ON THE RELATIONSHIP BETWEEN CATCHMENT SCALE AND CLIMATOLOGICAL VARIABILITY OF SURFACE-RUNOFF VOLUME

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ABSTRACT

The relationship between catchment scale and climatological variability of surface-runoff volume is evaluated through theoretical and empirical analyses. Given a point description of runoff volume due to infiltration excess under the time compression approximation, the climatological mean and variance of areal runoff volume for heterogeneous soil are evaluated by integrating second-order statistics of point runoff volume. The second-order statistics of rainfall duration and intensity, required for evaluation of those of point runoff volume, are obtained from mean and variance of areal rainfall volume under fractional coverage considerations. Hourly NEXRAD rainfall data were used to estimate rainfall-related parameters and statistics, and the two contrasting climate-soil systems were used to specify soil- and soil moisture-related parameters. The results show that 1) climatological variability of areal runoff volume, as represented by coefficient of variation, is significantly greater in semi-arid to arid climates, particularly over catchment scales exceeding 1,000 km^2, than that of areal rainfall volume, and

2) the scale-variability relationship of areal runoff volume has a more pronounced peak of maximum variability than that of areal rainfall volume. These results imply that, for large-scale hydrological models, extreme care must be taken in choosing grid size and integration time-step, and in assessing uncertainties associated with model input and output such as areal rainfall and areal runoff volumes.

INTRODUCTION

In this paper, we evaluate the relationship between catchment scale and climatological variability of areal surface-runoff volume. This is an important consideration in large-scale hydrological models in choosing grid size and integration time-step and in assessing variabilities or uncertainties associated with model input and output such as areal rainfall and runoff volumes. Because these models employ large catchment or grid scales, not only rainfall variability (i.e., inner variability and intermittency) but also heterogeneities of soil and soil moisture variables must be taken into account in evaluating the relationship.

In Seo and Smith (1994), the relationship between catchment scale and climatological variability of mean areal rainfall was investigated. It was shown that, under second-order homogeneity assumptions, variability of mean areal rainfall, as measured by its coefficient of variation (abbreviated as CV), is a function of mean fractional coverage, conditional (on occurrence of rain) CV of point rainfall, and two correlation-scales associated with inner variability and intermittency of point rainfall. Empirical analyses using hourly NEXRAD rainfall data were then performed to verify the scale-variability relationships of mean areal rainfall and fractional coverage.

One approach to evaluate the relationship between catchment scale and climatological variability of areal surface-runoff volume is to perform a simulation experiment using a space-time rainfall model and a rainfall-runoff model. To obtain climatologically representative statistics, however, such an experiment would be extremely costly. Also, the authors are not aware of any space-time rainfall models that can explicitly account for intermittency of rainfall. Instead, we have taken the following approach: 1) make simplifying assumptions on the structure of space-time rainfall fields, so that it is compatible with the time compression approximation (Reeves and Miller 1975, Milly and Eagleson 1988) used in describing point runoff volume following infiltration excess (Eagleson 1978e), 2) develop second-order statistics of point runoff volume in terms of rainfall, soil and soil moisture parameters, and 3) spatially integrate them to evaluate climatological mean and variance of areal runoff volume. Although runoff due to saturation excess is not explicitly considered in this work, its effect on the scale-variability relationship may be inferred from the limiting considerations of no infiltration (i.e., saturation excess only) and no saturation excess (i.e., infiltration only).

This paper is organized as follows. In the next section, we present expressions for climatological mean and variance of areal rainfall volume. They are 'true' statistics in that both inner variability and intermittency of rainfall are accounted for. In the following section, we derive analogous expressions for 'approximate' statistics under the assumption of no within-storm variability. Given the two sets of expressions, parameter estimation via method of moments allows preservation of total variability of areal rainfall volume in the approximate representation. In the following section, expressions for climatological mean and variance of areal runoff volume are derived, which may be evaluated via Monte-Carlo integration. In the following two sections, we describe how space-time correlation structures for inner variability and intermittency of rainfall are estimated, and how soil and soil moisture fields are specified. In the following two sections, results and conclusions are given.

MOMENTS OF AREAL RAINFALL VOLUME

Here we present expressions for climatological mean and variance of areal rainfall volume, in which both inner variability and intermittency are accounted for. Let us define the areal rainfall volume, V_a, as follows:

$$V_a \equiv \int_0^T \int_A R(u,t) \, du dt \tag{1}$$

where $R(u,t)$ is the rain rate at location u at time t, A denotes the catchment of area $\|A\|$ in km^2, and T is the time period in hours during which rainfall occurs continuously within A (i.e., fractional coverage of rainfall over A is always positive for $t \in [0,T]$). For stationary storms, T is bounded only by their lifetime. For an advecting storm, it is bounded by catchment and storm sizes as well. In the following developments, dependence of T on these variables is not shown for notational brevity.

In Seo and Smith (1994), expressions for the climatological mean and variance of mean areal rainfall were derived. Expressions for those of areal rainfall volume may be obtained in an analogous manner under the additional assumption that both conditional and indicator rainfall processes, $R(u,t)$ and $I(u,t)$, respectively, are stationary in $[0,T]$. The indicator random variable, $I(R(u,t);0)$, $u \in A$ and $t \in [0,T]$, has been defined in Eq.(3) of Seo and Smith (1994). The resulting expressions for the climatological mean and variance of V_a are:

$$E[V_a \mid T]$$
$$= \| A \| \; T \; m_R \; m_Z(A) \tag{2}$$

$$\mathrm{Var}[V_a \mid T]$$
$$= \sigma_R^2 \; m_Z(A) \; (1-m_Z(A)) \int_0^T\!\int_0^T\!\int_A\!\int_A \rho_R(R(u,s),R(v,t);L_{SR},L_{TR})$$
$$\cdot \rho_I(I(u,s),I(v,t);L_{SI},L_{TI}) \; dudvdtds$$
$$+ \sigma_R^2 \; m^2_{Fc}(A) \int_0^T\!\int_0^T\!\int_A\!\int_A \rho_R(R(u,s),R(v,t);L_{SR},L_{TR}) \; dudvdtds$$
$$+ m_R^2 \; m_Z(A) \; (1-m_Z(A))$$
$$\cdot \int_0^T\!\int_0^T\!\int_A\!\int_A \rho_I(I(u,s),I(v,t);L_{SI},L_{TI}) \; dudvdtds \tag{3}$$

where m_R and σ_R^2 are the conditional mean and variance of point rainfall, respectively, $m_Z(A)$ is the mean fractional coverage of rainfall over A, $\rho_R(R(u,s),R(v,t);L_{SR},L_{TR})$ is the conditional space-time correlation function of point rainfall with spatial and temporal correlation-scale parameters L_{SR} and L_{TR}, respectively, and $\rho_I(I(u,s),I(v,t);L_{SI},L_{TI})$ is the space-time indicator correlation function with analogously defined parameters L_{SI} and L_{TI}. Note that, following the stationarity assumption, the time-dependence notations in Eqs.(7) and (12) of Seo and Smith (1994) have been dropped in the above.

MOMENTS OF AREAL RAINFALL VOLUME ASSUMING NO WITHIN-STORM VARIABILITY

Here we derive new expressions for the climatological mean and variance of areal rainfall volume under the assumption that rain rate is constant in space and time within a storm (but varies from storm to storm). This enables utilization of the time compression approximation in describing runoff production via infiltration excess. We approximate the rainfall volume at location u from a storm, V(u), as follows:

$$V(u) = I_r\ T_r(u) \qquad (4)$$

where I_r is the rain rate in mm/hr, constant in space and time within a storm, and $T_r(u)$ is the rainfall duration in hours at location u. Then the areal rainfall volume, V_a, is given by:

$$V_a = I_r \int_A T_r(u)\ du \qquad (5)$$

Assuming that location u, $u \in A$, receives rainfall from a storm in the form of a single continuous pulse, we may write $T_r(u)$ in Eq.(5) as:

$$T_r(u) = \int_0^T I(R(u,t);0)\ dt \qquad (6)$$

Assuming independence between I_r and $T_r(u)$ (Eagleson 1978b,e), we have:

$$E[V_a \,|\, T] = m_{Ir} \int_A E[T_r(u) \,|\, T]\, du \tag{7}$$

$$\begin{aligned} Var[V_a \,|\, T] = &(\sigma_{Ir}^2 + m_{Ir}^2) \int_A \int_A Cov[T_r(u), T_r(v) \,|\, T]\, dudv \\ &- \sigma_{Ir}^2 \int_A \int_A E[T_r(u) \,|\, T]\, E[T_r(v) \,|\, T]\, dudv \end{aligned} \tag{8}$$

where m_{Ir} and σ_{Ir}^2 are the climatological mean and variance of I_r, respectively. In the above, $E[T_r(u) \,|\, T]$ and $Cov[T_r(u),T_r(v) \,|\, T]$ are written as:

$$E[T_r(u) \,|\, T] = T\, m_Z(A) \tag{9}$$

$$\begin{aligned} &Cov[T_r(u),T_r(v) \,|\, T] \\ &= m_Z(A)(1-m_Z(A)) \int_0^T \int_0^T \rho(I(u,s),I(v,t);L_{SI},L_{TI})\, dtds \end{aligned} \tag{10}$$

In writing Eqs.(9) and (10), we have used Eqs.(5) and (11) of Seo and Smith (1994), respectively. Finally, we have for the climatological mean and variance of areal rainfall volume:

$$E[V_a \,|\, T] = \|A\|\, T\, m_{Ir}\, m_Z(A) \tag{11}$$

$$\begin{aligned} &Var[V_a \,|\, T] \\ &= \sigma_{Ir}^2\, A^2\, T^2\, m_Z^2(A) \\ &+ (\sigma_{Ir}^2 + m_{Ir}^2)\, m_Z(A)(1-m_Z(A)) \cdot \int_0^T \int_0^T \int_A \int_A \rho(I(u,s),I(v,t);L_{SI},L_{TI})\, dudvdtds \end{aligned} \tag{12}$$

We may verify the validity of the above expressions by comparing them against Eqs.(2) and (3): Eq.(11) becomes identical to Eq.(2) if $m_{Ir}=m_R$, and Eq.(12) becomes identical to Eq.(3) if $L_{SR}\rightarrow\infty$ and $L_{TR}\rightarrow\infty$ in the latter (i.e., constant rain rate in space and time within a storm).

Because within-storm variability has been neglected in arriving at Eq.(12), a 'moment-matching' is necessary to reproduce the total variability in Eq.(3). For that, we replace m_{Ir} with m_R in Eq.(12), equate it with Eq.(3), and solve for σ_{Ir}^2 for each A. Physically, this amounts to substituting storm-to-storm variability of spatially averaged, constant intensity of rainfall for within-storm variability of point rainfall intensity. The above 'correction' also preserves the percentage contribution of intermittency to total variability because Eqs.(3) and (12) share the same intermittency term (i.e., the last term in Eq.(3)).

MOMENTS OF AREAL RUNOFF VOLUME FOLLOWING INFILTRATION EXCESS

We are now in a position to seek expressions for the climatological mean and variance of areal runoff volume following infiltration excess. Based on the Philip's equation (1957) and the time compression approximation, Eagleson (1978e) has shown that, neglecting surface retention, the point runoff volume at u, $R_s(u)$, may be approximated by the following:

$$R_s(u) = \begin{cases} (i_r-A_o(u))t_r(u) - S_i(u)(t_r(u)/2)^{1/2} \\ \qquad \text{for } t_r(u)>S_i^2(u)/\{2(i_r-A_o(u))\},\ i_r \geq A_o(u) \\ 0 \qquad \text{otherwise} \end{cases} \tag{13}$$

In the above, $A_o(u)$ and $S_i(u)$ are the infiltration sorptivity in cm/hr$^{1/2}$ and the gravitational infiltration rate as modified by capillary rise from water table (equal to infilration rate at large time) in cm/hr, respectively. They are given by (Eagleson 1978e):

$$A_o(u) = \tfrac{1}{2}\, K(u)\, (1+s_o(u)^{c(u)}) - w(u) \tag{14}$$

$$S_i(u) = 2(1-s_o(u)) \cdot \{5n(u)K(u)\Psi 1(u)\phi_i(d(u),s_o(u))/(3m(u)\pi)\}^{1/2} \tag{15}$$

where $K(u)$ is the saturated hydraulic conductivity in cm/hr at location u, $u \in A$, $s_o(u)$ is the time-averaged soil moisture saturation in the surface boundary layer, $c(u)$ is the pore connectivity index, $w(u)$ is the capillary rise from the groundwater table in cm/hr (ignored in this work), $n(u)$ is the porosity, $\Psi(u)$ is the saturated soil matrix potential in cm, $\phi_i(,)$ is the infiltration diffusivity function, $d(u)$ is the diffusivity index, and $m(u)$ is the pore size-distribution index. Given the soil and the climatic parameters, $s_o(u)$ can be obtained using the long-term water budget equation of Eagleson (1978f).

In the following developments, it is to be understood that all the statistical moments are conditional on T although they are not explicitly shown as such for notational brevity. With Eq.(13), expressions for the first two moments of areal runoff volume are written as:

$$E[\textstyle\int_A R_s(u)\, du] = \int_A \int^{\infty}_{Ao(u)} E[R_s(u) \,|\, I_r = i_r]\, f_{Ir}(i_r)\, di_r\, du \tag{16}$$

where

$$E[R_s(u) \,|\, I_r = i_r] = \textstyle\int^{\infty}_{d(u)} R_{sc}(u)\, f_{Tr(u)\,|\,Ir}(t_r(u) \,|\, i_r)\, dt_r(u) \tag{17}$$

$E[\int_A\int_A R_s(u)\ R_s(v)\ du\ dv]$

$$= \int_A\int_A\int^{\infty}_{Ax} E[R_s(u)\ R_s(v) \mid Ir=i_r]\ f_{Ir}(i_r)\ di_r\ du\ dv \tag{18}$$

where

$E[R_s(u)\ R_s(v) \mid Ir=i_r]$

$$= \int^{\infty}_{d(v)}\int^{\infty}_{d(u)} R_{sc}(u)\ R_{sc}(v) \cdot f_{Tr(u),Tr(v) \mid Ir}(t_r(u),t_r(v) \mid i_r)\ dt_r(u)dt_r(v) \tag{19}$$

In the above, we have defined $d(u) \equiv Si^2(u)/[2\{i_r-A_o(u)\}]$, $A_x \equiv \max\{A_o(u),A_o(v)\}$ and $R_{sc}(u) \equiv \{i_r-A_o(u)\}t_r(u)-S_i(u)\{t_r(u)/2\}^{1/2}$.

In this work, we assumed that the probability density functions of $T_r()$ and Ir are bivariate lognormal and two-parameter gamma, respectively. Then, for Eq.(17), we have:

$E[R_s(u) \mid I_r=i_r]$

$= \frac{1}{2}\ \{\ (i_r-A_o(u))\ \exp(m+0.5\sigma^2)\ \mathrm{erfc}((b(u)-\sigma)/\sqrt{2})$

$$-\ S_i(u)/\sqrt{2}\ \exp(0.5m+\tfrac{1}{8}\sigma^2)\ \mathrm{erfc}((b(u)-0.5\sigma)/\sqrt{2})\} \tag{20}$$

where m and σ are the mean and the standard deviation of ln(Tr(u)), respectively, and $b(u) \equiv \{\ln(S_i^2(u)/[2\{i_r-A_o(u)\}])-m\}/\sigma$. If $u \neq v$, we have for Eq.(19):

$2\sqrt{2\pi}\ E[R_s(u)\ R_s(v) \mid Ir=i_r]$

$= (i_r-A_o(v))(i_r-A_o(u))\ \exp(2m+\beta\sigma^2)$

$\cdot\int^{\infty}_{b(v)} \mathrm{erfc}[0.5\{b(u)-\rho z\}/\sqrt{\beta}-\sigma\sqrt{\beta}]\ \exp\{-0.5z^2+(1+\rho)\sigma z\}\ dz$

$-\ (i_r-A_o(v))\ S_i(u)/\sqrt{2}\ \exp(1.5m+0.25\beta\sigma^2)$

$$\cdot\int^{\infty}_{b(v)} \text{erfc}[0.5\{b(u)-\rho z\}/\sqrt{\beta}-0.5\sigma\sqrt{\beta}]\ \exp\{-0.5z^2+(1+0.5\rho)\sigma z\}\ dz$$

$$- S_i(v)/\sqrt{2}(i_r-A_o(v))\ \exp(1.5m+\beta\sigma^2)$$

$$\cdot\int^{\infty}_{b(v)} \text{erfc}[0.5\{b(u)-\rho z\}/\sqrt{\beta}-\sigma\sqrt{\beta}]\ \exp\{-0.5z^2+(0.5+\rho)\sigma z\}\ dz$$

$$+ \tfrac{1}{2}\ S_i(v)S_i(u)\ \exp(m+0.25\beta\sigma^2)$$

$$\cdot\int^{\infty}_{b(v)} \text{erfc}[0.5\{b(u)-\rho z\}/\sqrt{\beta}-0.5\sigma\sqrt{\beta}]\ \exp\{-0.5z^2+0.5(1+\rho)\sigma z\}\ dz \qquad (21)$$

where ρ is the spatial correlation coefficient between $\ln(T_r(u))$ and $\ln(T_r(v))$, $\beta\equiv(1-\rho^2)/2$, and erfc[$\cdot$] denotes the complementary error function. If u=v, we have for Eq.(19):

$$2\ E[R_s^2(u)\,|\,I_r=i_r]$$

$$= (i_r-A_o(u))^2\ \exp(2(m+\sigma^2))\ \text{erfc}((b(u)-2\sigma)/\sqrt{2})$$

$$- \sqrt{2}(i_r-A_o(u))S_i(u)\ \exp(1.5m+9\sigma^2/8)\ \text{erfc}((b(u)-1.5\sigma)/\sqrt{2})$$

$$+ \tfrac{1}{2}\ S_i^2(u)\ \exp(m+0.5\sigma^2)\ \text{erfc}((b(u)-\sigma)/\sqrt{2}) \qquad (22)$$

To evaluate the above expressions, we need to specify the second-order statistics of $T_r()$ and I_r, and the infiltration sorptivity and the gravitational infiltration rate, $A_o(u)$ and $S_i(u)$, $u\in A$. Also, to specify σ_{Ir}^2's that preserve the total variability of areal rainfall volume, we need to specify m_R, σ_R^2, $m_Z(A)$, L_{SI}, L_{SR}, L_{TI} and L_{TR} in Eq.(3). Estimation of m_R, σ_R^2, $m_Z(A)$, L_{SI} and L_{SR} has been described in Seo and Smith (1994). In the following sections, we describe how the remaining parameters are specified.

SPECIFICATION OF RAINFALL PARAMETERS

To specify the second-order statistics of $T_r()$ and I_r, one has to estimate the conditional space-time correlation function of point rainfall, $\rho_R(R(u,s),R(v,t);L_{SR},L_{TR})$, and the indicator space-time correlation function, $\rho_I(I(u,s),I(v,t);L_{SI},L_{TI})$, for evaluation of Eqs.(3), (10) and (12). To do so, we have made the following assumptions: 1) the space-time correlation functions are separable in Lagrangian domain, and 2) spatial correlation functions are Gaussian with no nugget effect (Journel and Huijbregts 1978, p165) , and 3) Lagrangian autocorrelation functions are exponential. The first assumption implies that, e.g.:

$$\rho(R(u,s),R(v,t);L_{SR},L_{TR}) = \rho_s(|v-u+U(s-t)|;L_{SR})\ \rho_L(|s-t|;L_{TR}) \tag{23}$$

where $\rho_s(|\cdot|;L_{SR})$ and $\rho_L(|\cdot|;L_{TR})$ are the spatial correlation and the Lagrangian autocorrelation functions of point rainfall, respectively, U is the advection vector, and $|\cdot|$ denotes the Euclidean distance. The second assumption implies that:

$$\begin{aligned}&\rho_s(|v-u+U(s-t)|;L_{SR})\\&= \exp(-[\{v_1-u_1+U_1(s-t)\}^2+\{v_2-u_2+U_2(s-t)\}^2]/L_{SR}^2)\end{aligned} \tag{24}$$

where (u_1,u_2) and (v_1,v_2) are the x- and the y-coordinates of the locations u and v, respectively, and U_1 and U_2 are the x- and the y-components of the climatological mean advection velocity, respectively. The third assumption implies that:

$$\rho_L(|s-t|;L_{TR}) = \exp(-|s-t|/L_{TR}) \tag{25}$$

The reason for the first two assumptions is primarily computational: they reduce sextuple integrals of the form, $\int_0^T\int_0^T\int_A\int_A \rho_R(R(u,s),R(v,t);L_{SR},L_{TR})$ dudvdtds in Eq.(3), to single integrals (see Appendix). Evaluation of $Cov[T_r(u),T_r(v)|T]$ in Eq.(10), on the other hand, requires numerical integration of a double integral because $\rho_s(|v-u+U(s-t)|;L_{SR})$ is not symmetric in (s-t). The choice of Lagrangian description stems from the high climatological mean advection speed observed in the Southern Plains (see Table 2), which makes parsimonious parameterization of space-time correlation structures impractical. The third assumption is used to minimize the data requirement and to circumvent difficulties associated with estimating Lagrangian autocorrelation at larger lags.

