Application of Relaxation Scheme to Wave-Propagation Simulation in Open-Channel Networks[a]

Discussion by Ming Jin,[4] D. L. Fread,[5] and J. M. Lewis[6]

The authors found in their test Example 2 that two implicit schemes, including the implicit four-point Preissmann scheme, did not generate proper solutions when the channel bed slope was increased to 0.002 and larger values, and the authors further suggested that the four-point implicit scheme is "suitable only for a relatively smooth or slowly varying flow regime with a mild slope." This is contrary to the discussers' experience with the NWS FLDWAV model, which is based on a weighted four-point implicit (Preissmann-type) scheme that yields two nonlinear, implicit finite-difference approximations of the Saint-Venant equations that are solved by the Newton-Raphson iterative technique. The discussers would not expect any modeling difficulties for the flow conditions in the authors' Example 2. However, the four-point implicit scheme does have numerical difficulties when the flow regime is essentially critical or is in the transcritical (subcritical-supercritical or supercritical-subcritical transition) state, and particularly when the flow has a transcritical moving interface in which a supercritical flow region travels with a very rapidly varying flood wave, e.g., a dam-break-induced flood wave. None of the above conditions exists in Example 2 for channel bed slopes less than about 0.006. Within the NWS FLDWAV model, the discussers have developed two techniques to solve this transcritical modeling difficulty. They are (1) an LPI (local partial inertia) modified implicit scheme (Fread et al. 1996), and (2) a characteristic-based upwind explicit scheme (Jin and Fread 1997). It has been found that the new LPI technique provides a very powerful enhancement to the weighted four-point implicit scheme, because it enables the implicit scheme to simulate any transcritical or mixed unsteady flow while retaining the four-point implicit scheme's inherent accuracy and computational efficiency.

The discussers tested the authors' Example 2 for a range of channel bed slopes varying from 0.0002 to 0.02, using the FLDWAV model. Fig. 10 shows the computed hydrographs at x = 4,000 m in the channel (branch) 3.

Fig. 11 shows the time-variation of the computed Froude numbers at x = 4,000 m. The conventional four-point implicit scheme had no numerical difficulties in modeling the entirely subcritical flow conditions of Example 2 with the bed slope less than about 0.006. For subcritical flow conditions throughout the routing reach for all times, a double-sweep solution algorithm is used along with a free flow downstream boundary condition consisting of the Manning equation with the friction slope lagged one time step. With the bed slope larger than about 0.008, the flow becomes supercritical throughout the entire reach for all times. For this flow condition, a cascading method of solution is used along with two upstream boundary conditions [$Q(t)$ and the Manning equation with friction slope determined by the water surface slope lagged one time step]. Only the LPI technique provided the proper (stable and con-

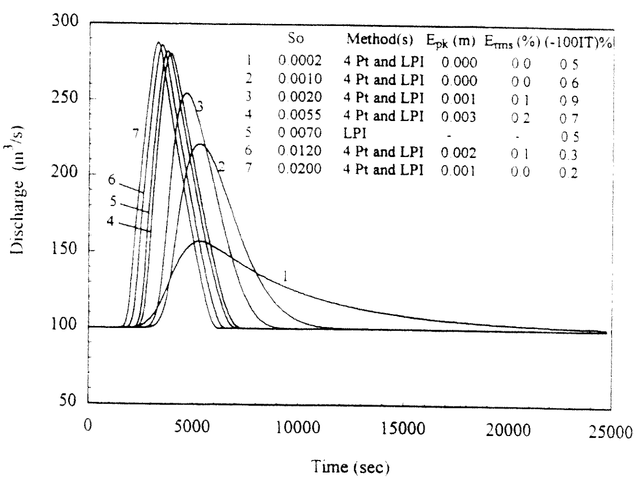

FIG. 10. Discharge Hydrographs at x = 4,000 (m)

[a]November 1998, Vol. 124, No. 11, by Mustafa M. Aral, Yi Zhang, and Shi Jin (Paper 12170).

[4]Visiting Sci., Hydrologic Res. Lab., National Weather Service, Silver Spring, MD 20910.

[5]Dir., Office of Hydrol., National Weather Service, Silver Spring, MD.

[6]Res. Hydrol., Hydrologic Res. Lab., National Weather Service, Silver Spring, MD.

