that the effective controls of the central portion of the annual peak distribution may be quite different from the effective controls of extreme floods. The Central Appalachian region, with its diversity of hydrologic processes, provides the type of setting in which this assumption is most likely to be met. We begin the section with a graphical description of extreme floods of the Central Appalachian region.

Figure 1 shows a plot of maximum unit discharge versus drainage area for 53 gaging stations in the Central Appalachian region. Maximum unit discharge is the largest instantaneous peak discharge during the period of record divided by drainage area. The period of record is 1935–1984 (each station has fewer than 10 years of missing data). Each of the stations is described in U.S. Geological Survey annual data reports as unaffected by regulation at high flow. Not surprisingly, maximum unit discharge generally decreases with drainage area. Also, variability in maximum unit discharge

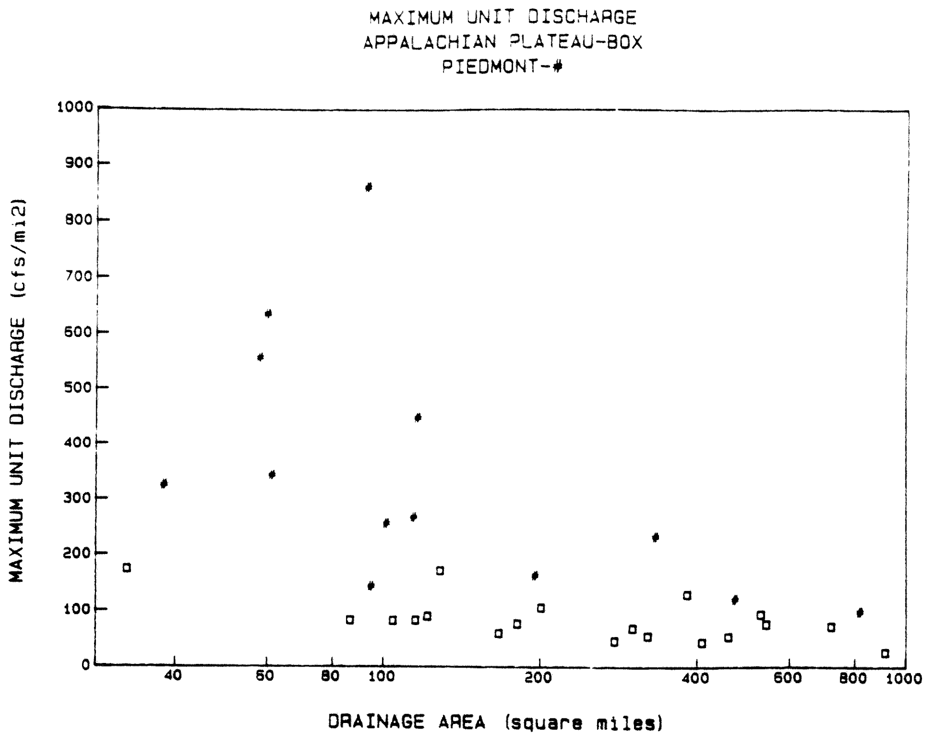

Fig. 2. Maximum unit discharge for Piedmont and Appalachian Plateau basins. Appalachian Plateau basins are represented by the number symbol. Piedmont basins are represented by boxes.

decreases with drainage area. The range in maximum unit discharge for the 26 basins larger than 150 mi^2 is only 40–250 cfsm. A plausible explanation for low variability in the maximum unit discharge plot is that flood peaks are bounded above. *Costa* [1987] notes that "drawing of envelope curves through time raises the question of the existence of expected limits to peak discharges in the United States. Perhaps some atmospheric limits are being approached and the limiting curve has begun to stabilize." Model assumptions concerning boundedness can have a pronounced effect on extreme flood quantile estimates [see *Hosking*, 1984; *Smith*, 1987].