L_{TI} and L_{TR} are estimated by computing lag-1 (hr) auto-correlation coefficients as functions of spatial displacement. In general, the two mean advection velocities that yield the maximum lag-1 (hr) Lagrangian conditional and indicator autocorrelation coefficients are not identical. In such cases, conditional autocorrelation was taken to be a better measure for estimation of advection velocity, and L_{TI} was specified accordingly. Table 1 summarizes climatological advection velocities and lag-1 (hr) temporal correlation coefficients as estimated from hourly NEXRAD rainfall data. The reader is referred to Fig. 1 of Seo and Smith (1994) for the area map.

SPECIFICATION OF SOIL AND SOIL MOISTURE PARAMETERS

Infiltration sorptivity and gravitational infiltration rate, $A_o(u)$ and $S_i(u)$, $u \in A$, were specified as follows. First, based on Milly and Eagleson (1987), fields of porosity, n(u), permeability, k(u), and pore size distribution index, m(u), were generated over the largest catchment area. The maximum domain of A considered in this work was a 256x256 km^2 area, represented by a 64x64 grid. The mesh size of $4x4km^2$ corresponds to that of radar rainfall data used in this work. Then, the time-averaged soil moisture saturation in the surface boundary layer, $s_o(u)$, was computed for all $u \in A$, using the long-term water budget equation of Eagleson (1978f).

In generating the soil parameters, we assumed that n(u), ln[k(u)] and ln[m(u)] are normally distributed, mutually independent, and spatially white-noise random processes (see Milly and Eagleson, 1987, for justification). Two types of soil, clay loam and silt loam, were considered. Given the standard deviation of ln(k(u)), $\sigma_{\ln(k(u))}$, those of n(u) and ln(m(u)) were specified according to $\sigma_{n(u)} = 0.05\ \sigma_{\ln(k(u))}$ and $\sigma_{\ln(m(u))} = 0.4\ \sigma_{\ln(k(u))}$ (Milly and Eagleson 1987). In solving for $s_o(u)$, $u \in A$, the two types of climate-soil systems of Eagleson (1978f) were considered; humid climate-clay loam and semi-arid climate-silt loam, as represented by Clinton, MA, and Santa Paula, CA, respectively. The reader is referred to Eagleson (1978f) or Milly and Eagleson (1987) for climatic parameters. Tables 2 and 3 summarize the sample statistics of soil and soil moisture parameters used in this work: they are based on $\sigma_{\ln(k(u))}=1$ and 1.4, respectively.

RESULTS

Before presenting the scale-variability relationship of areal runoff volume, it is informative to examine the effect of the parameters in the space-time correlation functions on the scale-variability relationship of areal rainfall volume. Figs. 1 and 2 show the effect of storm advection at FTG for various values of T. In producing the figures, we have used Eqs.(2), (3), (23), (24), (25), and rainfall statistics at FTG. In each figure, the upper- and the lowermost curves correspond to T=0.5 hr and T=3.5 hrs, respectively, with an increment of 0.5 hr. A 'dampening' effect of advection on the variability of areal rainfall volume is evident. Figs. 3 through 7 show sensitivities of the scale-variability relationship of areal rainfall volume on L_{SR}, L_{SI}, L_{TR}, L_{TI} and CV_R ($\equiv\sigma_R/m_R$), respectively. In each figure, the unconnected markers represent the reference relationship based on observed parameter values at LZK, whereas the rest of the curves are obtained by varying the value of the particular parameter in question. The increment is 10 km for L_{SR} and L_{SI}, 0.1 for $\rho_L(1\text{ (hr)};L_{TR})$ and $\rho_L(1\text{ (hr)};L_{TI})$, and 0.5 for CV_R. The figures illustrate that each parameter has a varying and unique effect on the scale-variability relationship of areal rainfall volume.

Figs. 8 through 13 show scale-variability relationships of areal runoff volume at various values of T, as obtained from Eqs.(16) and (18). They are based on rainfall statistics at ICT and LZK. Similar characteristics are observed when rainfall statistics at other sites are used. In each figure, the curves marked by 'N', 'H', and 'S' represent those under no infiltration, for the humid climate-clay loam system, and for the semi-arid climate-silt loam system, respectively (see Table 2 for the sample statistics of soil and soil moisture parameters). The figures may be summarized as follows. Climatological variability of areal runoff volume due to infiltration excess is greater than that of areal rainfall volume for both climate-soil systems. It is significantly greater for the semi-arid climate-silt loam system, particularly over catchment scales exceeding 1,000 km^2, than for the humid climate-clay loam system. Scale-variability relationships of areal runoff volume exhibit more pronounced peaks of maximum variability than those of areal rainfall volume. The catchment scale at which the peak variability of areal runoff volume occurs is larger than the catchment scale associated with that of areal rainfall volume.

Figs. 14 and 15 show examples of climatological mean and standard deviation of spatially averaged areal runoff volume (i.e., areal runoff volume divided by catchment area) as obtained from Eqs.(16) and (18), respectively. The ratio of the standard deviation to the mean produces Fig. 11. The figures indicate that infiltration reduces the absolute magnitude of climatological variability of areal runoff volume (as expressed by its standard deviation) over duration of 0.5 hr (or longer, not shown) for both climate-soil systems. However, the relative reduction in the standard deviation of areal runoff volume is smaller, particularly for the semi-arid climate-silt loam system, than that in the mean, hence resulting in amplification of CV.

Because the 'S' and the 'H' curves assume infiltration everywhere, whereas the 'N' curves assume no infiltration anywhere, each pair of 'S'-'N' and 'H'-'N' curves may be considered to form bounds for the scale-variability relationship of areal runoff volume due to infiltration and saturation excesses. Quantitative assessment of the effect of saturation excess was beyond the scope of this work. We only note here that, given a distributed hydrological model capable of delineating variable source areas, such an assessment can readily be accommodated in our formulation.

We now briefly turn our attention to the effect of increased spatial variability in soil parameters. The scale-variability relationships of areal runoff volume presented thus far have all been based on $\sigma_{\ln(k(u))}$=1.0. Fig. 16 shows an example of the scale-variability relationships based on $\sigma_{\ln(k(u))}$=1.4. The curves marked by 'h' and 's' correspond to the humid climate-clay loam and the semi-arid climate-silt loam systems, respectively (see Table 3 for the sample statistics of soil and soil moisture parameters). Also shown in the figure are those based on $\sigma_{\ln(k(u))}$=1.0 (marked by 'H' and 'S'). Similarly, Figs. 17 and 18 show examples of the mean and the standard deviation of spatially averaged areal runoff volume, respectively, based on $\sigma_{\ln(k(u))}$=1.0 and 1.4. Note that the increase in $\sigma_{\ln(k(u))}$ (and subsequent changes in $\sigma_{n(u)}$ and $\sigma_{\ln(m(u))}$) also increases not only the standard deviation of runoff volume but also the mean runoff volume. It is hence difficult to ascertain the pure effect of increased variability in soil parameters on the scale-variability relationship of areal runoff volume. Qualitatively, we may consider that an increase in spatial variability of soil parameters is equivalent to an increase in CV_R and/or a decrease in L_{SR}. The net effect, however, is not clear because the former would sharpen the scale-variability relationship (see Fig. 7) whereas the latter would flatten it (see Fig. 3). In Figs. 17 and 18, the increase in $\sigma_{\ln(k(u))}$ does not seem to affect the standard deviation of runoff volume as much as it does its mean: it is seen to suggest that spatial variability of soil parameters may not be as important a factor as their mean values in shaping the scale-variability relationship of areal runoff volume.

CONCLUSIONS

The relationship between catchment scale and climatological variability of areal surface-runoff volume is evaluated. The measure used to quantify the variability is coefficient of variation. Given the point description of runoff volume following infiltration excess under the time compression approximation (Eagleson 1978e), climatological mean and variance of areal runoff volume over heterogeneous soil are evaluated by integrating second-order statistics of point runoff volume. The second-order statistics of rainfall duration and intensity, required for evaluation of those of point runoff volume, are obtained from climatological mean and variance of areal rainfall volume under fractional coverage considerations. The two contrasting climate-soil systems of Eagleson (1978f) were assumed in evaluating the scale-variability relationship of areal runoff volume. Soil and soil moisture fields were specified based on Milly and Eagleson (1987) and the one-dimensional long-term water balance equation of Eagleson (1978f), respectively. Hourly NEXRAD rainfall data over the Southern Plains, U.S.A., were used to estimate rainfall-related statistics, including the space-time correlation functions for inner variability and intermittency of point rainfall.

The results show that 1) climatological variability of areal runoff volume is greater than that of areal rainfall volume, 2) it is significantly greater for the semi-arid climate-silt loam system, particularly over catchment scales greater than 1,000 km^2, than for the humid climate-clay loam system, 3) scale-variability relationships of areal runoff volume have more pronounced peaks of maximum variability than those of areal rainfall volume, and 4) the catchment scale at which the peak variability of areal runoff volume occurs is greater than the catchment scale associated with that of areal rainfall volume. They point out that extreme care must be taken, particularly for applications in semi-arid to arid climates, in choosing grid size and integration time-step in large-scale hydrological models, and in assessing uncertainties associated with model input and output such as areal rainfall and areal runoff volumes. Because of the time compression approximation used in describing infiltration, within-storm variability of rainfall could not be explicitly taken into account in this work. Its effect, and the effect of spatial variability of soil parameters, need further study.

ACKNOWLEDGEMENTS

This research was supported in part by NOAA (Climate and Global Change Research Program, Grant NA36690419). This support is gratefully acknowledged.

APPENDIX

Here we evaluate the sextuple integral of the following form:

$$J = \int_0^T \int_0^T \int_A \int_A \rho(R(u,s),R(v,t);L_{SR},L_{TR})\ dudvdtds \qquad \text{(A1)}$$

Assuming that the space-time correlation structure is separable in Lagrangian domain, we may write:

$$J = \int_0^T \int_0^T \int_A \int_A \rho_s(|v-u+U(s-t)|, L_s)\ dudv\ \rho_L(|s-t|, L_t)\ dsdt \tag{A2}$$

We first evaluate the inner integral by rewriting it as:

$$J_i(s-t) = \int_A \int_A \rho_s(|v-u+U(s-t)|, L_s)\ dudv \tag{A3a}$$

$$= \int_A \int_{A+h} \rho_s(|v-u|, L_s)\ dudv \tag{A3b}$$

where h=U(s-t).

Using the Cauchy-Gauss method for non-overlapping areas of integration (Journel and Huijbregts 1978, p99), we have:

$$J_i(s-t) = \int_{-l2}^{l2} \int_{-l1}^{l1} \rho_s(|u+h|, L_s)(l_1-|u_1|)(l_2-|u_2|)\ du_1 du_2 \tag{A4a}$$

$$\begin{aligned} = \int_0^{l2} \int_0^{l1} \ \{ & \rho_s(\{(u_1+h_1)^2+(u_2+h_2)^2\}^{1/2}, L_s) \\ & + \rho_s(\{(u_1-h_1)^2+(u_2-h_2)^2\}^{1/2}, L_s) \\ & + \rho_s(\{(u_1-h_1)^2+(u_2+h_2)^2\}^{1/2}, L_s) \\ & + \rho_s(\{(u_1+h_1)^2+(u_2-h_2)^2\}^{1/2}, L_s)\ \}\ (l_1-u_1)(l_2-u_2)\ du_1 du_2 \end{aligned} \tag{A4b}$$

The first term in (A4b), e.g., is given by:

$$\gamma(h_1,h_2) = \int_{-l2}^{l2} \int_{-l1}^{l1} \rho_s(\{(u_1+h_1)^2+(u_2+h_2)^2\}^{1/2}, L_s)\ (l_1-u_1)(l_2-u_2)\ du_1 du_2 \tag{A5a}$$

$$= \tfrac{1}{4}\ (l_1+h_1)\ (l_2+h_2)\ L_s^2\ \pi\ \{\mathrm{erfc}(h_1/L_s) - \mathrm{erfc}((l_1+h_1)/L_s)\}\ \{\mathrm{erfc}(h_2/L_s) - \mathrm{erfc}((l_2+h_2)/L_s)\}$$

$$- \tfrac{1}{4}\,(l_1+h_1)\, L_s^3\, \pi^{1/2}\, \{erfc(h_1/L_s) - erfc((l_1+h_1)/L_s)\}\, \{exp(-h_2^2/L_s^2 - exp(-(l_2+h_2)^2/L_s^2)\}$$

$$- \tfrac{1}{4}\,(l_2+h_2)\, L_s^3\, \pi^{1/2}\, \{erfc(h_2/L_s) - erfc((l_2+h_2)/L_s)\}\, \{exp(-h_1^2/L_s^2 - exp(-(l_1+h_1)^2/L_s^2)\}$$

$$+ \tfrac{1}{4}\, L_s^4\, \{exp(-h_1^2/L_s^2 - exp(-(l_1+h_1)^2/L_s^2)\}\, \{exp(-h_2^2/L_s^2 - exp((l_2+h_2)^2/L_s^2)\} \qquad (A5b)$$

where erfc(·) denotes the complementary error function.

(A2) is thus reduced to:

$$J = \int_0^T\!\int_0^T J_i(s-t)\, \rho_L(|s-t|, L_t)\, dtds \qquad (A6)$$

where

$$J_i(s-t) = \gamma(h_1,h_2) + \gamma(-h_1,-h_2) + \gamma(-h_1,h_2) + \gamma(h_1,-h_2) \qquad (A7)$$

By once again applying the Cauchy-Gauss method on (A6), (A1) is finally reduced to the following single integral:

$$J = 2\int_0^T J_i(u_1)\, \rho_L(u_1,L_t)\, (T - u_1)\, du_1 \qquad (A8)$$

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Table 1 Climatological mean advection velocities and lag-1 (hr) Lagrangian autocorrelation coefficients

Site	Advection Speed in km/hr (Downwind Direction)	$\rho_L(1\ (hr);L_{TR})$	$\rho_L(1\ (hr);L_{TI})$
FTG	47 (SE)	0.64	0.23
GLD	8 (E)	0.74	0.38
AMA	23 (NE)	0.68	0.43
DDC	40 (E)	0.59	0.39
FDR	32 (E)	0.58	0.45
ICT	18 (SE)	0.76	0.42
TLX	25 (SE)	0.62	0.47
INX	32 (E)	0.64	0.52
LZK	32 (E)	0.82	0.52

Table 2 Sample statistics of soil and soil moisture parameters for $\sigma_{\ln(k(u))}=1$

	Humid Climate - Clay Loam ('H')		Semi-Arid Climate - Silt Loam ('S')	
	mean	CV	mean	CV
$n(u)$	0.356	0.13	0.356	0.13
$m(u)$	0.297	0.36	0.693	0.36
$K(u)^a$	0.088	1.23	0.376	1.23
$s_o(u)$	0.719	0.08	0.486	0.16
$A_o(u)^a$	0.054	1.20	1.190	1.21
$S_i(u)^b$	0.316	0.55	2.270	0.57

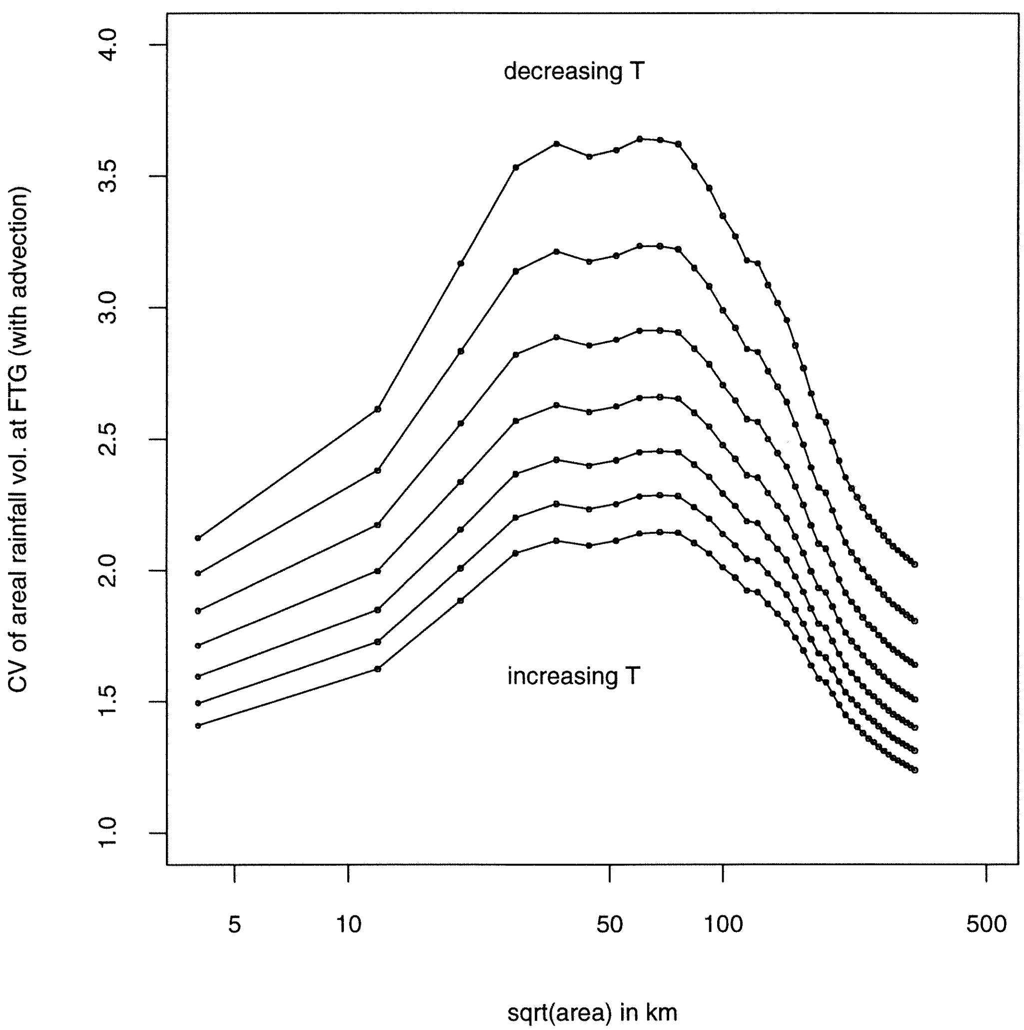
^{a} in cm/hr
^{b} in cm/hr$^{1/2}$

Table 3 Same as Table 2, but for $\sigma_{\ln(k(u))}=1.4$

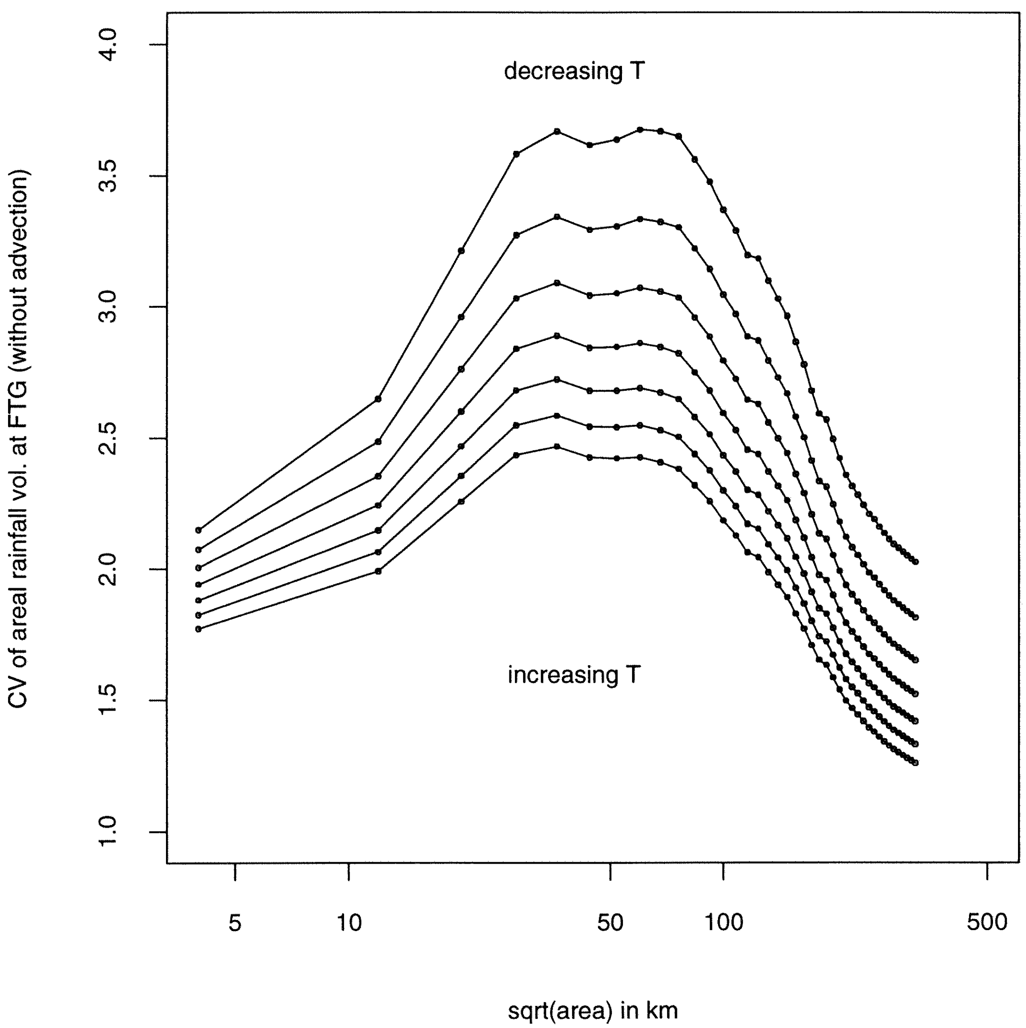
	Humid Climate - Clay Loam ('h')		Semi-Arid Climate - Silt Loam ('s')	
	mean	CV	mean	CV
$n(u)$	0.358	0.18	0.358	0.18
$m(u)$	0.298	0.54	0.696	0.54
$K(u)$ [a]	0.085	2.42	0.365	2.42
$s_o(u)$	0.722	0.10	0.520	0.19
$A_o(u)$ [a]	0.044	2.36	0.184	2.40
$S_i(u)$ [b]	0.280	0.87	1.997	0.88

