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Real-Time Dynamic Flood Routing with NWS FLDWAV Model using Kalman Filter Updating

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Abstract

Dynamic flood routing models based on the four-point implicit finite-difference solution of the complete one-dimensional Saint-Venant equations of unsteady flow are inherently deterministic. Such a model (FLDWAV) developed by the National Weather Service for real-time flood forecasting has been enhanced with a stochastic estimator based on an extended Kalman filter to provide optimal updating capabilities utilizing real-time observed river stages. The stochastic enhancement is described, and selected applications of the enhanced model spanning a wide range of unsteady flows are presented.

Introduction

Channel flood routing is important in improving the transport of water through man-made or natural waterways and in determining necessary actions to protect life and property from the effects of flooding. Many channel routing models have been developed, and those based on the complete one-dimensional hydrodynamic equations (Saint-Venant) have found increasing use in the engineering community. Such dynamic channel routing models are based entirely on deterministic considerations, and their accuracy is largely dependent on the accuracy of the model input, such as the specified hydraulic parameters within the mathematical equations used by the model, as well as boundary and initial conditions. Traditional deterministic methods cannot reflect the effects of possible inaccuracies in the equations, parameters, and boundary and initial conditions. When model results are applied to engineering practice, a margin of safety is often assigned to provide some degree of protection against the unknown effects. On the other hand, statistical models are receiving more attention because of their capability of reflecting the effects of uncertainties in the accuracy of the mathematical model, hydraulic parameters, and boundary and initial conditions. The Kalman filter is a

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statistical method that provides an updating technique to improve the simulation of unsteady flows for real-time river flood forecasting.

The U.S. National Weather Service (NWS) has been developing a new dynamic channel flood routing model (FLDWAV) to replace the popular DAMBRK and DWOPER dynamic models (Fread, 1985). A recent enhancement to the FLDWAV model is the addition of a stochastic real-time estimator for optimal updating of the model's predictions using real-time observations of river stages. In this paper, the technique of real-time dynamic flood routing using the NWS FLDWAV model with Kalman filter updating is presented. The FLDWAV model is based on implicit, nonlinear finite-difference approximations of the one-dimensional Saint-Venant equations of unsteady flow. The stochastic estimator uses an extended Kalman filter to provide optimal updating estimates. These are achieved by combining the predictions of the FLDWAV model with real-time observations modified by the Kalman filter gain factor. An efficient inverse matrix solution technique is used to determine the transition matrix in computing the Kalman filter gain factor. Selected applications of real-time estimation with the enhanced FLDWAV model, spanning a wide range of types of flood waves, are presented.

FLDWAV Algorithm and Features

Basic Equations. The FLDWAV (Fread, 1985) model is a generalized channel flood routing model. It is based on an implicit finite-difference solution of the conservation form of the extended Saint-Venant equations of unsteady flow. The basic equations are:

$$\partial Q/\partial x + \partial s_c(A+A_0)/\partial t - q = 0 \tag{1}$$

$$\partial s_m Q/\partial t + \partial(Q^2/A)/\partial x + gA\ (\partial h/\partial x + S_f + S_e) = 0 \tag{2}$$

where: $$S_f = n^2|Q|Q/(\lambda A^2 R^{4/3}) \text{ and } S_e = K_e/2g \cdot \partial(Q/A)^2/\partial x \tag{3}$$

in which x is distance along the longitudinal axis of the waterway, t is time, Q is discharge, A is active cross-sectional area, A_0 is inactive (off-channel storage) cross-sectional area, q is lateral inflow (positive) or outflow (negative), s_c and s_m are depth-dependent sinuosity correction factors, g is the gravity acceleration constant, h is water surface elevation, S_f is friction slope computed via the Manning formula, n is the Manning's resistance coefficient, R is the hydraulic radius, S_e is the local loss slope, K_e is an expansion (negative) or contraction (positive) coefficient, $\lambda = 1$ for the metric system of units and $\lambda = 2.21$ for the English system.

Solution Algorithm. The four-point weighted, implicit finite-difference approximation is used in FLDWAV to transform the continuous, nonlinear partial differential equations of Saint-Venant into nonlinear algebraic equations. For a river delineated with N cross-sections, the four-point discretization algorithm produces the following 2N-2 equations:

$$f_{2i}(Q_i^{j-1}, h_i^{j-1}, Q_{i+1}^{j-1}, h_{i+1}^{j-1}, Q_i^j, h_i^j, Q_{i+1}^j, h_{i+1}^j) = 0 \quad i=1, N-1 \tag{4}$$

$$f_{2i+1}(Q_i^{j-1}, h_i^{j-1}, Q_{i+1}^{j-1}, h_{i+1}^{j-1}, Q_i^j, h_i^j, Q_{i+1}^j, h_{i+1}^j) = 0 \quad i=1, N-1 \tag{5}$$

in which i refers to the i-th cross section along the river, j-1 and j denote the number of the time line in the x-t solution domain. Since the stages and discharges at (j-1)-th time-

line, $(Q_i, h_i, Q_{i+1}, h_{i+1})_{j-1}$ are known, the state variables in Eqs. (4-5) are Q and h at j-th time line $(Q_i, h_i, Q_{i+1}, h_{i+1})_j$.