The regional flood frequency model developed in section 2 accomodates both bounded and unbounded flood peak distributions. The generalized Pareto distribution is bounded above if the shape parameter k is positive. If k is negative, the distribution is unbounded and "thick-tailed." Exponential upper tails, the case in which k equals 0, represent a middle ground between the thick-tailed and bounded alternatives. By requiring the shape parameter k to be the same for all basins in the model of section 2, we force all basins to have the same upper tail thickness properties.

Figure 2 provides a closer look at variability in maximum unit discharge for the Central Appalachians. In this figure Appalachian Plateau basins, at the western end of the region, are distinguished from Piedmont basins, at the eastern end of the region. For the 11 Piedmont basins larger than 80 square miles, maximum unit discharge is greater than 200 cfsm for 5. For all Piedmont basins maximum unit discharge is greater than 100 cfsm. For the nine Appalachian Plateau basins with drainage area greater than 80 mi^2, seven have maximum unit discharge values less than 100 cfsm. The largest maximum unit discharge for Appalachian Plateau basins is 200 cfsm. It appears that big floods in the Appalachian Plateau are quite different from big floods in the Piedmont.

The apparent contrasts in extreme flood characteristics are surprising because the two regions are only separated by approximately 100 miles. A plausible explanation can be found in the typical paths of tropical storms [*Cry*, 1965]. Paths of tropical storms are such that their influence is generally greatest along the eastern edge of the Central Appalachians and decreases with distance from the Atlantic Ocean. Indeed, the flood of record for each of the Piedmont basins is associated with a tropical storm. For Appalachian Plateau basins the flood of record can come from a potpurri of events. Notable is the March 1936 flood, for which snowmelt was a major contributing factor [*Grover*, 1937].

The apparent contrasts in extreme floods are also surprising in view of geologic contrasts between the two regions. Piedmont basins are generally underlain by a thick mantle of saprolite with large subsurface storage capacity. Thin soils with small subsurface storage capacity form on sedimentary rocks of the Appalachian Plateau. Figure 3 shows the annual peak distribution for Seneca Creek in the Piedmont and Buffalo Creek in the Appalachian Plateau (drainage area for both basins is approximately 100 mi^2). The central portions of their annual peak distributions differ; the roles, however, are reversed from Figure 2. "Average" floods in the Appalachian Pleateau basin are larger than average floods in the Piedmont basin. As suggested above, the contrast between moderate floods in the Appalachian Plateau and Piedmont can be attributed in large part to geologic control. The effects of geologic control can also be clearly demonstrated in the Valley and Ridge province. *White* [1976] has shown that carbonate basins in the Valley and Ridge differ significantly from noncarbonate basins in the central portion of the annual peak distribution. Figure 4 shows sample distributions for Little Lehigh Creek, which is underlain by carbonates, and Passage Creek, which is underlain by sandstone and shale.