[a] in cm/hr
[b] in $cm/hr^{1/2}$

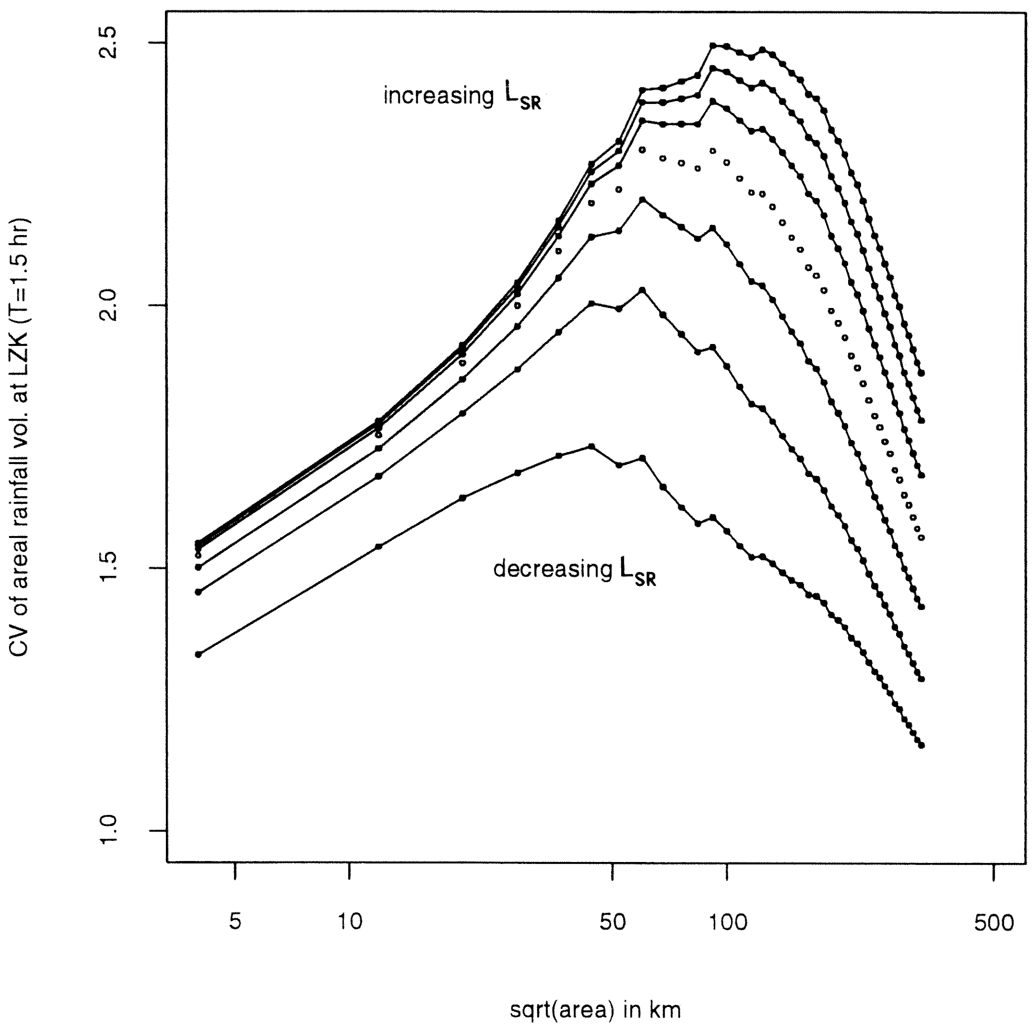
List of Figures and Their Captions



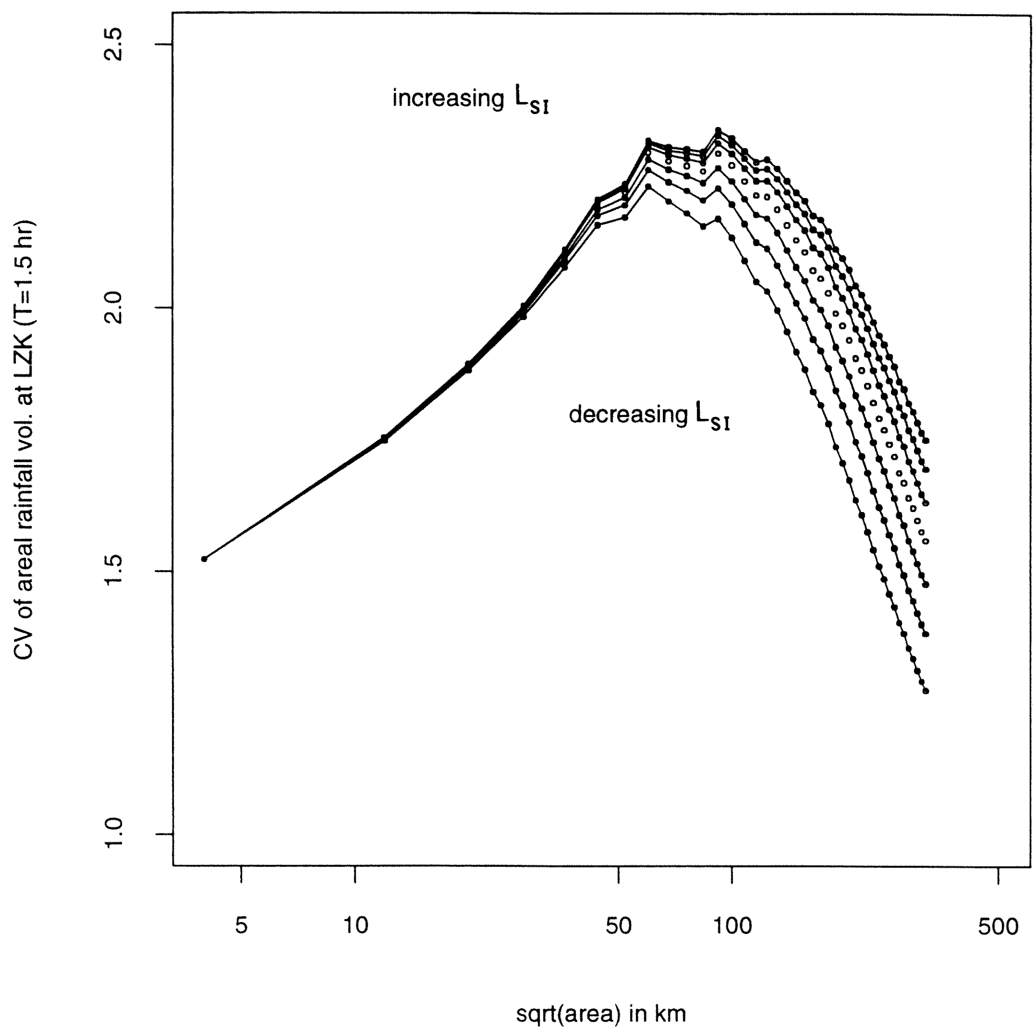
decreasing T
increasing T
CV of areal rainfall vol. at FTG (with advection)
1.0
1.5
2.0
2.5
3.0
3.5
4.0
5
10
50
100
500
sqrt(area) in km



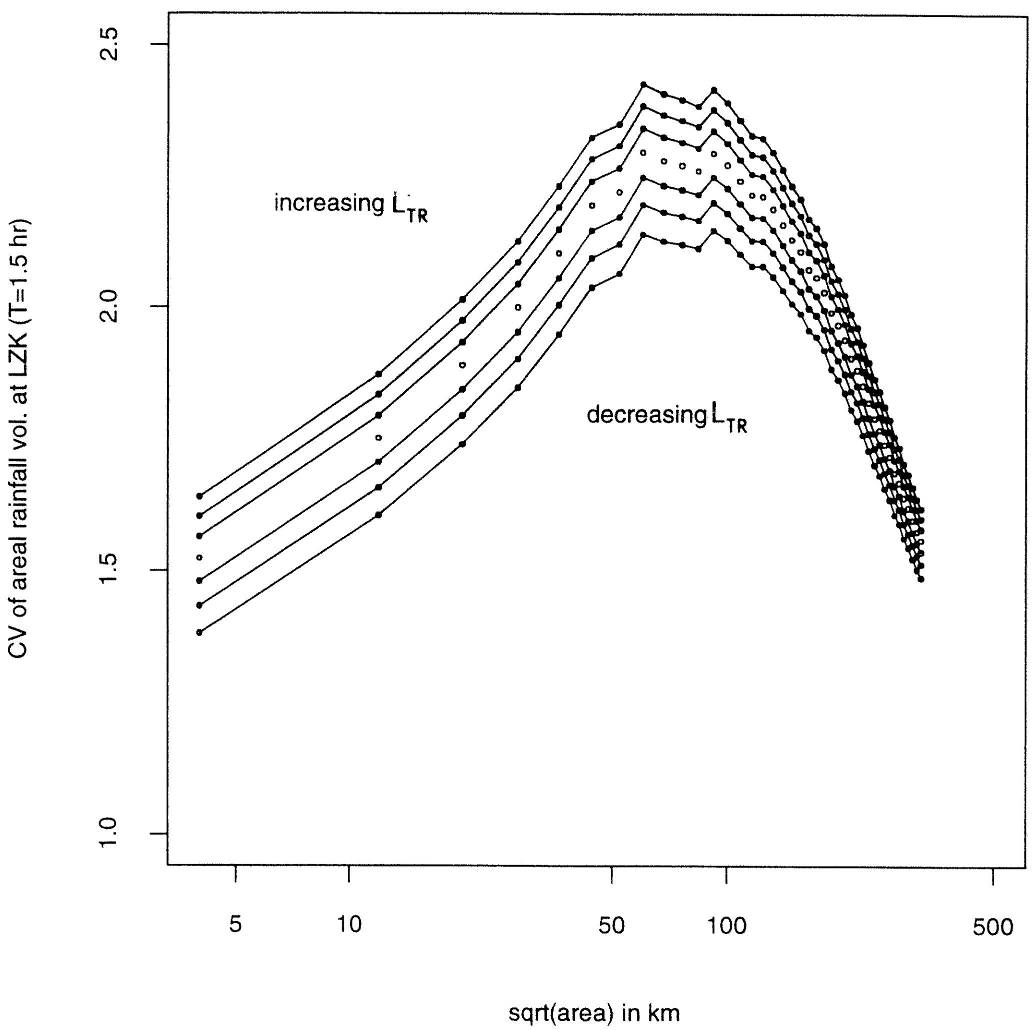
decreasing T
increasing T
CV of areal rainfall vol. at FTG (without advection)
1.0
1.5
2.0
2.5
3.0
3.5
4.0
5
10
50
100
500
sqrt(area) in km



increasing L_{SR}
decreasing L_{SR}
CV of areal rainfall vol. at LZK (T=1.5 hr)
1.0
1.5
2.0
2.5
5
10
50
100
500
sqrt(area) in km



increasing L_{SI}
decreasing L_{SI}
CV of areal rainfall vol. at LZK (T=1.5 hr)
2.5
2.0
1.5
1.0
5
10
50
100
500
sqrt(area) in km



CV of areal rainfall vol. at LZK (T=1.5 hr)
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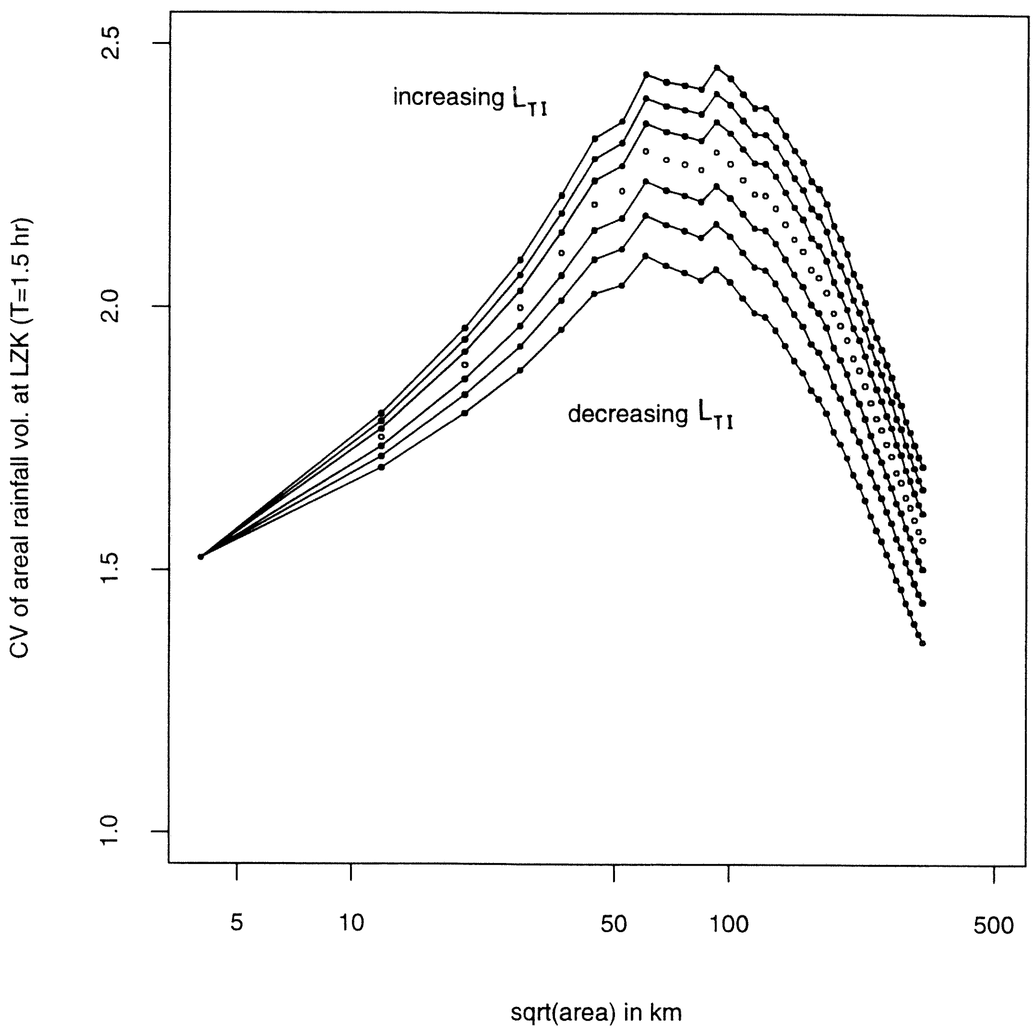
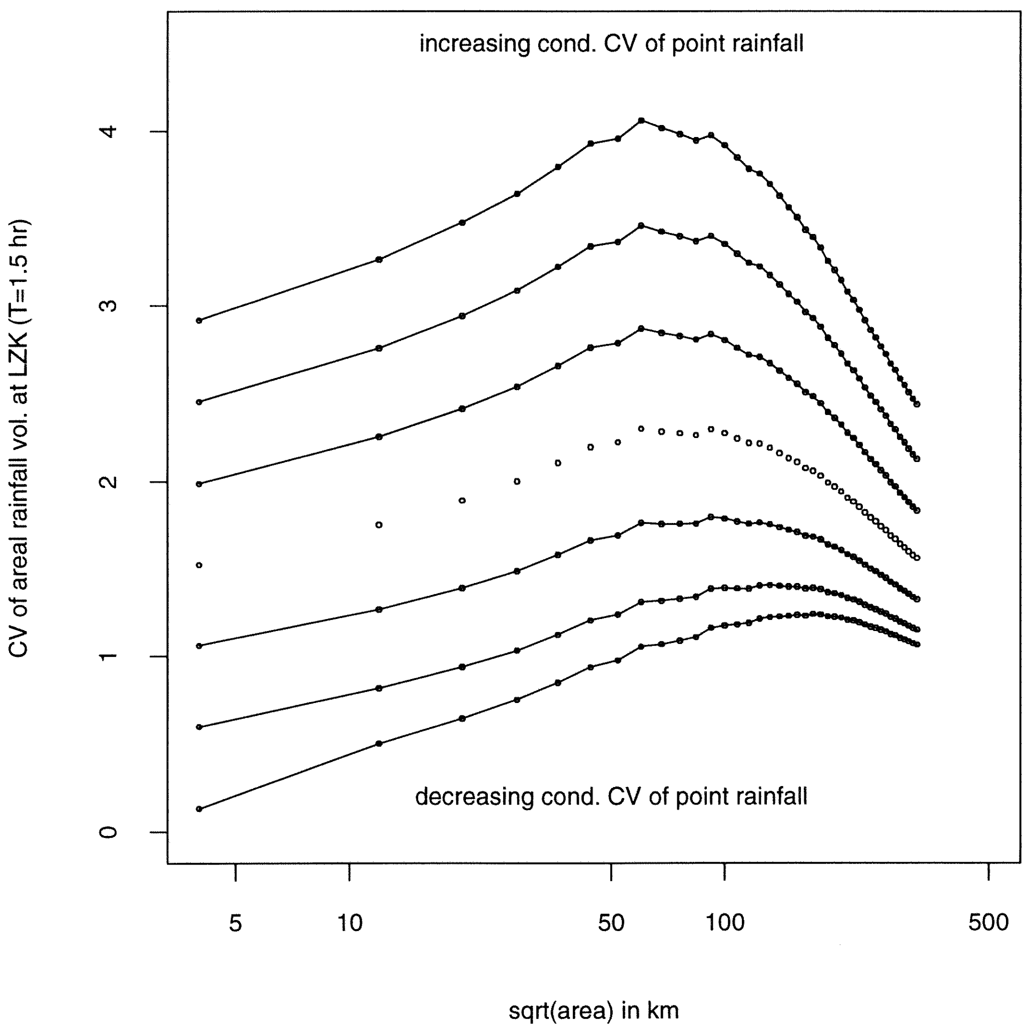
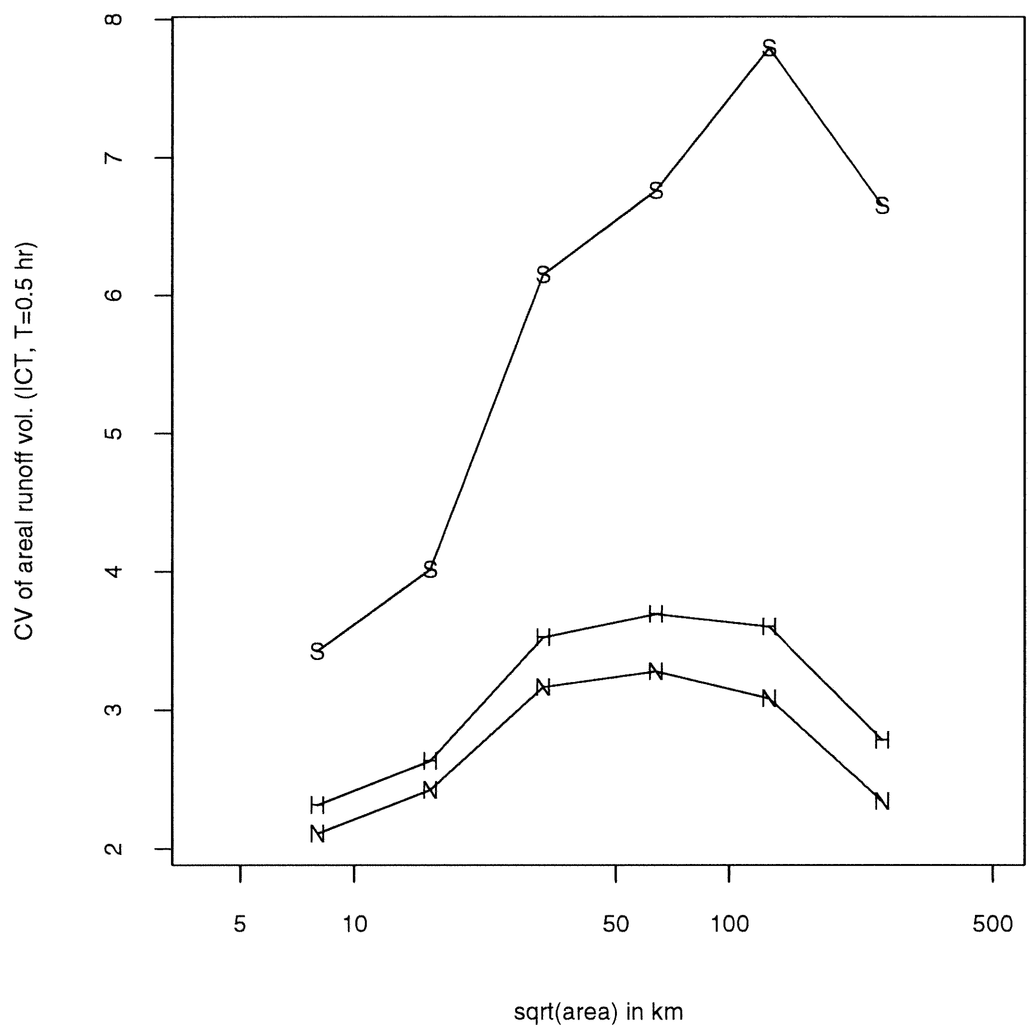
2.0
1.5
1.0
increasing L_{TR}
decreasing L_{TR}
5
10
50
100
500
sqrt(area) in km