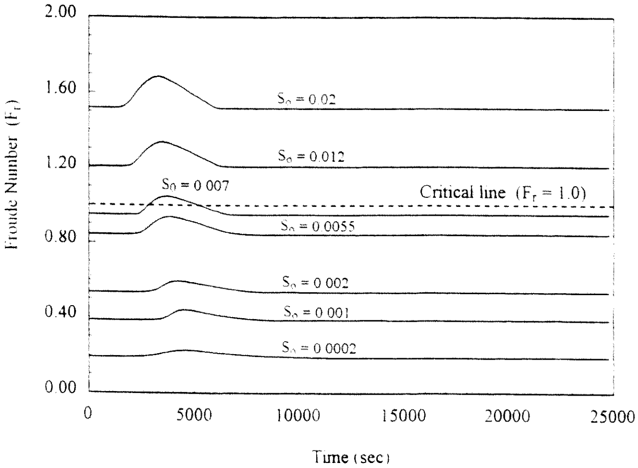

FIG. 11. Froude Numbers at x = 4,000 (m)

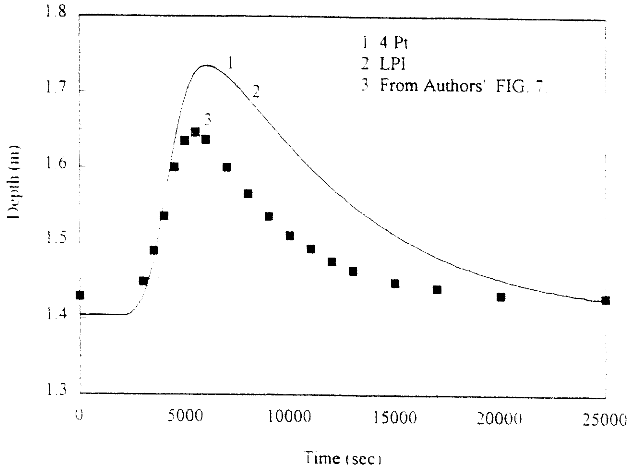

FIG. 13. Computed Depth Hydrographs at 4,000 (m)

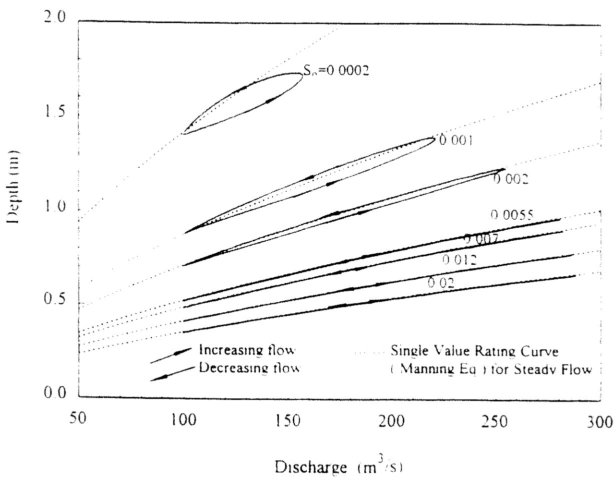

FIG. 12. Computed Rating Curves at x = 4,000 (m)

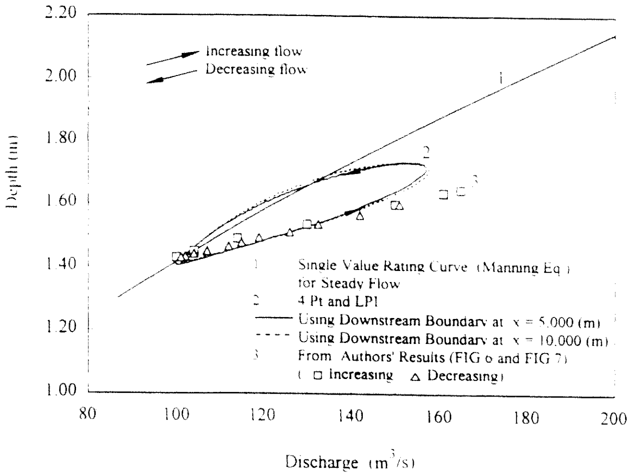

FIG. 14. Rating Curves at 4,000 (m) for S_0 0.0002

sistent) solution for the bed slope of 0.007, in which a transcritical flow regime existed. The LPI technique modifies the two inertial terms in the Saint-Venant momentum equation by multiplying them by a factor $(1 - F^m)$ in which F is the local time-dependent Froude number and m (varying between 1 and 10) is a coefficient controlling stability and accuracy. High values of m provide more accuracy while small values of m provide somewhat more stability in the numerical solutions. In all situations except the one involving the transcritical flow, the errors incurred in using the LPI factor (differences between the LPI solution and the implicit solution without the LPI factor) were measured. In Fig. 10, the errors in the computed peak depth (E_{pk}) and the relative RMS error in the computed depth hydrograph (E_{rms}) are listed. The discussers derived an indicator (IT) for the importance of the inertia terms in the momentum equation (as a ratio of the inertia terms to the water surface slope), i.e., $IT = -0.5F^2/(1 + 1.5\phi F^3)$, where $\phi = (n^2 g^{2/3} y^{1/6})/(\lambda^2 \partial y/\partial t)$ in which n is the Manning's resistance coefficient, g is the gravity constant, λ is a constant for Manning's equation (λ = 1.49 for the English system units and λ = 1.0 for SI units), and y is the flow depth. It was found that the expression ($-100IT$) can be used to approximate a conservative upper limit of the percentage errors incurred in using the LPI technique. This is also shown in Fig. 11.