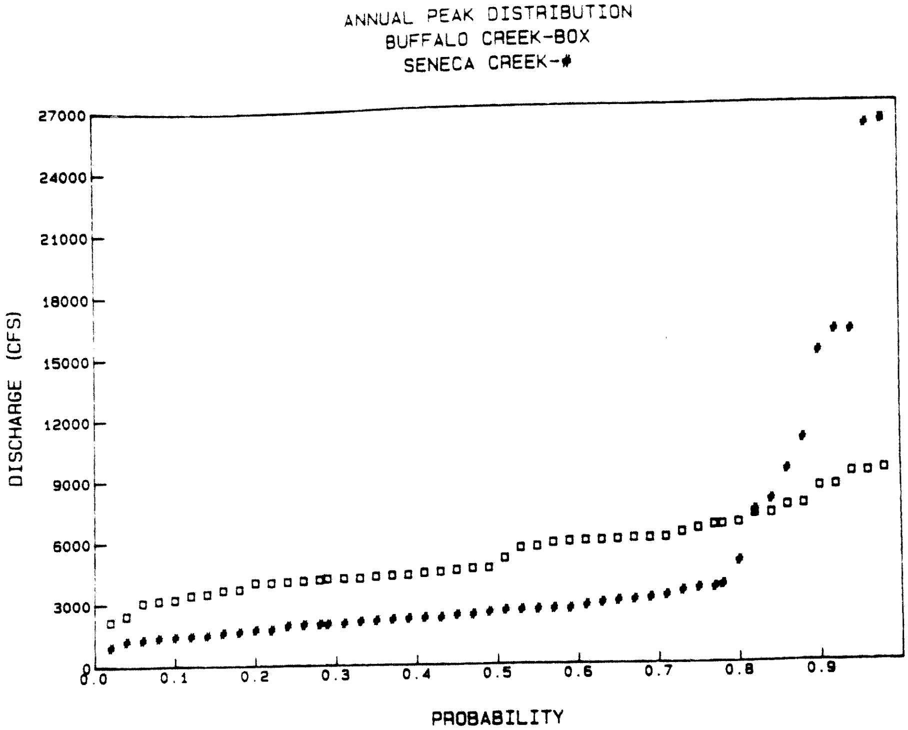

Fig. 3. Annual peak distributions for Buffalo Creek in the Appalachian Plateau (denoted by boxes) and Seneca Creek in the Piedmont province (denoted by the number symbol).

Median annual flood for the sandstone-shale basin is approximately twice median annual flood for the carbonate basin. The contrasts appear, however, to diminish in the upper tail. From big floods we obtain a very different picture of Central Appalachian floods than from smaller floods.

We illustrate below that the regional flood frequency model developed in section 2 can accomodate the diverse processes relevant to extreme floods in the Central Appalachians. The regression model for the scale parameter (equation (8)) is chosen to be

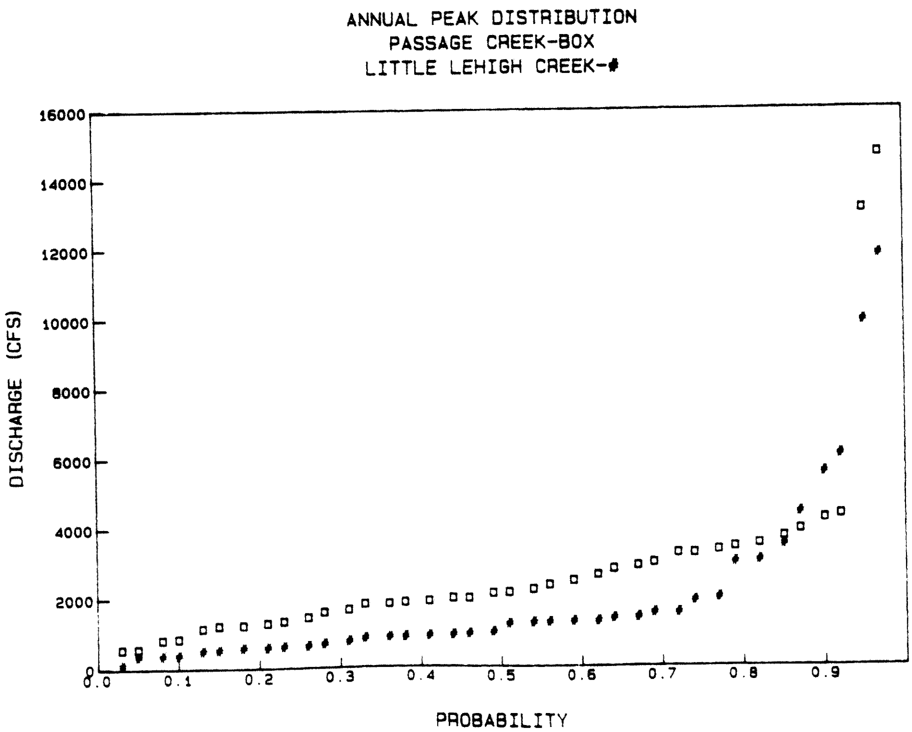

Fig. 4. Annual peak distributions for two Valley and Ridge basins, Passage Creek (underlain by sandstone and shale, boxes) and Little Lehigh Creek (underlain by carbonates, denoted by the number symbol).