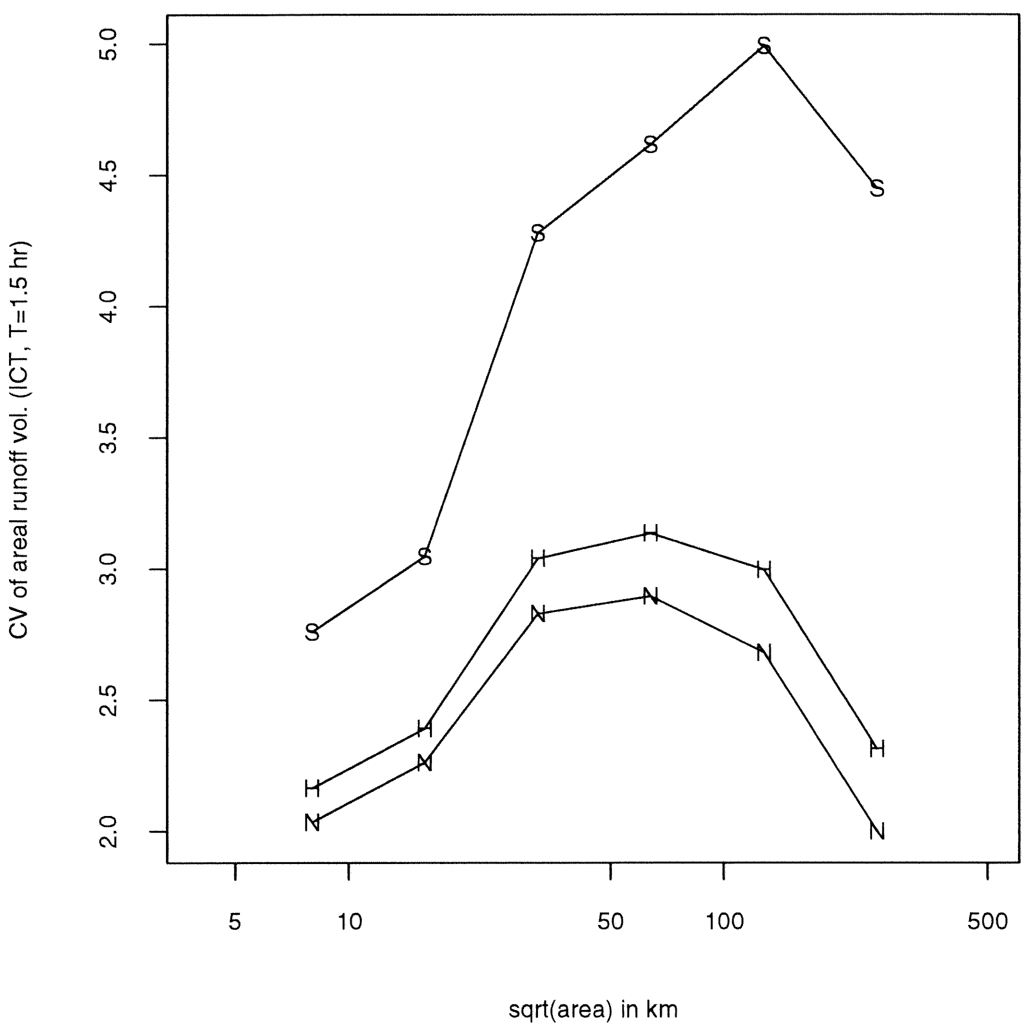


increasing L_{TI}
decreasing L_{TI}
CV of areal rainfall vol. at LZK (T=1.5 hr)
sqrt(area) in km
2.5
2.0
1.5
1.0
5
10
50
100
500

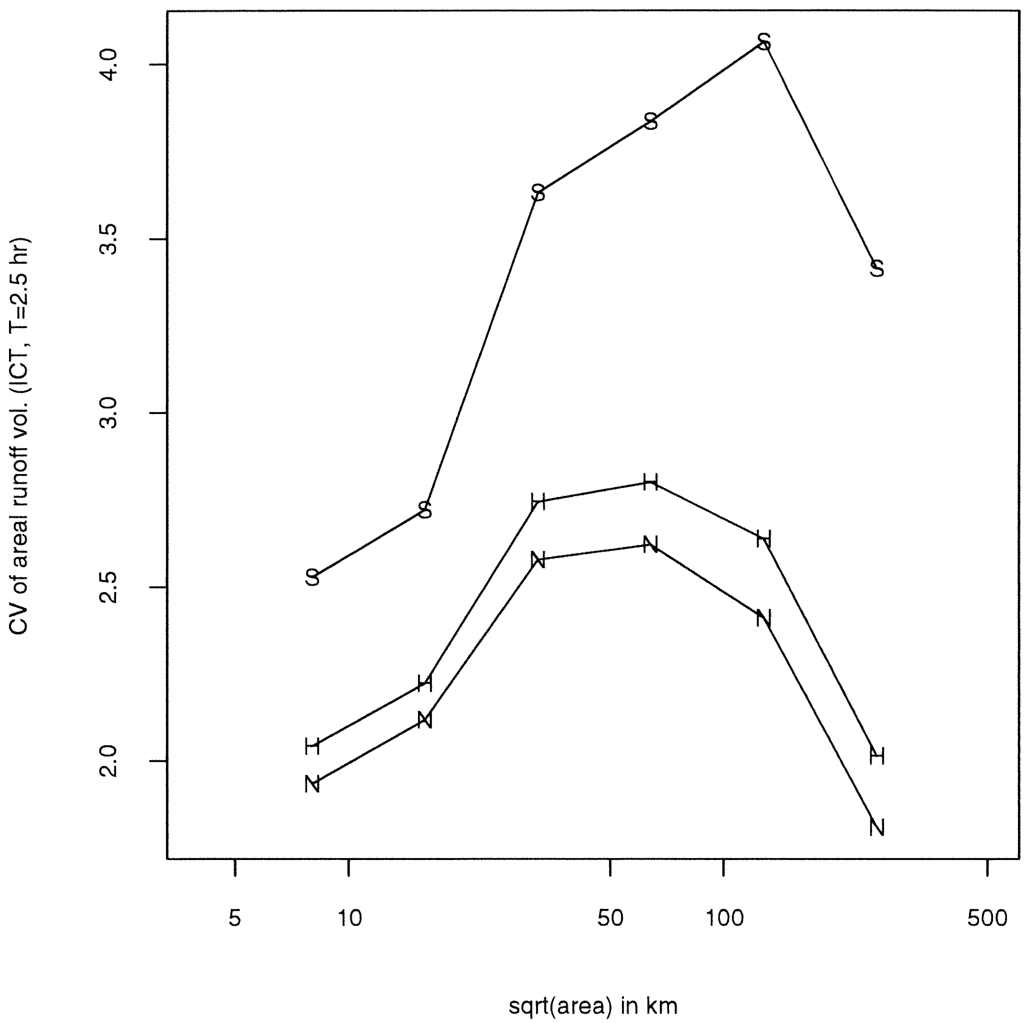

increasing cond. CV of point rainfall
decreasing cond. CV of point rainfall
CV of areal rainfall vol. at LZK (T=1.5 hr)
sqrt(area) in km
0
1
2
3
4
5
10
50
100
500

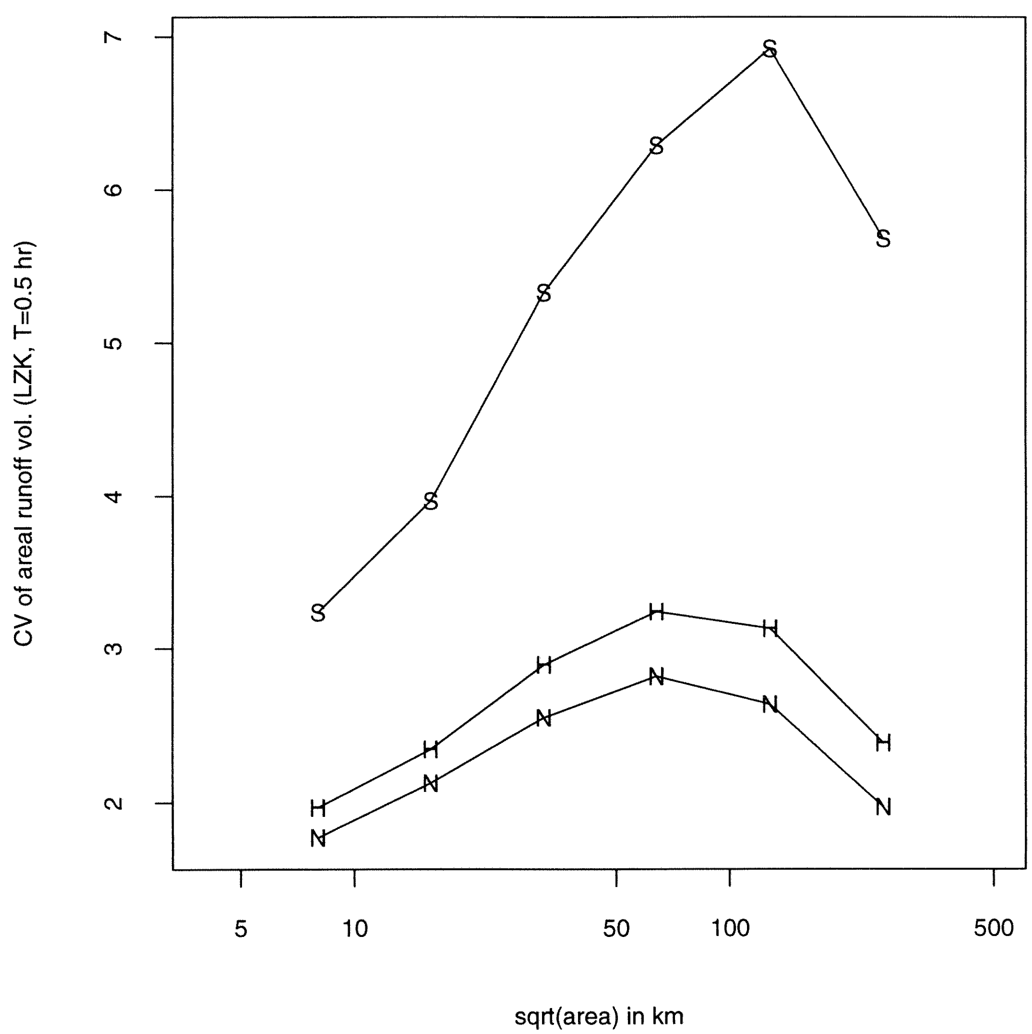


CV of areal runoff vol. (ICT, T=0.5 hr)
sqrt(area) in km
2
3
4
5
6
7
8
5
10
50
100
500
S
H
N



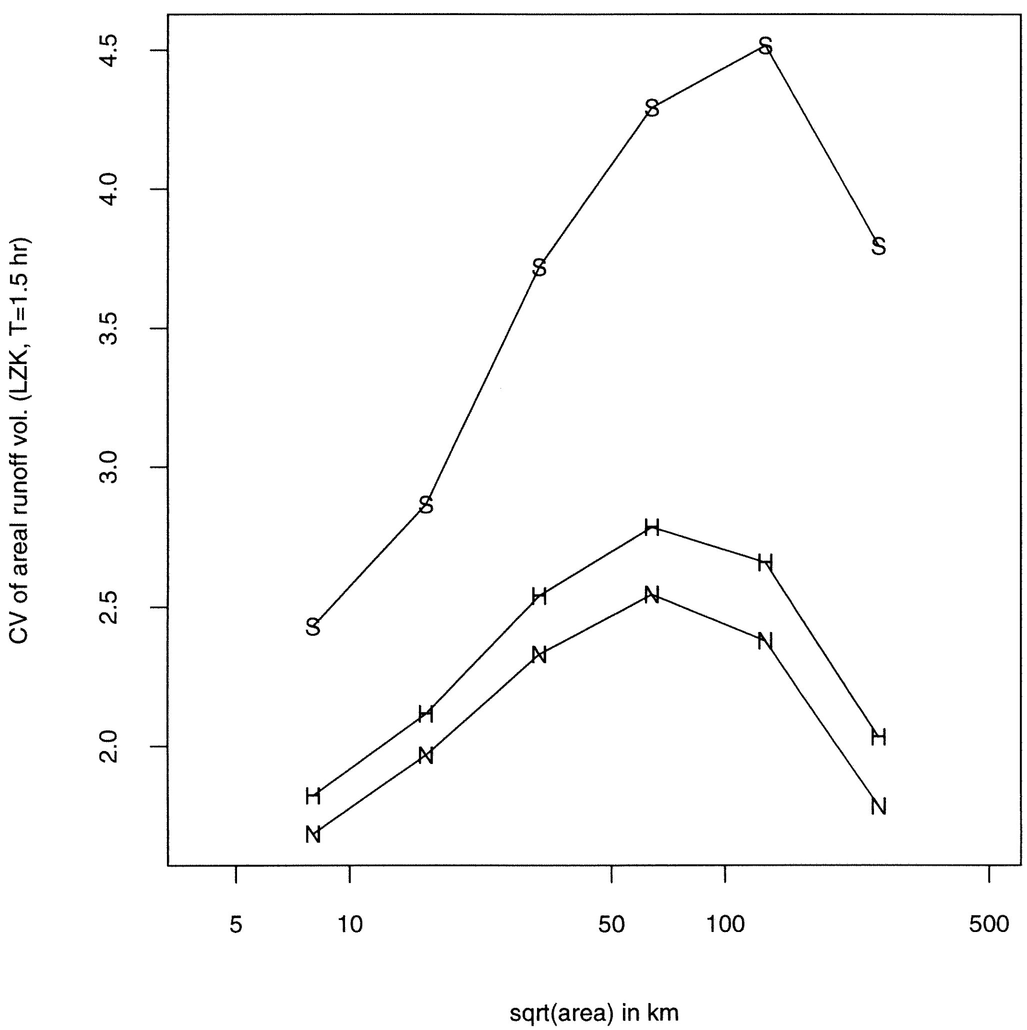
CV of areal runoff vol. (ICT, T=1.5 hr)
2.0
2.5
3.0
3.5
4.0
4.5
5.0
S
H
N
5
10
50
100
500
sqrt(area) in km


CV of areal runoff vol. (ICT, T=2.5 hr)
sqrt(area) in km
2.0
2.5
3.0
3.5
4.0
5
10
50
100
500
S
H
N

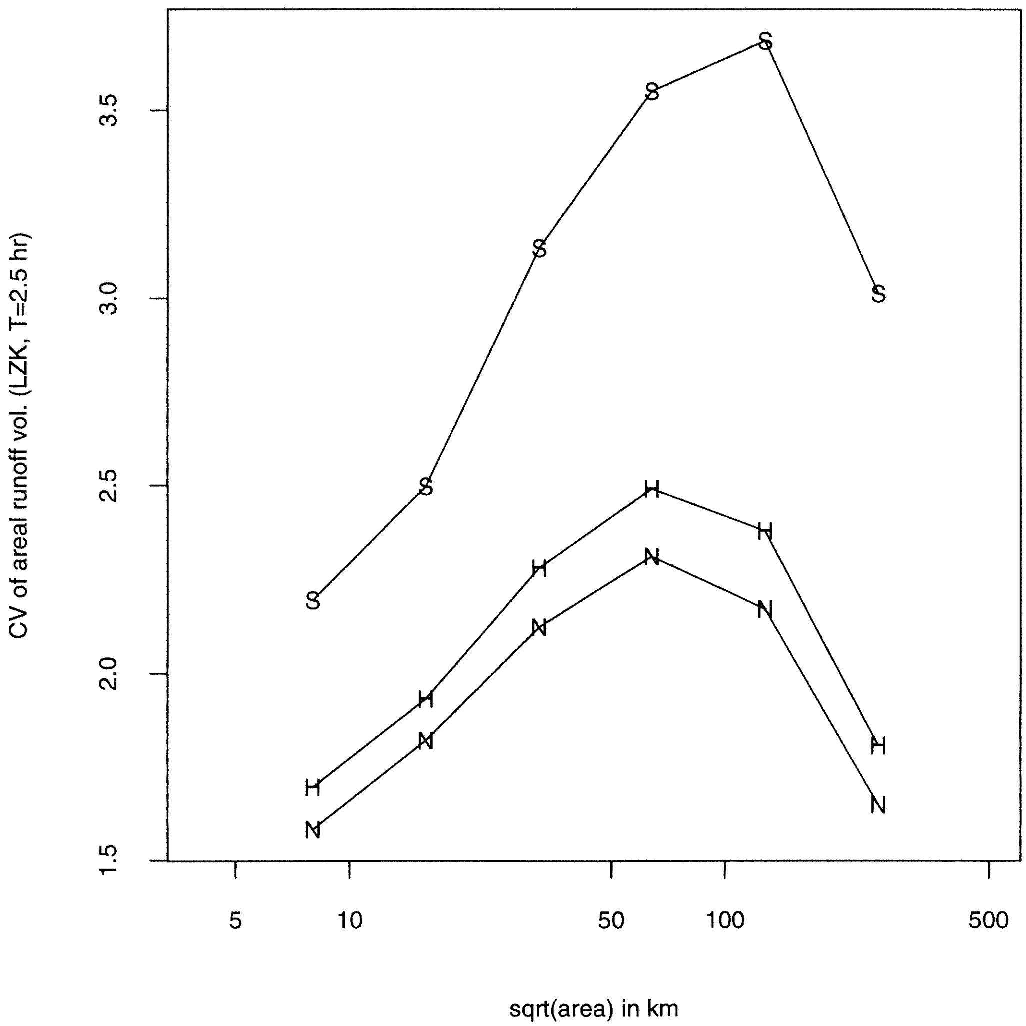


CV of areal runoff vol. (LZK, T=0.5 hr)
2
3
4
5
6
7
5
10
50
100
500
sqrt(area) in km
S
H
N