The computed rating curves (depth-discharge relations) are compared in Fig. 12 with the single-value rating curves derived from the Manning equation for steady flows for a range of channel slopes (0.0002–0.02). Also, it is observed from Fig. 12 that the hysteresis effect (loop) is always present for each channel slope; however, it becomes insignificant as the slope increases beyond 0.002 for the flow condition of Example 2.

Under the same conditions used by the authors in Example 2 with the channel bed slope of 0.0002, the discussers compared the results at x = 4,000 m from FLDWAV with the authors' results. As seen in Fig. 13, somewhat different results are obtained. Fig. 14 compares the depth-discharge relation at this location from the authors' Figs. 6 and 7 with that produced by FLDWAV. Also shown are the results from a FLDWAV simulation with an extended downstream boundary (increased from 5,000 to 10,000 m) to verify the FLDWAV model's performance in using the free-flow downstream boundary condition. As shown in Fig. 14, a significant loop exists in the computed depth-discharge relation using the FLDWAV model, whereas the authors' results reveal little if any hysteresis effects in the computed depth-discharge relation. This could be due in part to the authors' choice of an unspecified $h(t)$ as their downstream boundary conditions, as indicated in their Fig. 5. The discussers would like to know the basis for such an apparent choice and the authors' apparent inability to produce the loop in the depth-discharge relation at this location.

APPENDIX. REFERENCES

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Closure by Mustafa M. Aral,[7] Yi Zhang,[8] and Shi Jin[9]

The writers wish to thank the discussers for their comments and interest in the subject of wave propagation solutions in open-channel networks. As clearly indicated in our paper, our purpose was to provide a computational procedure that may be used in the solution of a wide variety of problems in open-channel networks. These problems, as demonstrated by several applications in the paper as well as in other reports published by the writers associated with the paper, range from subcritical to supercritical applications in open-channel networks including transcritical cases (Aral et al. 1996). For all these cases, the proposed method functions very well without any adjustments as required in most other methods. Further, the emphasis on our proposed method is on the solution of governing equations in conservative form. This is an essential component if the flow solutions are to be used in contaminant transport analysis, which is the case for our overall interest on the subject (Aral and Zhang 1998; Zhang 1998).

With these points in mind, we provide the following response to the comments of the discussers. In their discussion, Jin et al. posed some questions regarding our Example problem 2. The discussers indicate that they did not experience numerical difficulties in the solution of Example 2 when using conventional four-point implicit scheme until the channel slope was increased to 0.006 or larger, as opposed to our limiting slope of 0.002. They also compared their result with results presented in the paper for Example 2 and found some difference between the two results. Furthermore, they indicate that in their application a significant loop exists in the computed depth-discharge relation, while our results did not generate this loop. Probably they attribute these differences to some potential problem with the proposed method. The answer to the discrepancy is clearly in the application they have selected, and not in the method.

We anticipate that the difference between their results and ours is due to the use of a different downstream boundary condition in Example 2. In our application, we used a constant depth downstream boundary condition with depth $y = 1.43$ m throughout the whole simulation period, while the discussers used a free-flow downstream boundary condition consisting of the Manning equation. The confusion occurred because we did not clearly define the downstream boundary condition in our paper, and we apologize for that. Since different downstream boundary conditions were used in both applications, the results are, of course, different. Furthermore, since the discussers used a free-flow downstream boundary condition, the loop depth-discharge relationship may be generated by using a loop-rating curve based on the Manning equation in the downstream boundary condition (Fread 1992) or by the backwater due to the variant downstream water surface elevation. When a loop-rating curve is used at the downstream boundary, the loop is produced by using the friction slope S_f rather than the channel bottom slope S_0, with the friction slope being different for the rising and decreasing limbs of the hydrograph. In the paper, because we used constant depth downstream boundary condition, the friction slope is obtained by the Manning equation and the depth-discharge relationship should be a single valued function, not a loop function. Actually, we used several methods, including Preissmann scheme, to compute Example 2; the results were very close in each method. Finally, because downstream depth boundary condition is kept constant with a value equal to the initial value ($y = 1.43$ m) in the paper, the transcritical or supercritical flow regime may appear earlier (when channel slope S_0 equal to around 0.002). Our code using both the Preissmann scheme and the implicit finite-element method could not generate the proper result for this slope, while under the same condition the relaxation scheme still produced the correct results.

We hope that this response clarifies the questions posed by the discussers and we appreciate their interest in our paper.

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[7]Prof., School of Civ. and Envir. Engrg., Georgia Inst. of Technol., Atlanta, GA 30332.

[8]Grad. Student, School of Civ. and Envir. Engrg., Georgia Inst. of Technol., Atlanta, GA.

[9]Asst. Prof., School of Mathematics, Georgia Inst. of Technol., Atlanta, GA.