TABLE 1. Upper Order Statistics of Annual Flood Peaks From the Appalachian Plateau (Indicated by a 0 in the Second Column) and Piedmont (Indicated by a 1 in the Second Column)

USGS Gage No.		DA, mi^2	u_i, cfs	$Y_{(i,37)}$, cfs	$Y_{(i,38)}$, cfs	$Y_{(i,39)}$, cfs	$Y_{(i,40)}$, cfs
03061500	0	115.	8,460.	8,520.	9,150.	9,170.	9,300.
03080000	0	121.	6,350.	6,820.	6,910.	9,930.	10,900.
03208500	0	286.	28,900.	31,800.	33,800.	46,600.	59,000.
03079000	0	382.	15,500.	15,600.	16,600.	16,800.	50,000.
03066000	0	86.2	4,410.	4,510.	4,900.	4,920.	6,800.
03058500	0	181.	10,400.	11,100.	11,400.	12,400.	18,000.
03053500	0	277.	10,300.	10,500.	11,600.	12,000.	13,000.
01645000	1	101.	15,000.	16,000.	16,000.	25,900.	26,100.
02030000	1	116.	7,470.	11,100.	21,600.	23,000.	52,000.
02030500	1	226.	10,900.	11,500.	14,100.	16,000.	42,200.
02028500	1	94.6	12,200.	13,700.	17,500.	30,000.	70,000.
01663500	1	287.	25,000.	26,900.	28,800.	33,700.	60,000.
01644000	1	332.	20,300.	27,600.	35,600.	48,700.	78,100.
01662000	1	195.	10,500.	14,200.	18,000.	21,900.	32,000.

One square mile equals 2.590 km^2; 1 cubic foot per second equals 0.0283 m^3/s.

$$S_i = \exp\{c + b_1 \log(A_i) + b_2 X_i\} \tag{17}$$

where A_i is drainage area of basin i and X_i is a 0-1 indicator function which takes the value 1 if the basin lies in the Piedmont and 0 if the basin lies in the Appalachian Plateau. Equation (17) states that the scale parameter is a power function of drainage area and that Piedmont basins differ from Appalachian Plateau basins by a multiplicative factor of exp (b_2) in the scale parameter.

Fourteen stations from the Appalachian Plateau and Piedmont (see Table 1) are used to estimate parameters for the model of (17). The period of record is 1942–1981 (40 years). The threshold value is the 0.88 quantile (u_i equals the order statistic $Y_{(i,36)}$). The estimated parameters are $k = -0.29$, $c = 3.06$, $b_1 = 0.93$, and $b_2 = 1.52$.

The estimate of the shape parameter k is negative, suggesting that flood peaks are unbounded and thick-tailed. On the basis of this evidence, we certainly can not attribute behaviour of the maximum unit discharge plot to boundedness of flood peaks. A likelihood ratio test can be constructed to determine whether we can rule out the bounded and exponential tail alternatives. To do so we test whether the estimate of k is significantly different from 0. The likelihood ratio statistic is of the form

$$\Lambda_n = -2[(L_n(\tilde{c}, \tilde{b}_1, \tilde{b}_2, 0) - L_n(\hat{c}, \hat{b}_1, \hat{b}_2, \hat{k})] \tag{18}$$

where $\tilde{c}$, $\tilde{b}_1$, and $\tilde{b}_2$ are maximum likelihood estimators obtained from (12) under the constraint that k equal 0. The likelihood ratio statistic has a limiting chi-squared distribution with 3 degrees of freedom [see *Bickel and Doksum*, 1977]. The value of the likelihood ratio statistic (3.6) has a significance level of less than 0.90. The evidence for unbounded thick-tailed flood peaks is thus not particularly strong.

