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1. INTRODUCTION

Operational experience and systematic evaluation indicate that the biggest sources of systematic biases in Weather Surveillance Radar - 1988 Doppler version (WSR-88D) rainfall products are nonuniform reflectivity gradient in the vertical, inaccurate Z-R relationships, and lack of radar calibration (see, e.g., Smith et al. 1996, Seo et al. 1997).

1.1 Mean Field Bias

In order to deal with the latter two (and, to an extent, the first as well) sources of error, the precipitation estimation stream in the National Weather Service (NWS) (Hudlow 1998, Fulton et al. 1998) employs procedures that estimate mean field bias in real time. Because it multiplicatively affects the entire radar umbrella, mean field bias adjustment has probably the biggest quantitative impact on radar-based rainfall estimates in the post-Radar Product Generator (RPG) data processing stream. Hence, performance of the adjustment procedure is critical to quantitative use of radar rainfall data in operational hydrologic forecasting.

In the first half of this paper, we introduce a new procedure for real-time adjustment of mean field bias in WSR-88D rainfall products. The procedure, based on operational experience at the River Forecast Centers (RFC) and a critical examination of the existing procedures in NWS (Seo et al. 1997), is designed to be simple, unbiased, parsimonious, and intuitive.

1.2 Range-Dependent Biases

In an attempt to account for reflectivity gradient in the vertical, the WSR-88D rainfall algorithm (Fulton et al. 1998), also known as the Precipitation Processing Subsystem (PPS), employs nonlinear adjustment of radar rain rate as a function of range.

sloping reflectivity gradient in the vertical (Seo et al. 1998), and hence may not be applicable to stratiform storms with bright band enhancement. Also, even if the correction coefficients are estimated with reasonable accuracy (by no means a trivial task), it would be extremely difficult to ascertain when or when not to apply the correction, without real-time guidance from on-line analysis of the real-time data themselves.

In the second half of this paper, we describe the ongoing effort in NWS to develop and implement a procedure for real-time adjustment of range-dependent biases (due, e.g., to bright band enhancement, radar sampling of ice crystals/particles above the freezing level, gently-sloping vertical reflectivity gradient in tropical storm, partial beam filling at far ranges).

2. REAL-TIME ADJUSTMENT OF MEAN FIELD BIAS

The new procedure (Seo et al. 1998) is a conceptual extension of the current Stage II procedure (Seo et al. 1997). Rather than estimating the mean field bias directly, however, the new procedure separately estimates the spatial averages of positive gage and radar rainfall over the area commonly identified as raining by the two sensors, and approximates the mean field bias by the ratio of the two spatial averages estimated.

Recursive estimation of the spatial averages is achieved by exponential smoothing (Schweppe 1973), for which one only requires specification of the size of the temporal moving-average window. Under a pair of lognormality assumptions, it is also possible to obtain a relative measure of uncertainty associated with the mean field bias thus estimated at that memory span.

Owing to the computational economy, it is possible in the new procedure to parallel-run the above recursive filter at several or more choices of memory span. Such an implementation yields, e.g., hourly, daily, weekly, monthly, and annual mean field bias estimates and the relative measures of uncertainty associated with them. Given the level of uncertainty tolerance, one may then arrive at the 'best' mean field bias estimate for the particular hour by applying a set of decision rules (Seo et al. 1998).

* Corresponding author address: D.-J. Seo, Hydrologic Research Laboratory, Office of Hydrology, National Weather Service, Silver Spring, MD 20910; e-mail: dongjun.seo@noaa.gov. The existing procedure is intended primarily for tropical storms (e.g., hurricanes and tropical depressions), which are characterized by gently

It is shown (Seo et al. 1998) that the relative error variance associated with the estimates of spatial averages of positive gage and radar rainfall may also be used as a measure of uncertainty in the mean field bias estimates. The inverse of the relative error variance is nothing but a counter for the (age-weighted) number of positive radar-gage pairs available within the temporal moving-average window, and hence is amenable to, if necessary, manual adjustment by the forecasters.

An example of the hourly output from the procedure, referred to as the bias table, is shown in Table 1.

Table 1

| index | α | $\beta(k k)$ | $p^{-1}(k k)$ | $g_m(k k)$ | $r_m(k k)$ |
|-------|----------|--------------|---------------|------------|------------|
| 1 | 1.0 | 1.53 | 6.3 | 1.94 | 1.27 |
| 2 | 5.0 | 1.45 | 23.1 | 3.71 | 2.56 |
| 3 | 10.0 | 1.44 | 30.6 | 3.83 | 2.65 |
| 4 | 20.0 | 1.43 | 36.2 | 3.87 | 2.70 |
| 5 | 50.0 | 1.40 | 43.5 | 3.90 | 2.80 |
| 6 | 100.0 | 1.29 | 63.6 | 3.98 | 3.08 |
| 7 | 200.0 | 1.20 | 116.5 | 4.05 | 3.39 |
| 8 | 500.0 | 1.13 | 316.8 | 4.11 | 3.63 |
| 9 | 1000.0 | 1.11 | 741.7 | 4.13 | 3.71 |
| 10 | 2000.0 | 1.11 | 1438.4 | 4.14 | 3.74 |

In the table, α , $\beta(k|k)$, and $p^{-1}(k|k)$ denote the memory span (in hrs), the estimated bias, the effective number of positive radar-gage pairs within the memory span, respectively, and $g_m(k|k)$ and $r_m(k|k)$ denote the estimated spatial averages of positive gage and radar, respectively, rainfall over the area commonly identified as raining by the two sensors.