12

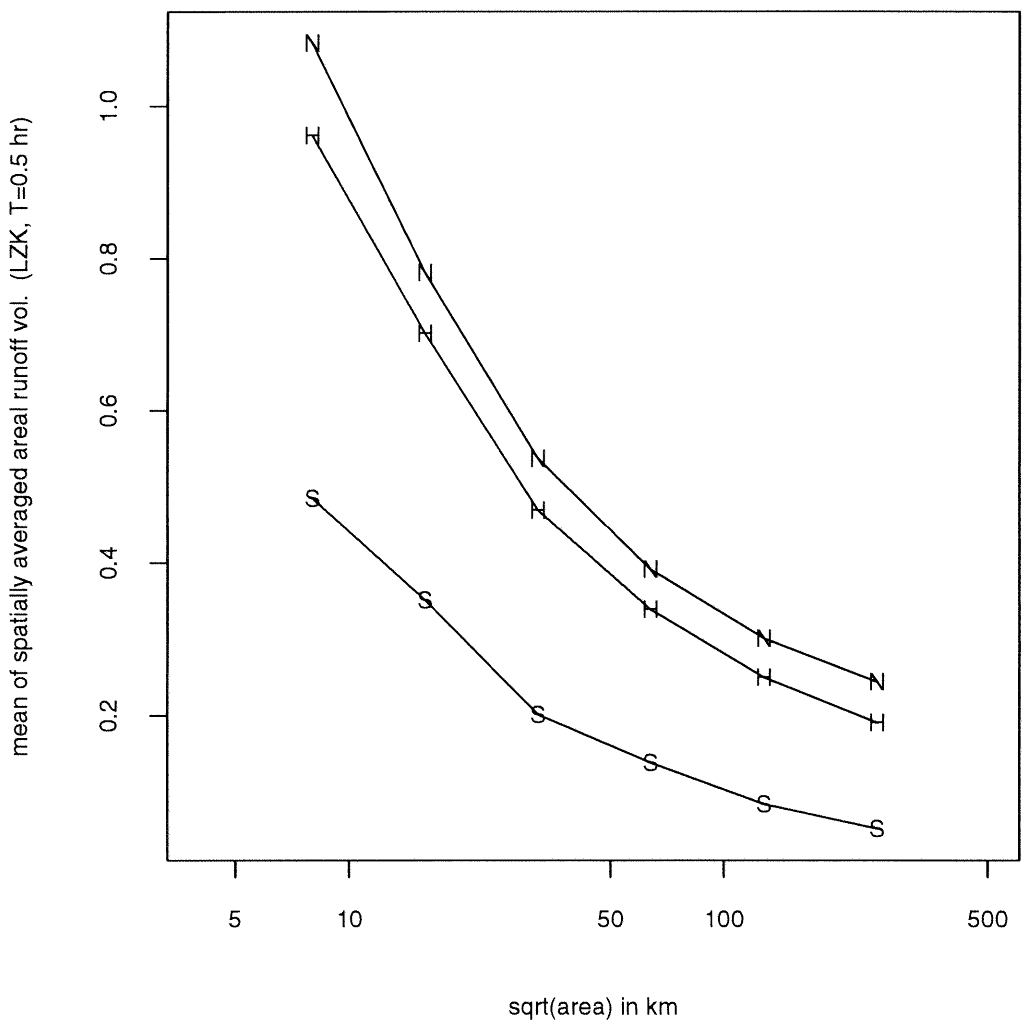


CV of areal runoff vol. (LZK, T=1.5 hr)
4.5
4.0
3.5
3.0
2.5
2.0
5
10
50
100
500
sqrt(area) in km
S
H
N

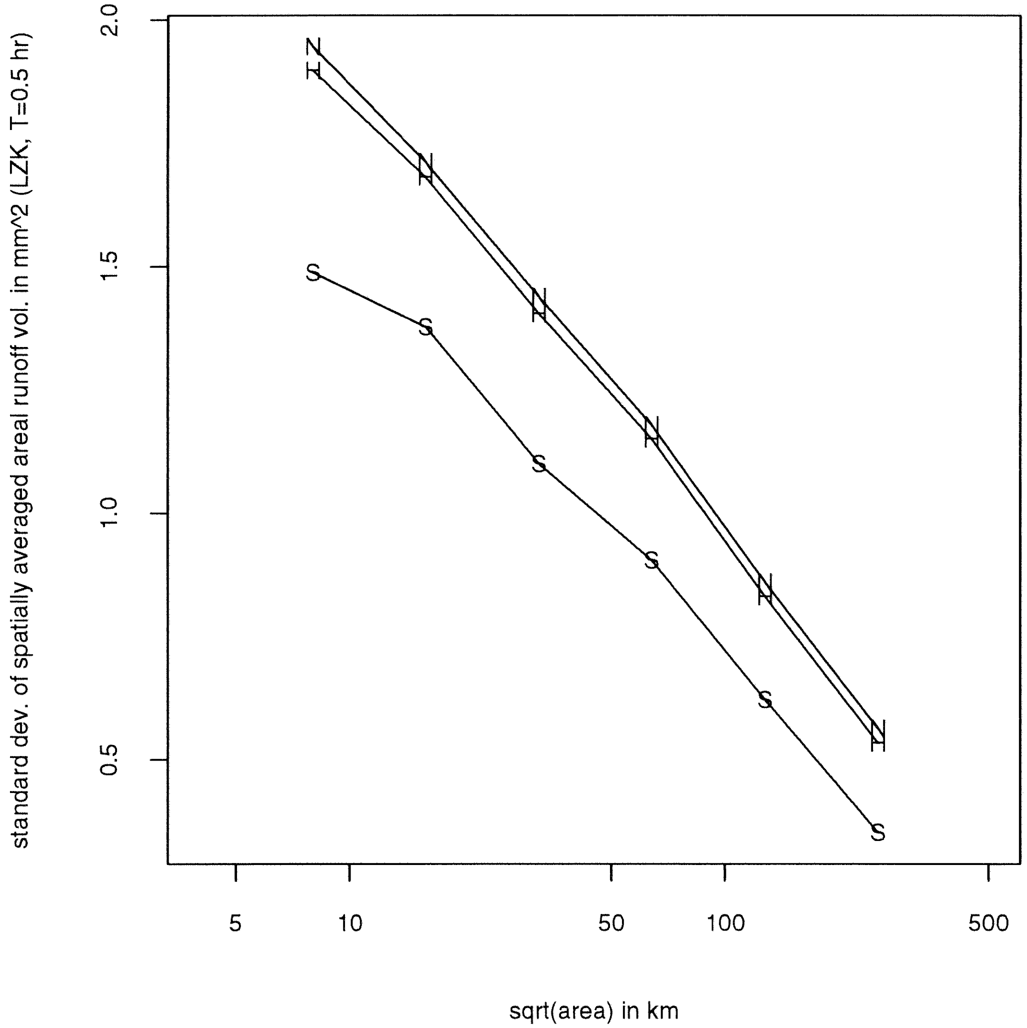


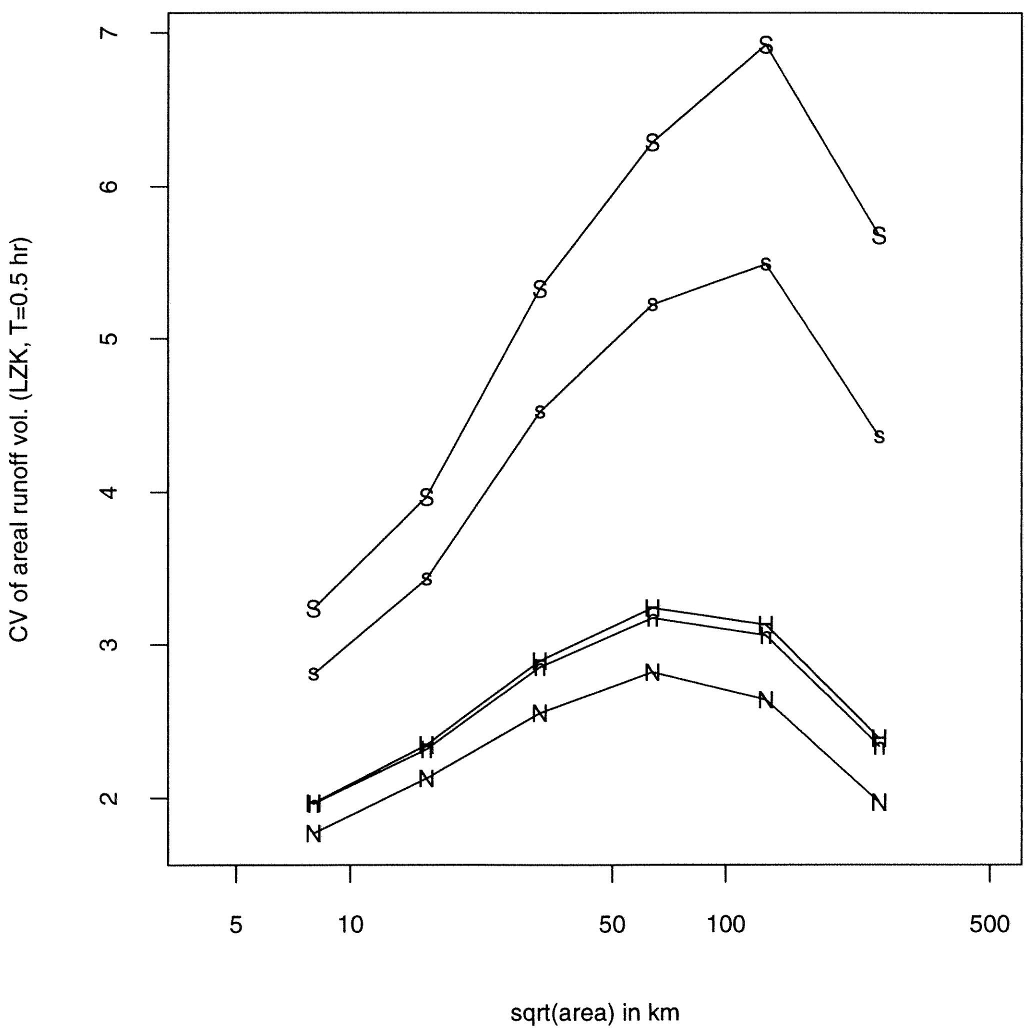
CV of areal runoff vol. (LZK, T=2.5 hr)
1.5
2.0
2.5
3.0
3.5
5
10
50
100
500
sqrt(area) in km
S
H
N

14

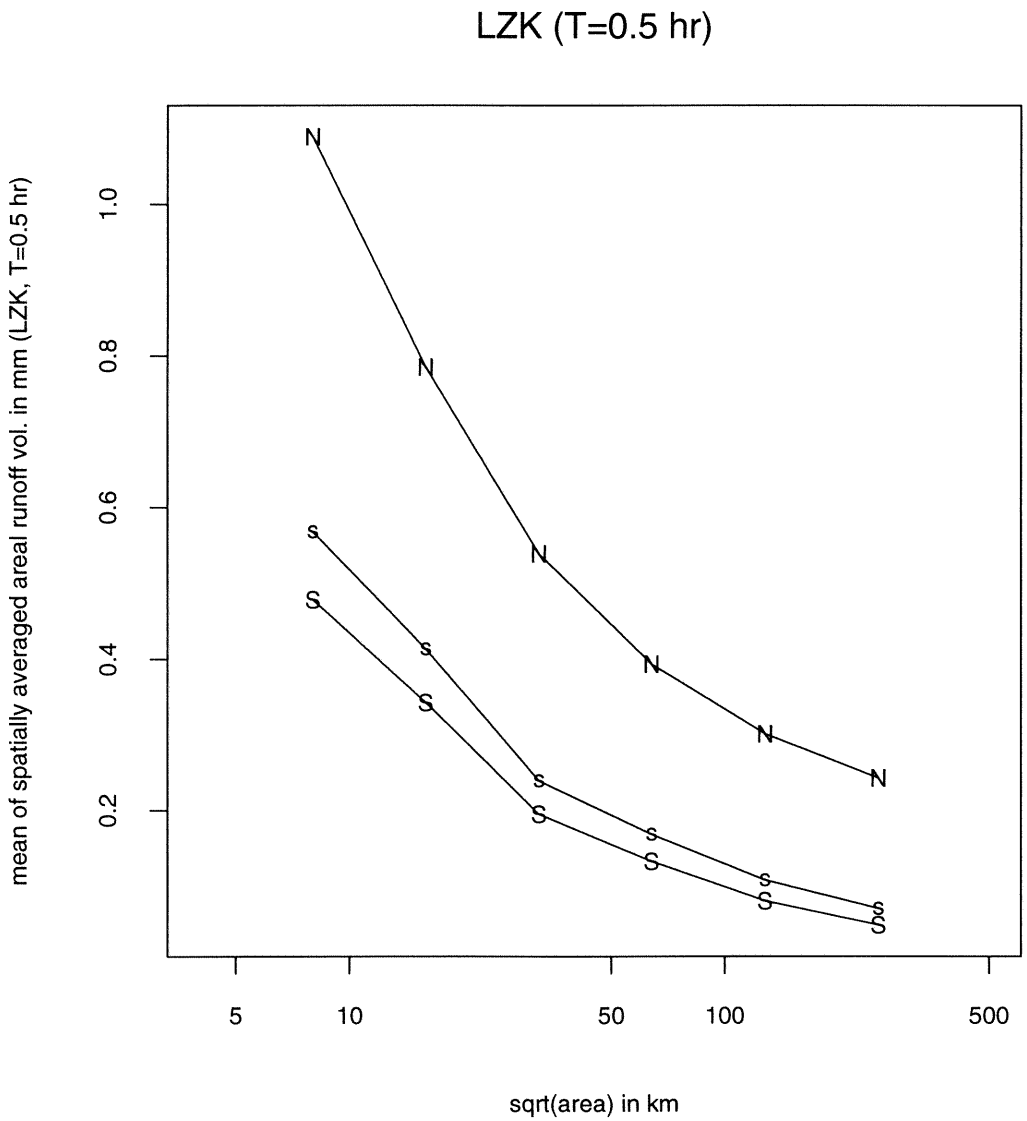


mean of spatially averaged areal runoff vol. (LZK, T=0.5 hr)
0.2
0.4
0.6
0.8
1.0
5
10
50
100
500
sqrt(area) in km
N
H
S


standard dev. of spatially averaged areal runoff vol. in mm^2 (LZK, T=0.5 hr)
2.0
1.5
1.0
0.5
5
10
50
100
500
sqrt(area) in km
N
H
S

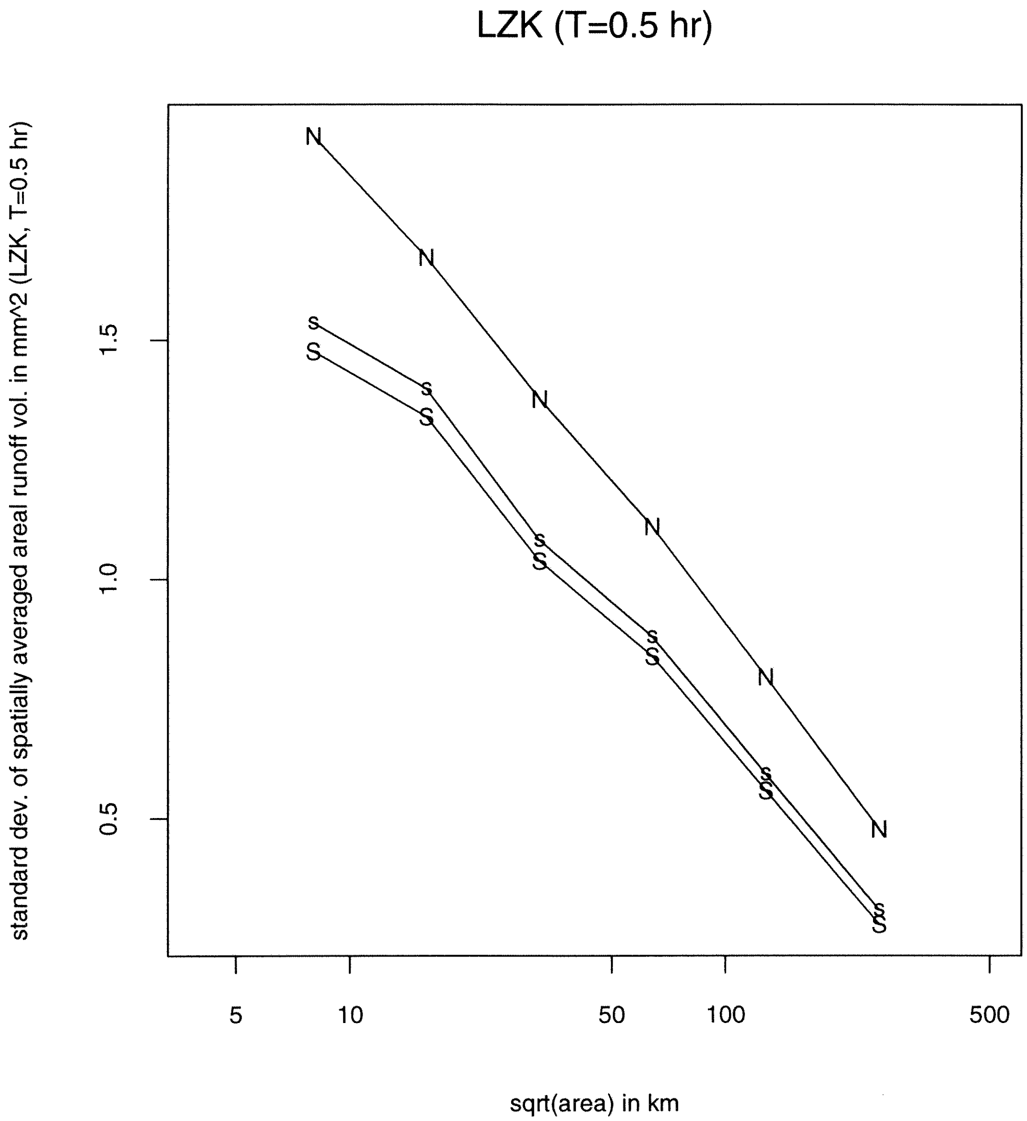

CV of areal runoff vol. (LZK, T=0.5 hr)
sqrt(area) in km
2
3
4
5
6
7
5
10
50
100
500
S
s
H
N

17



LZK (T=0.5 hr)
mean of spatially averaged areal runoff vol. in mm (LZK, T=0.5 hr)
1.0
0.8
0.6
0.4
0.2
5
10
50
100
500
sqrt(area) in km
N
s
S

190



LZK (T=0.5 hr)
standard dev. of spatially averaged areal runoff vol. in mm^2 (LZK, T=0.5 hr)
0.5
1.0
1.5
5
10
50
100
500
sqrt(area) in km
N
s
S

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On the relationship between catchment scale and climatological variability of surface-runoff volume

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Abstract. The relationship between catchment scale and climatological variability of surface-runoff volume is evaluated through theoretical and empirical analyses. Using a point description of surface-runoff volume following infiltration excess under the time compression approximation, climatological mean and variance of areal runoff volume over heterogeneous soil are evaluated by integrating second-order statistics of point runoff volume. Second-order statistics of rainfall duration and intensity, required for evaluation of those of point runoff volume, are obtained from mean and variance of areal rainfall volume under fractional coverage considerations. Hourly Weather Surveillance Radar-1988 Doppler rainfall data were used to estimate rainfall parameters following a set of simplifying assumptions on the space-time correlation structure. Two contrasting climate-soil systems were considered in specifying soil and soil moisture parameters following a set of assumptions on spatial variability and interdependency of the soil parameters.

Introduction

In this paper we evaluate the relationship between catchment scale and climatological variability of areal surface-runoff volume. The measure used to quantify the variability is coefficient of variation (CV). Because large-scale hydrologic models employ large grid (or catchment) scales, subgrid-scale variabilities due to within-storm variability and intermittency of rainfall as well as spatial variabilities of soil and soil moisture are an important modeling consideration [*Entekhabi and Eagleson*, 1989; *World Meteorological Organization and International Council of Scientific Unions (WMO and ICSU)*, 1992]. The scale variability relationships of areal rainfall and runoff volumes, as estimated in this work, quantify the scale dependence of long-term subgrid-scale variabilities of areal rainfall volume and resultant areal runoff volume over heterogeneous soil, respectively. They hence serve an important information on scale-dependent variabilities (or uncertainties) associated with model input and output, such as areal rainfall and runoff volumes, and in choosing grid size and integration time interval of the model in relation with the particular subgrid-scale parameterization used.