The estimate of the coefficient b_2 implies that the scale parameter for Piedmont basins is nearly 5 times (exp (1.52)) larger than the scale parameter for Appalachian Plateau basins of the same drainage area. A likelihood ratio test is constructed to test whether the estimate of b_2 is significantly different from 0. The likelihood ratio statistic is of the form

$$\Lambda_n = -2[(L_n(\tilde{c}, \tilde{b}_1, 0, \tilde{k}) - L_n(\hat{c}, \hat{b}_1, \hat{b}_2, \hat{k})] \tag{19}$$

The value of the likelilhood ratio statistic (29.4) has a significance level greater than 0.99. This result strongly supports the need to distinguish extreme floods in the Piedmont from extreme floods in the Appalachian Plateau in our regional frequency analysis.

Table 2 shows quantile estimates for Seneca Creek (the Piedmont station of Figure 3) obtained using the regional model of equation (17) and an at-site lognormal model (using the maximum likelihood estimators recommended in the National Research Council report "Estimating Probabilities of Extreme Floods"). For Seneca Creek the regional generalized Pareto model yields significantly higher estimates of extreme floods than the at-site lognormal method. The fundamental assumption of our regional model is that the large flood values shown in Figure 3 for Seneca Creek likely reflect different processes than the smaller flood values. By including data from extreme floods at other sites in our regional analysis, we are more confident that the large floods of Figure 3 are not atypically large. Furthermore, by excluding data from smaller floods we are more confident that our estimates of extreme flood quantiles truly reflect extreme flood processes.

TABLE 2. Estimated Flood Quantiles for Seneca Creek Obtained From the Regional Generalized Pareto Model and the At-Site Lognormal model

	Estimated Discharge, cfs	
Return Interval, years	Generalized Pareto Model	Lognormal Model
10	16,300	8,800
100	40,900	18,200
1,000	89,000	32,600

The sample annual peak distribution for Seneca Creek is shown in Figure 3. One cubic foot per second equals 0.023 m^3/s.

5. Summary and Conclusions

A regional flood frequency model is developed for estimating recurrence intervals of extreme floods. The regionalization procedure developed in this paper differs notably from previous procedures in that the index flood is a large quantile rather than the mean annual flood. Based on *Pick-*

ands's [1975] characterization of the upper tail of distribution functions, exceedances of the specified quantile are assumed to have a generalized Pareto distribution. The generalized Pareto distribution has two parameters: a shape parameter and a scale parameter. The shape parameter does not vary from basin to basin, implying that annual peak distributions for all basins have the same upper tail thickness. Covariate information, such as drainage area or indicator functions for basin geology, is incorporated in the flood frequency model through a regression equation for the scale parameter (equation (8)). The particular form of the regression model chosen for the scale parameter is dictated in large part by computational tractability.

An at-site estimator is used for the threshold quantile at each site. Exceedances of the estimated quantile, for all sites, are pooled in order to estimate the unknown scale and shape parameters. Likelihood-based inference procedures are developed in section 3 for parameter estimation and hypothesis testing. Inference procedures are derived under a site to site independence assumption. At the end of section 3 it is noted that inference procedures are valid under a weaker "conditional independence" assumption.

The regional flood frequency model is applied in section 4 to annual flood peak data from 14 stations in the Central Appalachian region. The example is artificial in restricting attention to Appalachian Plateau and Piedmont basins. The example does illustrate the contrasts that arise in comparing extreme floods with smaller floods in the Central Appalachians and their potential consequences for estimating extreme flood quantiles (similar issues are raised by *Waylen and Woo* [1982] for western Canada).

A limitation on the practical utility of the procedure developed in this paper is imposed by flood measurement error. In situations where measurements of extreme floods are poor (for example, high gradient mountain streams, as documented by *Jarrett* [1987] and *Wolman and Costa* [1984]; see also *Potter and Walker* [1985]), "tail procedures," like the method developed in this paper, should not be used.

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