Results from true validation at Tulsa (KINX) and Twin Lakes (KTLX), OK, and the Advanced Weather Interactive Processing System (AWIPS)-Open Radar Product Generator (ORPG) implementation schedule will be given in the oral presentation.

3. REAL-TIME ADJUSTMENT OF RANGE-DEPENDENT BIASES

In Seo et al. (1997), a prototype procedure for real-time adjustment of range-dependent biases was developed: in essence, it estimates in real time the correction coefficients in the current PPS procedure. Though the procedure, which does not require explicit retrieval of the vertical profile of reflectivity (VPR), performs well for tropical storms, it is found to be susceptible to errors for stratiform storms with strong bright band enhancement (Seo et

al. 1998), and hence may not be considered an 'all-purpose' procedure for adjustment of range-dependent biases (whatever the sources of such biases may be).

3.1 Alternative Approach

As an alternative, a VPR-based procedure is formulated (Seo et al. 1998). Though numerous VPR-based procedures appear in the literature (see, e.g., Seo et al. 1997 for references), they do not share the same objectives or operating conditions with WSR-88D (the Volume Coverage Patterns, in particular), and hence are not necessarily applicable to WSR-88D rainfall estimation. In order to formulate the alternative procedure, below we first define the problem in the context of WSR-88D rainfall estimation.

The apparent radar rainrate at some mid- to far range, R (in mm/hr), is to be adjusted by a single multiplicative factor, F_a :

$$R_{\text{corr}} = F_a R \quad (1)$$

where R_{corr} denotes the corrected radar rainrate (in mm/hr). The adjustment factor, which is to be estimated (presumably) from the mean VPR, is given by:

$$F_a = E[R_{\text{ta}}] / E[R] \quad (2)$$

where R_{ta} is the radar rainrate (in mm/hr) at the target altitude (in km), and $E[\]$ denotes the expectation operator. The target altitude can be set at anywhere as long as it is well-sampled by the radar so that $E[R_{\text{ta}}]$ (and any other statistics that may be needed) may be directly estimated.

The significance of Eq.(2) is that it renders R_{corr} to be unbiased with respect to R_{ta} in the mean sense, a prerequisite for the adjustment procedure to be hydrologically viable.

In Eq.(2), R_{ta} and R are given by:

$$R_{\text{ta}} = \alpha Z_{\text{ta}}^\beta \quad (3)$$

$$R = \alpha Z_e^\beta \quad (4)$$

where α and β are the constant and the exponent, respectively, in the 'R-Z' relationship, and Z_{ta} and Z_e are the equivalent reflectivity factor at the target altitude and at the mid- to far range, respectively. In Eq.(4), Z_e may be approximated by:

$$Z_e = \int_{\theta_0}^{\infty} g_n^2(\theta - \theta_0) z(\theta) d\theta \quad (5)$$

5)

$-\infty$

where $g_n^2(\theta-\theta_0)$ is the normalized two-way gain at elevation angle θ in the WSR-88D beam centered at elevation angle θ_0 , and $z(\theta)$ is the (true) reflectivity factor at elevation angle θ .

Because the Z-R relationship is nonlinear, estimation of $E[R]$ from measurements of equivalent reflectivity factor is not trivial. Here, we propose a second-order approximation via the Taylor-series expansion:

$$R \approx \alpha Z_{eN}^b + (Z_e - Z_{eN}) \alpha \beta Z_{eN}^{\beta-1} + 0.5 (Z_e - Z_{eN})^2 \alpha \beta (\beta-1) Z_{eN}^{\beta-2} \quad (6)$$

where Z_{eN} denotes the 'nominal' equivalent reflectivity factor at the mid- to far range. Choosing Z_{eN} to be $E[Z_e]$, and taking expectations on both sides of Eq.(6), we have:

$$E[R] \approx \alpha E^\beta [Z_e] + 0.5 \alpha \beta (\beta-1) E^{\beta-2} [Z_e] \text{Var}[Z_e] \quad (7a)$$

$$= \alpha (1 + 0.5 \beta (\beta-1) CV^2 [Z_e]) E^\beta [Z_e] \quad (7b)$$

where $\text{Var}[Z_e] = E[Z_e^2] - E^2[Z_e]$ and $CV[\]$ denotes the coefficient of variation of the variable bracketed. Analogously applying the second-order approximation to $E[R_{ta}]$, we have for the adjustment factor, F_a :

$$F_a \approx \frac{(1 + 0.5 \beta (\beta-1) CV^2 [Z_{ta}]) \uparrow E[Z_{ta}] \uparrow^\beta}{(1 + 0.5 \beta (\beta-1) CV^2 [Z_e]) \downarrow E[Z_e] \downarrow} \quad (8)$$

Noting that $\alpha=0.017$ and $\beta=0.714$ for $Z=300R^{1.4}$ (i.e., the PPS default) and that $CV[Z_e]$ tends to vary rather significantly in the vertical (Seo et al. 1997), we conclude that the first term in Eq.(8) cannot be ignored. In other words, to evaluate the adjustment factor even approximately (i.e., within the accuracy of the second-order approximation), it is necessary to evaluate not only $E[Z_e]$ and $E[Z_{ta}]$ but also $\text{Var}[Z_e]$ and $\text{Var}[Z_{ta}]$ (i.e., estimation of mean VPR does not suffice).

Estimation of (true) mean VPR is a topic of ongoing research (see, e.g., Andrieu and Creutin 1995, Andrieu et al. 1995, Vignal et al. 1997). The variance terms may be estimated via the moving-average approximation (Seo et al. 1997).

Preliminary evaluation of the above alternative procedure and the Open Radar Product Generator (ORPG) implementation schedule will be given in the oral presentation.

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