In work by *Seo and Smith* [1996], the relationship between catchment scale and climatological variability of mean areal rainfall was investigated. It was shown that under second-order homogeneity assumptions, climatological variability of mean areal rainfall (as measured by CV), given raining somewhere in the area, is a function of mean fractional coverage, conditional (on occurrence of rain) CV of point rainfall, and two correlation scales associated with inner variability (or within-storm variability) and intermittency of rainfall. Empirical analyses using hourly Weather Surveillance Radar-1988 Doppler (WSR-88D) rainfall data from nine sites in the southern plains of the United States were then performed to verify the scale-variability relationships of mean areal rainfall under fractional coverage considerations. It was shown that at most sites in the southern plains, the predicted relationships between catchment scale and climatological variability of mean areal rainfall agree well with the empirical relationships. It is also an indication that the assumption of second-order homogeneity of rainfall is reasonable in the study area.

Because empirical estimation of scale-variability relationships of areal runoff volume (accumulated over a half hour to several hours) is practically impossible due to lack of data, one is left with either a numerical or an analytical approach. The numerical approach would be to perform a simulation experiment using a space-time rainfall model and a distributed-parameter rainfall-runoff model. To the best of the authors' knowledge, however, a space-time rainfall model that can explicitly account for intermittency of rainfall does not exist. Also, such an experiment would be extremely costly to obtain climatologically representative statistics. Instead, the approach taken in this work is as follows: (1) make simplifying but reasonable (in the climatological sense) assumptions concerning the structure of space-time rainfall fields, so that the structure is compatible with the time compression approximation to be used in estimating point runoff volume following infiltration excess [*Reeves and Miller*, 1975; *Eagleson*, 1978b; *Milly and Eagleson*, 1988; *Chow et al.*, 1988]; (2) develop expressions for the second-order statistics of point runoff volume in terms of rainfall, soil, and soil moisture parameters; and (3) spatially integrate the expressions to evaluate climatological mean and variance of areal runoff volume. Although runoff due to saturation excess is not explicitly considered in this work, its effect on the scale-variability relationship can easily be inferred from the limiting considerations of no infiltration (i.e., saturation excess only) and no saturation excess (i.e., infiltration only).

This paper is organized into sections as follows. In the next section we present expressions for climatological mean and

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variance of areal rainfall volume. They are "true" statistics in that both inner variability and intermittency of rainfall are explicitly accounted for. Then we derive analogous expressions for "alternative" statistics under the assumption of no within-storm variability. Given the two sets of expressions, parameter estimation via the method of moments allows the alternative statistics to exactly reproduce the total variability of areal rainfall volume as would be given by the true statistics. Next, expressions for the climatological mean and variance of areal runoff volume are derived, which may be evaluated via Monte Carlo integration. In the next two sections, we describe how space-time correlation structures for inner variability and intermittency of rainfall are estimated, and how soil and soil moisture fields are specified. In the final two sections prior to the appendix, results and conclusions are given.

Moments of Areal Rainfall Volume

Here we present expressions for the climatological mean and variance of areal rainfall volume in which both inner variability and intermittency are accounted for. Let us define the areal rainfall volume, V_a, as follows:

$$V_a = \int_0^T \int_A R(u, t)\, du\, dt \tag{1}$$

where $R(u, t)$ is the rain rate at location u at time t, A denotes the catchment of area $\|A\|$ in square kilometers, and T is the time period in hours during which rainfall occurs somewhere in A (i.e., fractional coverage of rainfall over A is positive for $t \in [0, T]$). For stationary storms, T is bounded by their lifetime. For advecting storms it is bounded by catchment and storm sizes as well. In the following developments, dependence of T on these variables is not shown for notational brevity.

In work by *Seo and Smith* [1996], expressions for climatological mean and variance of mean areal rainfall, given raining somewhere in A, were derived. Expressions for those of areal rainfall volume given raining somewhere in A over $[0, T]$ may be obtained in an analogous manner under the additional assumption that both conditional and indicator rainfall processes, $R(u, t)$ given $R(u, t) > 0$ and $I(R(u, t); 0)$, respectively, are stationary in $[0, T]$. The indicator random variable, $I(R(u, t); 0)$, $u \in A$, and $t \in [0, T]$, was defined by *Seo and Smith* [1996] as

$$\begin{aligned} i(R(u, t); 0) &= 1 \qquad R(u, t) > 0 \\ i(R(u, t); 0) &= 0 \qquad \text{otherwise} \end{aligned} \tag{2}$$

where $i(R(u, t); 0)$ denotes the experimental value of the random variables $I(R(u, t); 0)$. The resulting expressions for the climatological mean and variance of V_a given raining somewhere in A over $[0, T]$ are

$$E[V_a|T] = \|A\| T m_R m_Z(A) \tag{3}$$

$$\begin{aligned} \operatorname{var}[V_a|T] = {} & \sigma_R^2 m_Z(A)\{1 - m_Z(A)\} \\ & \cdot \int_0^T \int_0^T \int_A \int_A \rho_R(R(u, t), R(v, s); L_{SR}, L_{TR}|R(u, t) > 0, \\ & R(v, s) > 0) \cdot \rho_I(I(u, t), I(v, s); L_{SI}, L_{TI})\, du\, dv\, dt\, ds \\ & + \sigma_R^2 m_Z^2(A) \int_0^T \int_0^T \int_A \int_A \rho_R(R(u, t), R(v, s); \\ & L_{SR}, L_{TR}|R(u, t) > 0, R(v, s) > 0)\, du\, dv\, dt\, ds \\ & + m_R^2 m_Z(A)\{1 - m_Z(A)\} \\ & \cdot \int_0^T \int_0^T \int_A \int_A \rho_I(I(u, t), I(v, s); L_{SI}, L_{TI})\, du\, dv\, dt\, ds \end{aligned} \tag{4}$$

where m_R and σ_R^2 are the conditional mean and variance of point rainfall, respectively; $m_Z(A)$ is the mean fractional coverage of rainfall over A and $[0, T]$; $\rho_R(R(u, t), R(v, s); L_{SR}, L_{TR}|R(u, t) > 0, R(v, s) > 0)$ is the conditional space-time correlation function of point rainfall with spatial and temporal correlation-scale parameters L_{SR} and L_{TR}, respectively; and $\rho_I(I(u, t), I(v, s); L_{SI}, L_{TI})$ is the space-time indicator correlation function with analogously defined parameters L_{SI} and L_{TI}.

Equation (4) states that three terms contribute to the climatological variance of areal rainfall volume: inner variability of rainfall (the second term), intermittency of rainfall (the third term), and the cross term between the two (the first term). The reader is kindly referred to *Seo and Smith* [1996] for derivation and verification of a set of analogous expressions for mean areal rainfall. It is noted that following the stationarity assumption, the time-dependence notations in (7) and (12) of *Seo and Smith* [1996] have been dropped in the above.

Moments of Areal Rainfall Volume Assuming No Within-Storm Variability

Equations (3) and (4) are accurate representations of the climatological mean and variance of areal rainfall volume in that both inner variability and intermittency of rainfall are explicitly accounted for. They do not, however, accommodate the time compression approximation to be used in describing surface runoff via infiltration excess. In order to do so, we derive new expressions for the climatological mean and variance of areal rainfall volume under the assumption that within a storm, rain rate is constant in space and time (but varies from storm to storm).

We approximate the rainfall volume at location u from a single storm, $V(u)$, as follows:

$$V(u) = I_r T_r(u) \tag{5}$$

where I_r is the rain rate in millimeters per hour, constant in space and time within a storm; and $T_r(u)$ is the rainfall duration in hours at location u. Then the areal rainfall volume, V_a, is given by

$$V_a = I_r \int_A T_r(u)\, du \tag{6}$$

Assuming that location u, $u \in A$, receives rainfall from a storm in the form of a single continuous pulse, we may write $T_r(u)$ in (6) as

$$T_r(u) = \int_0^T I(R(u, t); 0)\, dt \tag{7}$$

Assuming independence between I_r and $T_r(u)$ (see *Eagleson* [1978a, b] for justification), we have for the areal rainfall volume given raining somewhere in A over $[0, T]$:

$$E[V_a|T] = m_{Ir} \int_A E[T_r(u)|T]\, du \tag{8}$$

$$\mathrm{var}[V_a|T] = (\sigma_{Ir}^2 + m_{Ir}^2) \int_A \int_A \mathrm{cov}[T_r(u), T_r(v)|T]\, du\, dv$$

$$+ \sigma_{Ir}^2 \int_A \int_A E[T_r(u)|T] E[T_r(v)|T]\, du\, dv \tag{9}$$

where m_{Ir} and σ_{Ir}^2 are the climatological mean and variance of I_r, given raining somewhere in A over $[0, T]$, respectively. Following the definition of $T_r(u)$ in (7), $E[T_r(u)|T]$ and $\mathrm{cov}[T_r(u), T_r(v)|T]$ are written as

$$E[T_r(u)|T] = Tm_Z(A) \tag{10}$$

$$\mathrm{cov}[T_r(u), T_r(v)|T] = m_Z(A)\{1 - m_Z(A)\}$$

$$\cdot \int_0^T \int_0^T \rho(I(u, s), I(v, t); L_{SI}, L_{TI})\, dt\, ds \tag{11}$$

The reader is kindly referred to (6) and (12) of *Seo and Smith* [1996] for further details. Then we have

$$E[V_a|T] = \|A\| T m_{Ir} m_Z(A) \tag{12}$$

$$\mathrm{var}[V_a|T] = \sigma_{Ir}^2 A^2 T^2 m_Z^2(A) + (\sigma_{Ir}^2 + m_{Ir}^2) m_Z(A)\{1 - m_Z(A)\}$$

$$\cdot \int_0^T \int_0^T \int_A \int_A \rho(I(u, s), I(v, t); L_{SI}, L_{TI})\, du\, dv\, dt\, ds \tag{13}$$

We may verify the validity of the above expressions by comparing them with (3) and (4): (12) becomes identical to (3) if $m_{Ir} = m_R$, and (13) becomes identical to (4) if $L_{SR} \to \infty$ and $L_{TR} \to \infty$ in the latter (i.e., if positive rain rate is constant in space and time within a storm).

Equations (12) and (13) are alternative expressions to (3) and (4), respectively, developed to accommodate the time compression approximation. Preservation of $E[V_a|T]$ by (12) is easily achieved by setting $m_{Ir} = m_R$ in (12). Preservation of $\mathrm{var}[V_a|T]$ by (13) is achieved by finding the value of σ_{Ir}^2 with which (13) yields the same variance as (3). Physically, this amounts to substituting storm-to-storm variability of storm duration-averaged mean areal rainfall for within-storm space-time variability of point rainfall. Because (4) and (13) share the same intermittency term (the last term in (4)), the only consequence of the assumption of no within-storm variability, insofar as its effect on climatological mean and variance of mean areal rainfall is concerned, is that (13) does not allow an exact quantification of the contribution from inner variability.

Moments of Areal Runoff Volume Following Infiltration Excess

We are now in a position to seek expressions for climatological mean and variance of areal runoff volume, given raining somewhere in A over $[0, T]$, following infiltration excess over heterogeneous soil. Based on *Philip*'s [1957] equation and the time compression approximation, *Eagleson* [1978b] has shown that neglecting surface retention, point runoff volume at u, $R_s(u)$, may be approximated by the following:

$$R_s(u) = \{I_r - A_o(u)\}T_r(u) - S_i(u)\{T_r(u)/2\}^{1/2}$$

$$T_r(u) > S_i^2(u)/[2\{I_r - A_o(u)\}], \; I_r \geq A_o(u) \tag{14}$$

$$R_s(u) = 0 \qquad \text{otherwise}$$

In the above, $S_i(u)$ and $A_o(u)$ are the infiltration sorptivity in cm/h$^{1/2}$ and the gravitational infiltration rate as modified by capillary rise from the water table (equal to infiltration rate at large time) in centimeters per hour, respectively. They are given by [*Eagleson*, 1978b]

$$A_o(u) = \tfrac{1}{2} K(u)\{1 + s_o(u)^{c(u)}\} - w(u) \tag{15}$$

$$S_i(u) = 2\{1 - s_o(u)\}$$

$$\cdot [5n(u)K(u)\Psi(u)\phi_i(d(u), s_o(u))/\{3m(u)\pi\}]^{1/2} \tag{16}$$

where $K(u)$ is the saturated hydraulic conductivity in centimeters per hour at location u, $u \in A$, $s_o(u)$ is the time-averaged soil moisture saturation in the surface boundary layer, $c(u)$ is the pore connectivity index, $w(u)$ is the capillary rise from the groundwater table in centimeters per hour (ignored in this work), $n(u)$ is the porosity, $\Psi(u)$ is the saturated soil matrix potential in centimeters, $\phi_i(\quad)$ is the infiltration diffusivity function, $d(u)$ is the diffusivity index, and $m(u)$ is the pore-size distribution index. Given the soil and climatic parameters, $s_o(u)$ can be estimated from the long-term water budget equation of *Eagleson* [1978c].

In the following developments it is to be understood that all the statistical moments are conditional on T although for notational brevity they are not explicitly shown as such. With (14), $h(u) = Si^2(u)/[2\{i_r - A_o(u)\}]$, $A_x \equiv \max\{A_o(u), A_o(v)\}$, and $R_{sc}(u) \equiv \{i_r - A_o(u)\}t_r(u) - S_i(u)\{t_r(u)/2\}^{1/2}$, expressions for the first two moments of areal runoff volume given raining somewhere in A over $[0, T]$ are written as

$$E\left[\int_A R_s(u)\, du\right] = \int_A \int_{A_o(u)} E[R_s(u)|I_r = i_r] f_{I_r}(i_r)\, di_r\, du \tag{17}$$

where

$$E[R_s(u)|I_r = i_r] = \int_{h(u)}^{\infty} R_{sc}(u) f_{T_r(u)|I_r}(t_r(u)|i_r)\, dt_r(u) \tag{18}$$

$$E\left[\int_A \int_A R_s(u) R_s(v)\, du\, dv\right]$$

$$= \int_A \int_A \int_{A_x}^{\infty} E[R_s(u) R_s(v)|Ir = i_r] f_{I_r}(i_r)\, di_r\, du\, dv \tag{19}$$

where

$$E[R_s(u) R_s(v)|Ir = i_r] = \int_{h(v)}^{\infty} \int_{h(u)}^{\infty} R_{sc}(u) R_{sc}(v)$$

$$\cdot f_{T_r(u), T_r(v)|I_r}(t_r(u), t_r(v)|i_r)\, dt_r(u)\, dt_r(v) \tag{20}$$

In this work it is assumed that the probability density functions of $T_r(\ )$ and Ir are bivariate lognormal and two-parameter gamma, respectively: empirical analyses of hourly WSR-88D rainfall data in the southern plains indicate that the assumption is reasonable. Then, for (18) we have

$$E[R_s(u)|I_r = i_r] = \tfrac{1}{2}\{(i_r - A_n(u)) \exp(m + 0.5\sigma^2) \cdot \operatorname{erfc}((b(u) - \sigma)/\sqrt{2}) - S_t(u)/\sqrt{2} \exp(0.5m + (\tfrac{1}{8})\sigma^2) \cdot \operatorname{erfc}((b(u) - 0.5\sigma)/\sqrt{2})\} \tag{21}$$

where m and σ are the mean and standard deviation of ln $(Tr(u))$, respectively, and $b(u) = \{\ln(S_t^2(u)/[2\{i_r - A_n(u)\}]) - m\}/\sigma$. When $u \neq v$, we have for (20) the following:

$$\begin{aligned} 2\sqrt{2}\pi E[R_s(u)R_s(v)|Ir = i_r] &= (i_r - A_n(v))(i_r - A_n(u)) \exp(2m + \beta\sigma^2) \\ &\cdot \int_{b(v)}^{\infty} \operatorname{erfc}[0.5\{b(u) - \rho z\}/\sqrt{\beta} - \sigma\sqrt{\beta}] \\ &\cdot \exp\{-0.5z^2 + (1 + \rho)\sigma z\}\, dz \\ &- (i_r - A_n(v))S_t(u)/\sqrt{2} \exp(1.5m + 0.25\beta\sigma^2) \\ &\cdot \int_{b(v)}^{\infty} \operatorname{erfc}[0.5\{b(u) - \rho z\}/\sqrt{\beta} - 0.5\sigma\sqrt{\beta}] \\ &\cdot \exp\{-0.5z^2 + (1 + 0.5\rho)\sigma z\}\, dz \\ &- S_t(v)/\sqrt{2}(i_r - A_n(v)) \exp(1.5m + \beta\sigma^2) \\ &\cdot \int_{b(v)}^{\infty} \operatorname{erfc}[0.5\{b(u) - \rho z\}/\sqrt{\beta} - \sigma\sqrt{\beta}] \\ &\cdot \exp\{-0.5z^2 + (0.5 + \rho)\sigma z\}\, dz \\ &+ (\tfrac{1}{2})S_t(v)S_t(u) \exp(m + 0.25\beta\sigma^2) \\ &\cdot \int_{b(v)}^{\infty} \operatorname{erfc}[0.5\{b(u) - \rho z\}/\sqrt{\beta} - 0.5\sigma\sqrt{\beta}] \\ &\cdot \exp\{-0.5z^2 + 0.5(1 + \rho)\sigma z\}\, dz \end{aligned} \tag{22}$$

where ρ is the spatial correlation coefficient between ln $(T_r(u))$ and ln $(T_r(v))$, $\beta \equiv (1 - \rho^2)/2$, and erfc [ ] denotes the complementary error function. When $u = v$, we have for (20) the following:

$$\begin{aligned} 2E[R_s^2(u)|I_r = i_r] &= (i_r - A_n(u))^2 \exp(2(m + \sigma^2)) \\ &\cdot \operatorname{erfc}((b(u) - 2\sigma)/\sqrt{2}) - \sqrt{2}(i_r - A_n(u))S_t(u) \\ &\cdot \exp(1.5m + 9\sigma^2/8) \operatorname{erfc}((b(u) - 1.5\sigma)/\sqrt{2}) \\ &+ \tfrac{1}{2}S_t^2(u) \exp(m + 0.5\sigma^2) \operatorname{erfc}((b(u) - \sigma)/\sqrt{2}) \end{aligned} \tag{23}$$

To evaluate the above expressions, we need to specify the second-order statistics of $T_r(\ )$ and I_r, and the infiltration sorptivity and the gravitational infiltration rate, $S_t(u)$ and $A_n(u)$, $u \in A$. Also, to specify $\sigma_{I_r}^2$'s that preserve the total variability of areal rainfall volume given raining in A over $[0, T]$, we need to specify m_R, σ_R^2, $m_Z(A)$, L_{SI}, L_{SR}, L_{TI}, and L_{TR} in (4). For estimation of m_R, σ_R^2, $m_Z(A)$, L_{SI}, and L_{SR}, the reader is kindly referred to *Seo and Smith* [1996]. In the following sections, we describe how the remaining parameters may be specified.

Specification of Rainfall Parameters

To specify the second-order statistics of $T_r(\ )$ and I_r, one has to estimate the conditional space-time correlation function of point rainfall, $\rho_R(R(u, s), R(v, t); L_{SR}, L_{TR}|R(u, s) > 0, R(v, t) > 0)$, and the indicator space-time correlation function, $\rho_I(I(u, s), I(v, t); L_{SI}, L_{TI})$, for evaluation of (4), (11), and (13). To do so, we have made the following assumptions: (1) the space-time correlation functions are separable in Lagrangian domain; (2) spatial correlation functions are Gaussian with no nugget effect; and (3) Lagrangian autocorrelation functions are exponential. The first assumption implies that, for example, the conditional space-time correlation function may be rewritten as

$$\rho(R(u, s), R(v, t); L_{SR}, L_{TR}|R(u, s) > 0, R(v, t) > 0) = \rho_s(|v - u + U(s - t)|; L_{SR})\rho_I(|s - t|; L_{TR}) \tag{24}$$

where $\rho_s(|\ |; L_{SR})$ and $\rho_I(|\ |; L_{TR})$ are the spatial correlation and the Lagrangian autocorrelation functions of positive point rainfall, respectively; U is the advection vector; and $|\ |$ denotes the Euclidean distance. Equation (24) is a very reasonable assumption in that it offers a more realistic space-time correlation structure than that under Taylor's hypothesis [*Taylor*, 1935; *Bras and Rodriguez-Iturbe*, 1976] or the assumption of frozen field with dissipation and random generation [*Gupta and Waymire*, 1986].

The second assumption implies that

$$\rho_s(|v - u + U(s - t)|; L_{SR}) = \exp(-[\{v_1 - u_1 + U_1(s - t)\}^2 + \{v_2 - u_2 + U_2(s - t)\}^2]/L_{SR}^2) \tag{25}$$

where (u_1, u_2) and (v_1, v_2) are the x and y coordinates of the locations u and v, respectively, and U_1 and U_2 are the x and y components of the climatological mean advection velocity, respectively. Observational evidences [*Seo and Smith*, 1996] suggest that the spatial correlation functions are closer to exponential than Gaussian. However, there exists a compelling computational reason (see below) to use the Gaussian model. To justify the substitution, we performed a numerical experiment which indicates that the Gaussian model introduces negligible error.

The third assumption implies that

$$\rho_I(|s - t|; L_{TR}) = \exp(-|s - t|/L_{TR}) \tag{26}$$

This assumption is based on empirical analyses of hourly radar rainfall data, which indicate that the Lagrangian autocorrelation function is approximately exponential. The first two assumptions greatly reduce computational requirements: sextuple integrals of the form, $\int_0^T \int_0^T \int_A \int_A \rho_R(R(u, s), R(v, t); L_{SR}, L_{TR})\, du\, dv\, dt\, ds$ in (4) are reduced to single integrals (see appendix). Evaluation of cov $[T_r(u), T_r(v)|T]$ in (11), on the other hand, requires numerical integration of a double integral because $\rho_s(|v - u + U(s - t)|; L_{SR})$ is not symmetric in $(s - t)$.

Table 1. Climatological Mean Advection Velocities and Lag-1 (Hour) Lagrangian Autocorrelation Coefficients

Site	Advection Speed (Downwind Direction), km/hr	ρ_I (1 (hour); L_{TR})	ρ_I (1 (hour); L_{TI})
FTG (Denver, Colo.)	47 (SE)	0.64	0.23
GLD (Goodland, Kansas)	8 (E)	0.74	0.38
AMA (Amarillo, Tex.)	23 (NE)	0.68	0.43
DDC (Dodge City, Kansas)	40 (E)	0.59	0.39
FDR (Frederick, Okla.)	32 (E)	0.58	0.45
ICT (Wichita, Kansas)	18 (SE)	0.76	0.42
TLX (Twin Lakes, Okla.)	25 (SE)	0.62	0.47
INX (Tulsa, Okla.)	32 (E)	0.64	0.52
LZK (Little Rock, Ark.)	32 (E)	0.82	0.52

L_{TI} and L_{TR} are estimated by computing lag-1 (hour) autocorrelation coefficients as functions of spatial displacement. In general, the two mean advection velocities that yield the maximum lag-1 (hour) Lagrangian conditional and indicator autocorrelation coefficients are not identical. In such cases, conditional autocorrelation was taken to be a better measure for estimating advection velocity, and L_{TI} was specified accordingly. Table 1 summarizes climatological advection velocities and lag-1 (hour) temporal correlation coefficients as estimated from hourly WSR-88D rainfall data. The reader is referred to Figure 1 of *Seo and Smith* [1996] for the area map.

Specification of Soil and Soil Moisture Parameters

Infiltration sorptivity and gravitational infiltration rate, $S_i(u)$ and $A_o(u)$, $u \in A$, were specified as follows. First, based on work by *Milly and Eagleson* [1987], fields of porosity, $n(u)$, permeability, $k(u)$, and pore-size distribution index, $m(u)$, were generated over the largest catchment area. The maximum domain of A considered in this work was a 256 × 256 km^2 area, represented by a 64 × 64 grid. The mesh size of 4 × 4 km^2 corresponds to that of the radar rainfall data used in this work. Then the time-averaged soil moisture saturation in the surface boundary layer, $s_o(u)$, was computed for all $u \in A$ using the long-term water budget equation of *Eagleson* [1978c].

In generating the soil parameters we assumed that $n(u)$, ln $[k(u)]$ and ln $[m(u)]$ are normally distributed, mutually independent, and spatially white noise random processes (see work by *Milly and Eagleson* [1987] for justification). Two types of soil, clay loam and silt loam, were considered. Given the standard deviation of ln $(k(u))$, or $\sigma_{\ln(k(u))}$, those of $n(u)$ and ln $(m(u))$ were specified according to $\sigma_{n(u)} = 0.05\sigma_{\ln(k(u))}$ and $\sigma_{\ln(m(u))} = 0.4\sigma_{\ln(k(u))}$ [*Milly and Eagleson*, 1987]. In solving for $s_o(u)$, $u \in A$, the two types of climate-soil systems of *Eagleson* [1978c] were considered; humid climate–clay loam and semiarid climate–silt loam, as represented by Clinton, Massachusetts, and Santa Paula, California, respectively. The reader is referred to work by *Eagleson* [1978c] or *Milly and Eagleson* [1987] for climatic parameters. Tables 2 and 3 summarize the sample statistics of soil and soil moisture parameters used in this work: they are based on $\sigma_{\ln(k(u))} = 1$ and 1.4, respectively; $\sigma_{\ln(k(u))} = 1.4$ represents the maximum spatial variability of soil parameters for which the long-term water balance equation converged to solutions at all grid points.

Results

Before presenting the scale-variability relationship of areal runoff volume, it is informative to examine the effect of the parameters in the space-time correlation functions on the scale-variability relationship of areal rainfall volume. Figures 1 and 2 show the effect of storm advection at Denver, Colorado (FTG), for various values of the time interval, T. In producing the figures we have used (3), (4), (24), (25), and (26), and rainfall statistics at FTG. The reader is referred to Figure 5 of *Seo and Smith* [1996] for the scale-mean fractional coverage relationship, $m_Z(A)$, at this site. In each figure the upper- and the lowermost curves correspond to $T = 0.5$ and $T = 3.5$ hours, respectively, with an increment of 0.5 hour. As intuition

Table 2. Sample Statistics of Soil and Soil Moisture Parameters for $\sigma_{\ln(k(u))} = 1$

	Humid Climate Clay Loam (H)		Semiarid Climate Silt Loam (S)	
	Mean	CV	Mean	CV
$n(u)$	0.356	0.13	0.356	0.13
$m(u)$	0.297	0.36	0.693	0.36
$K(u)$*	0.088	1.23	0.376	1.23
$s_o(u)$	0.719	0.08	0.486	0.16
$A_o(u)$*	0.054	1.20	1.190	1.21
$S_i(u)$†	0.316	0.55	2.270	0.57

*In cm/h.
†In cm/h$^{1/2}$.

Table 3. Sample Statistics of Soil and Soil Moisture Parameters for $\sigma_{\ln(k(u))} = 1.4$

	Humid Climate Clay Loam (h)		Semiarid Climate Silt Loam (s)	
	Mean	CV	Mean	CV
$n(u)$	0.358	0.18	0.358	0.18
$m(u)$	0.298	0.54	0.696	0.54
$K(u)$*	0.085	2.42	0.365	2.42
$s_o(u)$	0.722	0.10	0.520	0.19
$A_o(u)$*	0.044	2.36	0.184	2.40
$S_i(u)$†	0.280	0.87	1.997	0.88

*In cm/h.
†In cm/h$^{1/2}$.

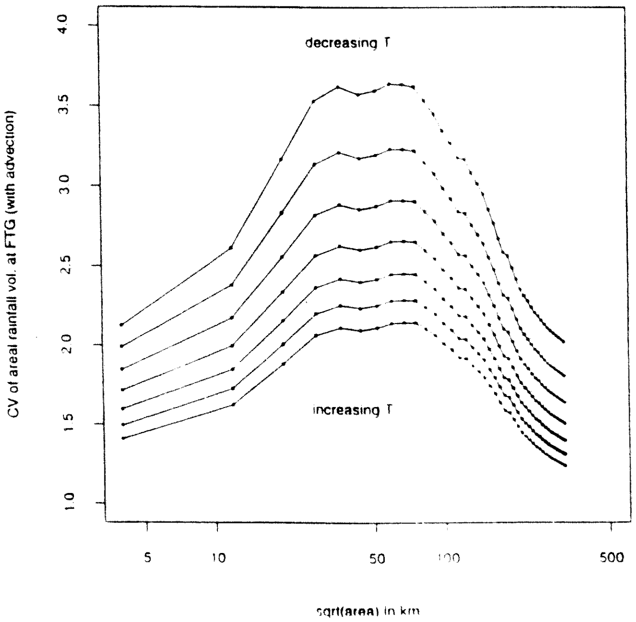

Figure 1. Scale-variability relationship of areal runoff volume at FTG with storm advection taken into account. The upper- and the lowermost curves correspond to T = 0.5 and 3.5 hours, respectively. The increment is 0.5 hour.

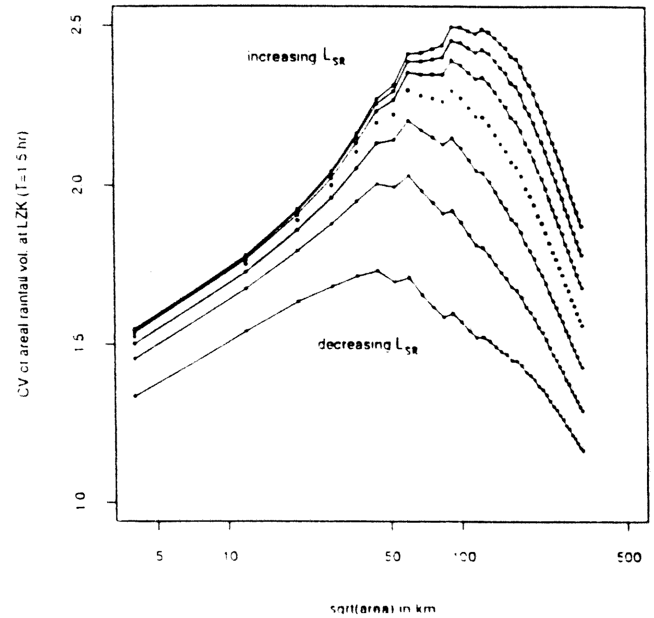

Figure 3. Sensitivity of scale-variability relationship of areal runoff volume at LZK on L_{SR}. The unconnected markers are based on observed parameter values. The increment is 10 km.

would suggest, advection tends to dampen the variability of areal rainfall volume.

Figures 3 through 7 show sensitivities of the scale-variability relationship of areal rainfall volume on L_{SR}, L_{SI}, L_{TR}, L_{TI}, and CV_R ($\equiv \sigma_R/m_R$), respectively. In each figure the unconnected markers represent the relationship based on observed parameter values at Little Rock, Arkansas (LZK), whereas the rest of the curves are obtained by varying the value of the particular parameter in question. The reader is referred to Figure 5 of *Seo and Smith* [1996] for the scale-mean fractional coverage relationship, $m_Z(A)$, at this site. The increment is 10 km for L_{SR} and L_{SI}, 0.1 for ρ_L(1 (hour); L_{TR}) and ρ_L(1 (hour); L_{TI}), and 0.5 for CV_R. The figures serve to illustrate intricate dependencies of the scale-variability relationship of areal rainfall volume on the five parameters, L_{SR}, L_{SI}, L_{TR}, L_{TI}, and CV_R, that collectively characterize the local rainfall climatology.

Figures 8 through 10 show scale-variability relationships of areal runoff volume at various values of T, as obtained from (17) and (19). They are based on rainfall statistics at LZK. Similar characteristics are observed when rainfall statistics at

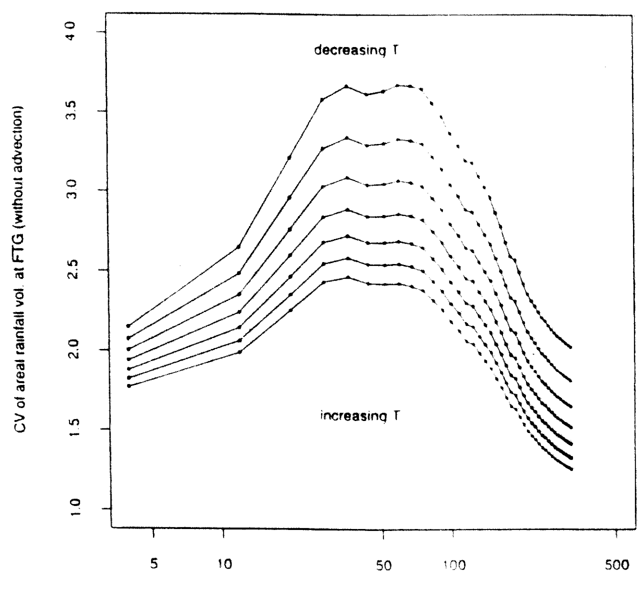

Figure 2. Same as Figure 1, but with no advection assumed.

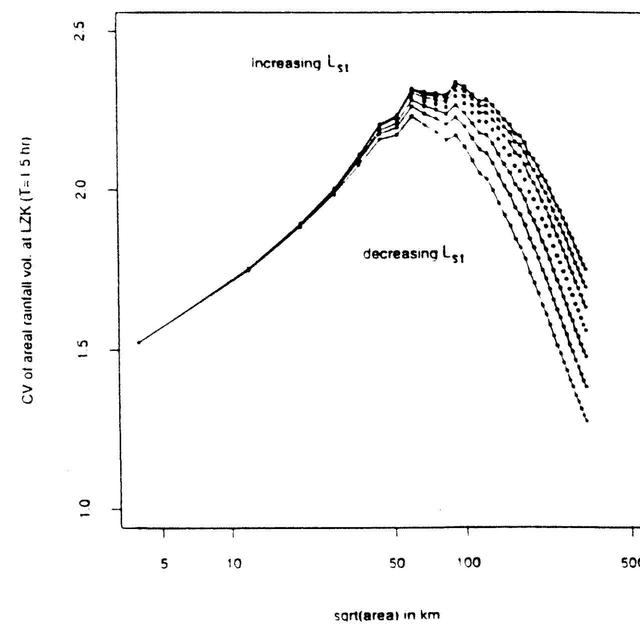

Figure 4. Same as Figure 3, but on L_{SI}.

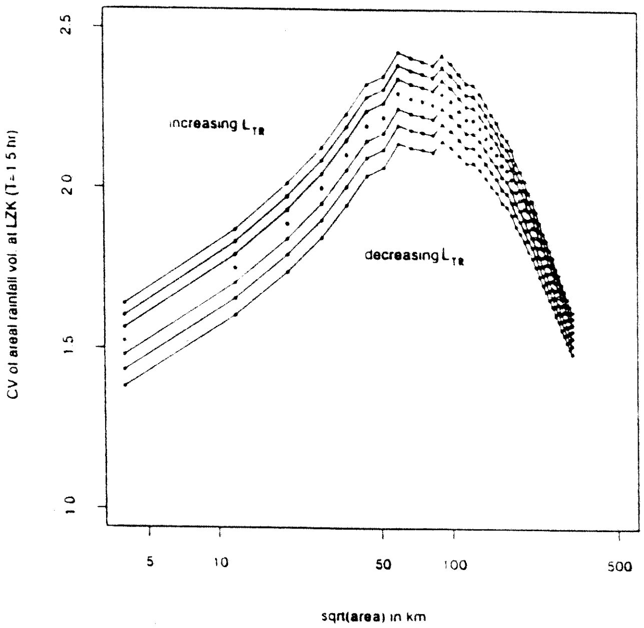

Figure 5. Same as Figure 3, but on L_{TR}. The increment is 0.1.

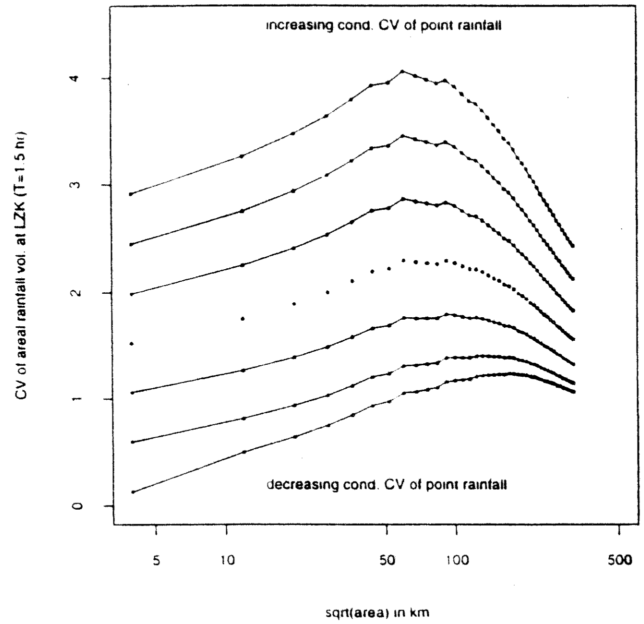

Figure 7. Same as Figure 3, but on CV_R. The increment is 0.5.

other sites are used. In each figure the curves marked by N, H, and S represent those under no infiltration, for the humid climate–clay loam system, and for the semiarid climate–silt loam system, respectively (see Table 2 for the sample statistics of soil and soil moisture parameters). The figures may be summarized as follows. Climatological variability of areal runoff volume due to infiltration excess over heterogeneous soil is greater than that of areal rainfall volume for both climate-soil systems. It is significantly greater for the semiarid climate–silt loam system, particularly over catchment scales exceeding 1,000 km^2, than for the humid climate–clay loam system. For semiarid climate the scale-variability relationship of areal runoff volume exhibits a more pronounced peak of maximum variability than that of areal rainfall volume. Also, the catchment scale at which the peak variability of areal runoff volume occurs is larger than the catchment scale associated with that of areal rainfall volume.

By using radar rainfall data from the southern plains, we are by necessity assuming that the rainfall climatology in California and Massachusetts shares the same relationship between catchment scale and mean fractional coverage, space-time con-

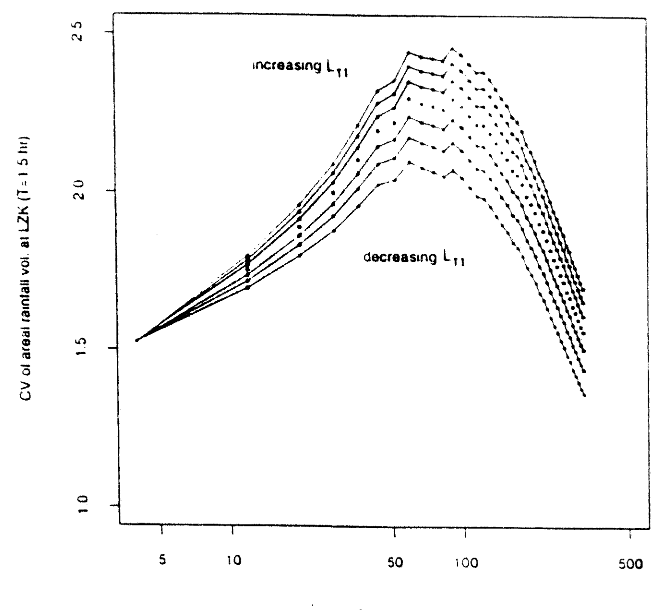

Figure 6. Same as Figure 5, but on L_{11}.

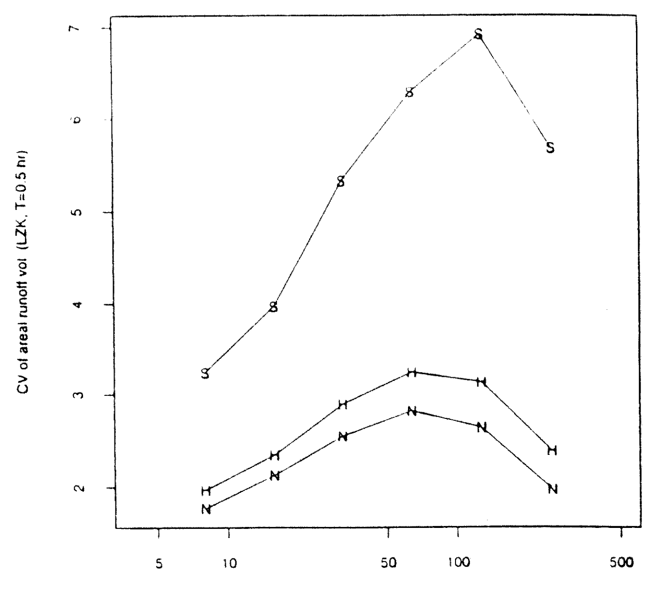

Figure 8. Scale-variability relationships of areal runoff volume for T = 0.5 hours, based on rainfall statistics at LZK. N, H, and S represent no infiltration, humid climate–clay loam system, and semiarid climate–silt loam system, respectively.

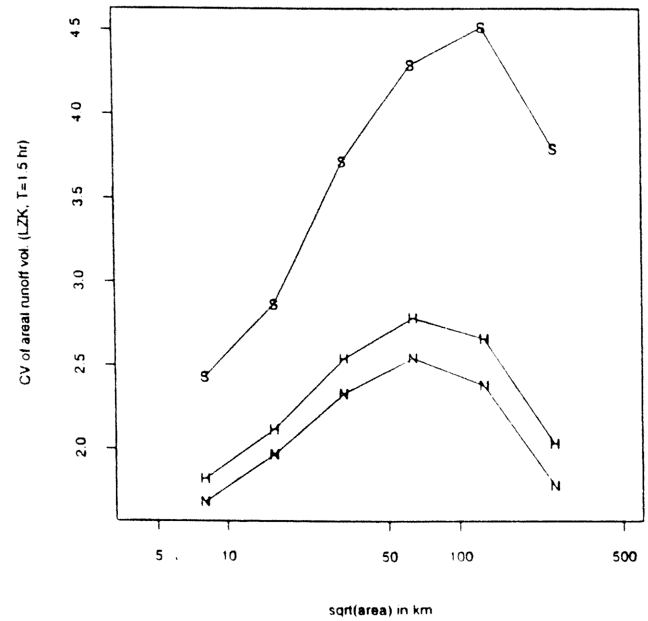

Figure 9. Same as Figure 8, but for $T = 1.5$ hours.

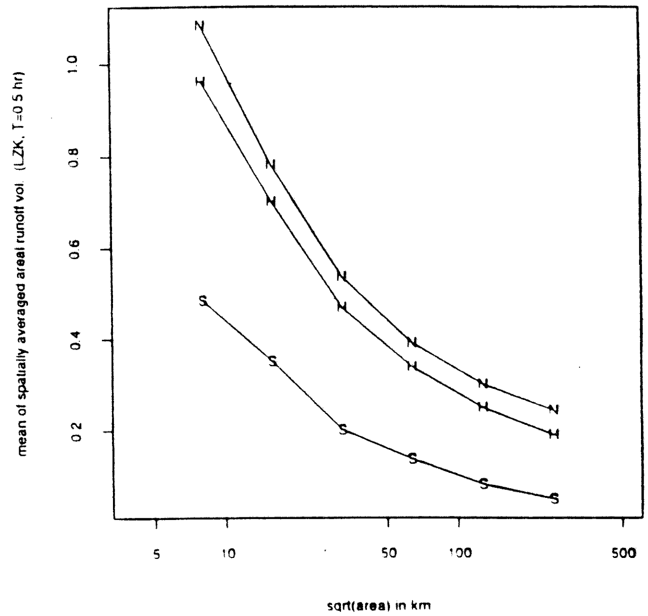

Figure 11. Same as Figure 8, but for spatially averaged climatological mean of areal runoff volume.

ditional and indicator correlation structures, and coefficient of variation of positive point rainfall. It is argued here that the resulting scale-variability relationships of areal runoff volume are not only correct qualitatively but also reasonable quantitatively for the following reasons: (1) regardless of climate regime, mean fractional coverage is a monotonically decreasing function of catchment scale bounded between 0 and 1 [see *Seo and Smith*, 1996]; (2) regardless of climate regime, space-time correlation structures are monotonically decreasing functions of spatial lag distance and time lag bounded between 0 and 1; and (3) coefficient of variation of positive point rainfall, which is the only non-normalized parameter, depends largely on microphysics of rain formation (and hence is less susceptible to regional variations than the other parameters).

Figures 11 and 12 show examples of climatological mean and standard deviation of spatially averaged areal runoff volume (i.e., areal runoff volume divided by the catchment area), given raining somewhere in A over $[0, T]$, as obtained from (17) and (19), respectively. The ratio of the standard deviation to the mean produces Figure 8. The figures indicate that infiltration reduces standard deviation of areal runoff volume for both climate-soil systems. However, the relative reduction in the standard deviation of areal runoff volume is smaller, particu-

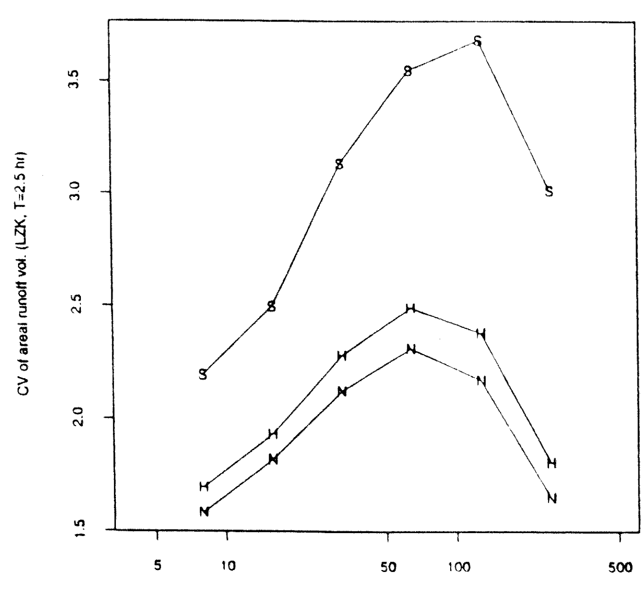

Figure 10. Same as Figure 8, but for $T = 2.5$ hours.

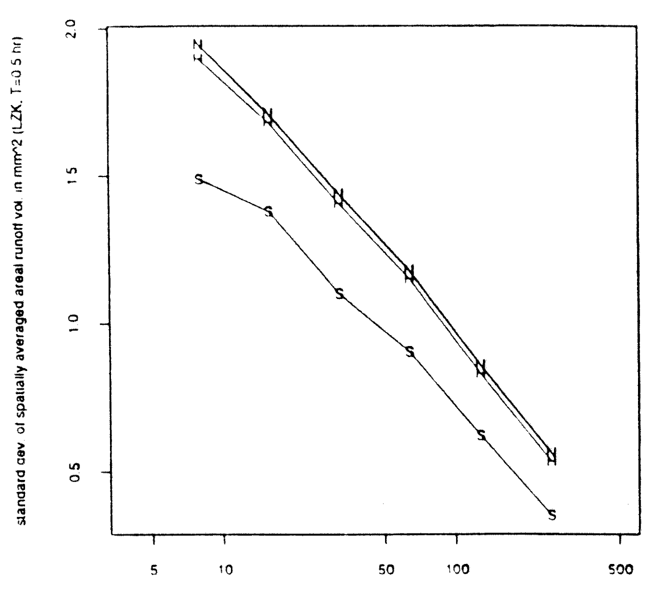

Figure 12. Same as Figure 11, but for spatially averaged climatological standard deviation of areal runoff volume.

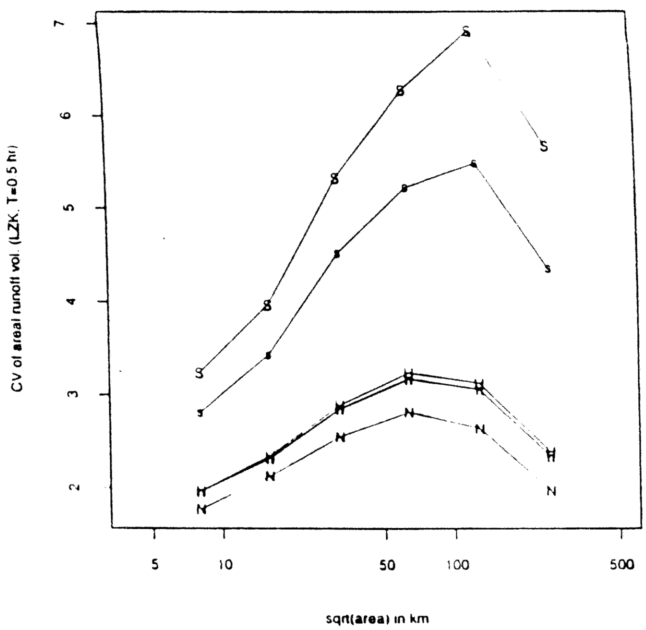

Figure 13. Same as Figure 8, but based on $\sigma_{\ln(k(u))} = 1.4$; n and s correspond to no infiltration and semiarid climate silt loam system for $\sigma_{\ln(k(u))} = 1.4$.

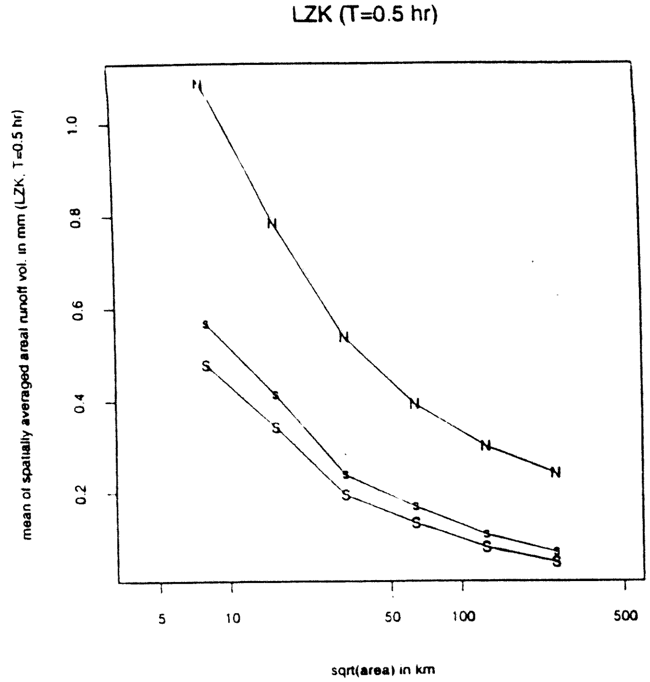

Figure 14. Same as Figure 13, but for spatially averaged climatological mean of areal runoff volume.

larly for the semiarid climate–silt loam system, than that in the mean, hence resulting in amplification of CV.

Because the S and the H curves in the figures assume infiltration everywhere in the rain area, whereas the N curves assume no infiltration anywhere, each pair of S-N and H-N curves may be considered to form bounds for the scale-variability relationship of areal runoff volume due to infiltration and saturation excesses combined. Quantitative assessment of the effect of saturation excess was beyond the scope of this work. We only note here that given a distributed-parameter hydrological model capable of delineating variable source areas, such an assessment can be readily accommodated in the formulation described in this work.

We now briefly turn our attention to the effect of increased spatial variability in soil parameters. The scale-variability relationships of areal runoff volume presented thus far are all based on $\sigma_{\ln(k(u))} = 1.0$. Figure 13 shows an example of the scale-variability relationships based on $\sigma_{\ln(k(u))} = 1.4$. The curves marked by h and s correspond to the humid climate–clay loam and the semiarid climate–silt loam systems, respectively (see Table 3 for the sample statistics of soil and soil moisture parameters). Also shown in the figure are those based on $\sigma_{\ln(k(u))} = 1.0$ (marked by H and S) for comparison purposes. Similarly, Figures 14 and 15 show examples of the mean and the standard deviation, respectively, of spatially averaged areal runoff volume for the semiarid climate–silt loam system based on $\sigma_{\ln(k(u))} = 1.0$ and 1.4. Note that the increase in $\sigma_{\ln(k(u))}$ (and subsequent changes in $\sigma_{n(u)}$ and $\sigma_{\ln(m(u))}$) also increases not only the standard deviation of runoff volume but also the mean runoff volume. It is hence difficult to ascertain the pure effect of increased variability in soil parameters on the scale-variability relationship of areal runoff volume. Qualitatively, we may consider that an increase in spatial variability of soil parameters is equivalent to an increase in CV_R and/or a decrease in L_{sR}. The net effect, however, is not clear because the former would sharpen the scale-variability relationship (see Figure 7), whereas the latter would flatten it (see Figure 3). In Figures 14 and 15, the increase in $\sigma_{\ln(k(u))}$ does not seem to affect the standard deviation of runoff volume as much as it does its mean: it is seen to suggest that increased spatial variability of soil parameters beyond a certain level may not be as important a factor as

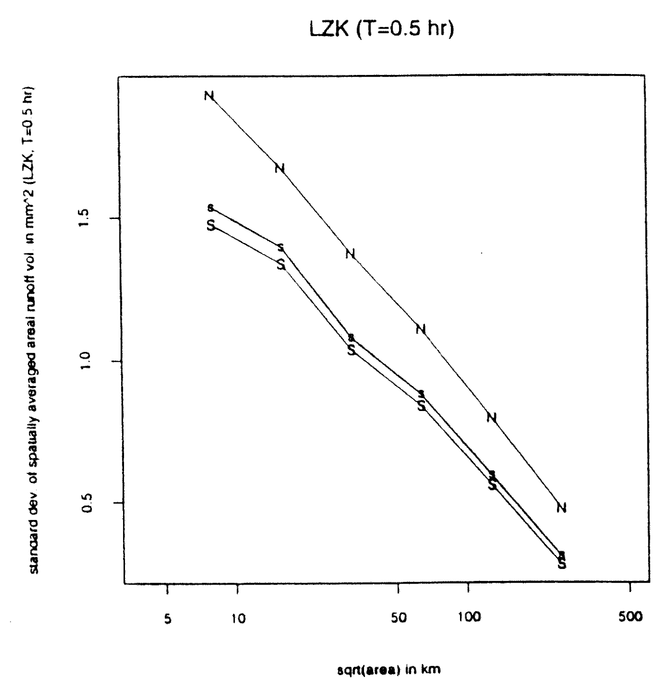

Figure 15. Same as Figure 14, but for spatially averaged climatological standard deviation of areal runoff volume.

their mean values in shaping the scale-variability relationship of areal runoff volume.

Summary and Conclusions

The relationship between catchment scale and climatological variability of areal surface-runoff volume is evaluated. The measure used to quantify the variability is coefficient of variation. Given the point description of runoff volume following infiltration excess under the time compression approximation [*Eagleson*, 1978b], climatological mean and variance of areal runoff volume over heterogeneous soil are evaluated by integrating second-order statistics of point runoff volume. The second-order statistics of rainfall duration and intensity, required for evaluation of those of point runoff volume, are obtained from climatological mean and variance of areal rainfall volume under fractional coverage considerations. Hourly WSR-88D rainfall data over the southern plains of the United States were used to estimate rainfall-related statistics, including space-time correlation functions for inner variability and intermittency of rainfall, following a set of simplifying assumptions. The two contrasting climate-soil systems of *Eagleson* [1978c] were used in evaluating the scale-variability relationship of areal runoff volume. Soil fields were specified based on work by *Milly and Eagleson* [1987] following a set of assumptions on spatial variability and interdependency of soil parameters. Soil moisture fields are specified by solving the one-dimensional long-term water balance equation of *Eagleson* [1978c]. The scale-variability relationships of areal runoff volume thus obtained provides, as a function of catchment scale and integration time interval, quantitative estimates of long-term subgrid scale variability of areal runoff volume following rainfall over heterogeneous soil.

The results show that given that it rains somewhere in the area over the integration time interval, (1) the climatological variability of areal runoff volume over heterogeneous soil, as measured by its coefficient of variation, is significantly greater in semiarid to arid climates, particularly over catchment scales exceeding 1,000 km^2, than that of areal rainfall volume; and (2) the scale-variability relationship of areal runoff volume has a more pronounced peak of maximum variability than that of areal rainfall volume. Stated differently, the results indicate that in a humid climate, climatological variability of areal surface-runoff volume and its dependence on catchment scale are attributable largely to subgrid-scale variability of rainfall, whereas in semiarid to arid climates, they are attributable to subgrid-scale variabilities of rainfall, soil, and soil moisture. The implication in large-scale hydrological modeling is that extreme care must be taken in choosing grid size and integration time interval, in relation to the particular subgrid-scale parameterization used, to realistically capture natural variabilities of areal rainfall and runoff volumes and in ascertaining scale-dependent variabilities (or uncertainties) associated with model input and output, such as areal rainfall and runoff volumes, from those associated with model errors.

Because of the time compression approximation used in describing infiltration, within-storm variability of rainfall could not be explicitly taken into account in this work. Its effect and the effect of spatial variability of soil parameters need further study.

Appendix

Here we evaluate the sextuple integral of the following form:

$$J = \int_0^T \int_0^T \int_A \int_A \rho(R(u, s), R(v, t);$$

$$L_{SR}, L_{TR})\, du\, dv\, dt\, ds \qquad \text{(A1)}$$

Assuming that the space-time correlation structure is separable in Lagrangian domain, we may write

$$J = \int_0^T \int_0^T \int_A \int_A \rho_s(|v - u + U(s - t)|, L_s)\, du\, dv$$

$$\cdot \rho_T(|s - t|, L_t)\, ds\, dt \qquad \text{(A2)}$$

We first evaluate the inner integral by rewriting it as

$$J_i(s - t) = \int_A \int_A \rho_s(|v - u + U(s - t)|, L_s)\, du\, dv \qquad \text{(A3a)}$$

$$= \int_A \int_{A+h} \rho_s(|v - u|, L_s)\, du\, dv \qquad \text{(A3b)}$$

where $h = U(s - t)$.

Using the Cauchy-Gauss method for nonoverlapping areas of integration [*Journel and Huijbregts*, 1978, p. 99], we have

$$J_i(s - t) = \int_{-l_2}^{l_2} \int_{-l_1}^{l_1} \rho_s(|u + h|, L_s)(l_1 - |u_1|)(l_2 - |u_2|)\, du_1\, du_2 \qquad \text{(A4a)}$$

$$= \int_0^{l_2} \int_0^{l_1} \{\rho_s(\{(u_1 + h_1)^2 + (u_2 + h_2)^2\}^{1/2}, L_s)$$

$$+ \rho_s(\{(u_1 - h_1)^2 + (u_2 - h_2)^2\}^{1/2}, L_s)$$

$$+ \rho_s(\{(u_1 - h_1)^2 + (u_2 + h_2)^2\}^{1/2}, L_s)$$

$$+ \rho_s(\{(u_1 + h_1)^2 + (u_2 - h_2)^2\}^{1/2}, L_s)\}$$

$$\cdot (l_1 - u_1)(l_2 - u_2)\, du_1\, du_2 \qquad \text{(A4b)}$$

The first term in (A4b), for example, is given by

$$\gamma(h_1, h_2) = \int_{l_2}^{l_2} \int_{l_1}^{l_1} \rho_s(\{(u_1 + h_1)^2 + (u_2 + h_2)^2\}^{1/2}, L_s)$$

$$\cdot (l_1 - u_1)(l_2 - u_2)\, du_1\, du_2 \qquad \text{(A5a)}$$

$$= (\tfrac{1}{4})(l_1 + h_1)(l_2 + h_2) L_s^2 \pi$$

$$\cdot \{\operatorname{erfc}(h_1/L_s) - \operatorname{erfc}((l_1 + h_1)/L_s)\}$$

$$\cdot \{\operatorname{erfc}(h_2/L_s) - \operatorname{erfc}((l_2 + h_2)/L_s)\}$$

$$- (\tfrac{1}{4})(l_1 + h_1) L_s^3 \pi^{1/2}$$

$$\cdot \{\operatorname{erfc}(h_1/L_s) - \operatorname{erfc}((l_1 + h_1)/L_s)\}$$

$$\cdot \{\exp(-h_2^2/L_s^2 - \exp(-(l_2 + h_2)^2/L_s^2)\}$$

$$
\begin{aligned}
&- (\tfrac{1}{4})(l_2 + h_2) L_s^3 \pi^{1/2} \{\mathrm{erfc}\ (h_2/L_s) - \mathrm{erfc}\ ((l_2 + h_2)/L_s)\} \\
&\cdot \{\exp\ (-h_1^2/L_s^2 - \exp\ (-(l_1 + h_1)^2/L_s^2)\} \\
&+ (\tfrac{1}{4}) L_s^4 \{\exp\ (-h_1^2/L_s^2 - \exp\ (-(l_1 + h_1)^2/L_s^2)\} \\
&\cdot \{\exp\ (-h_2^2/L_s^2 - \exp\ ((l_2 + h_2)^2/L_s^2)\} \qquad \text{(A5b)}
\end{aligned}
$$

where erfc ( ) denotes the complementary error function.

(A2) is thus reduced to

$$J = \int_0^T \int_0^T J_t(s - t) \rho_L(|s - t|, L_t)\, dt\, ds \qquad \text{(A6)}$$

where

$$
\begin{aligned}
J_t(s - t) = {} & \gamma(h_1, h_2) + \gamma(-h_1, -h_2) \\
& + \gamma(-h_1, h_2) + \gamma(h_1, -h_2) \qquad \text{(A7)}
\end{aligned}
$$

By once again applying the Cauchy-Gauss method to (A6), (A1) is finally reduced to the following single integral:

$$J = 2 \int_0^T J_t(u_1) \rho_L(u_1, L_t)(T - u_1)\, du_1 \qquad \text{(A8)}$$

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