Equations (4-5), together with two boundary equations, form a system of discrete, implicit, nonlinear equations which define the relationship of the state variables $(Q_1,h_1, \ldots. Q_N,h_N)$ between the j-th time line and the (j-1)-th time line; this system can be expressed as:

$$\mathbf{F}(\mathbf{Y}_{j-1}, \mathbf{Y}_j, t_{j-1}, t_j) = 0 \tag{6}$$

in which **F** is a vector of functions as defined by Eqs. (4-5) and the two boundary equations, and **Y** is a vector of state variables with 2N components, i.e.,

$$y_j(2i-1) = (Q_i)_j; \quad y_j(2i) = (h_i)_j \qquad (i=1,\ldots N) \tag{7}$$

$$\mathbf{Y}_j = (Q_1, h_1, \ldots., Q_N, h_N)_{t=t_j} \tag{8}$$

The Newton-Raphson functional iterative method is used in FLDWAV to solve Eq. (6). The initial conditions, Y(t=0), are automatically obtained within FLDWAV via a steady flow backwater computation or specified as data input for unsteady flows occurring at t=0.

Special features. The FLDWAV model has several special features including: (1) simulation of flows in a single channel or multiple interconnected channels (network); (2) simulation of flows that change from subcritical to supercritical or conversely; (3) simulation of flows that change from free surface to pressurized or conversely; (4) treatment of sinuosity effects of meandering rivers; (5) an assortment of internal boundary conditions to simulate time-dependent dam breaches, gate controlled flows, assorted spillway flows, bridge/roadway-embankment overtopping flows, and levee overtopping and crevasse flows; (6) an assortment of specified external boundary conditions for discharge time-series, water surface elevation time-series, stage-discharge single-valued or looped relations; (7) automatic calibration of Manning's n; (8) automatic selection of computational time and distance steps; and (9) the metric or English system of units.

Extended Kalman Filter Enhancement of FLDWAV

Eqs. (4-5), along with two boundary equations, form a system of discrete, implicit nonlinear equations as represented by Eq. (6). In order to account for the uncertainties existing in the mathematical equations, as well as boundary and initial conditions, one can transform this deterministic dynamic system into a stochastic dynamic system by adding Gaussian white noise processes to the equations. Eq. (6) can be rewritten in a stochastic sense as:

$$\mathbf{F}(\mathbf{Y}_{j-1}, \mathbf{Y}_j, t_{j-1}, t_j) = \mathbf{W}_{j-1} \tag{9}$$

in which **Y** is the vector of system state variables defined by Eq. (8), j refers to the state at j-th time line $(t=t_j)$, **W** is a Gaussian white noise vector with covariance **Q**, i.e.,

$$\mathbf{E}(\mathbf{W}_j) = 0; \quad \mathbf{cov}(\mathbf{W}_j, \mathbf{W}_k) = \mathbf{E}(\mathbf{W}_j \mathbf{W}_k^T) = \mathbf{Q}_j \delta_{jk} \tag{10}$$

where δ is the Krnoecker operator.

Assuming that real-time measurements (observations) of river water stages and/or discharges at gaging stations along the river are available at discrete times $t_j (j=0,1,2,....)$ and the errors in the measurements are represented by white noise processes, one can derive a system of measurement equations at the x-coordinates corresponding to the locations of gaging stations. The measurement equations consist of a linear combination of the system state variables (corrupted by uncorrelated noise) which can be written in vector-matrix notation as:

$$\mathbf{Z}_j = \mathbf{H}_j \mathbf{Y}_j + \mathbf{V}_j \tag{11}$$

where $\mathbf{Z}_j$ is the set of measurements at time t_j, and $\mathbf{H}_j$ is the measurement matrix at time t_j; it describes the linear combination of state variables which comprise $\mathbf{Z}_j$ in the absence of noise. $\mathbf{V}_j$ is the noise associated with the errors of the measurements; it has the following statistics:

$$E(\mathbf{V}_j) = 0; \quad \mathrm{cov}(\mathbf{V}_j, \mathbf{V}_k) = E(\mathbf{V}_j \mathbf{V}_k^T) = \mathbf{R}_j \delta_{jk} \tag{12}$$

in which $\mathbf{R}$ is the covariance of $\mathbf{V}$.

In order to apply a linear Kalman filtering algorithm to the discrete nonlinear, implicit dynamic system (Eq. (9)) to obtain a practical optimal estimation for $\mathbf{Y}_j$ using updated information (the new measurement, $\mathbf{Z}_j$), one can expand $\mathbf{F}$ in Eq. (6) in a Taylor series about a discrete reference state trajectory to get a linearized system. Using the 4-pt implicit finite-difference solution algorithm of the FLDWAV model, the predictive estimation for the state variables at $t=t_j$ (denoted as $\mathbf{Y}_{j|j-1}$) is obtained from Eq. (6) with $\mathbf{Y}_{j-1}$ replaced by the previous optimal estimation $\mathbf{Y}_{j-1|j-1}$. The nonlinear, implicit equation (left hand of Eq. (9)) can be linearized about $\mathbf{Y}_{j-1|j-1}$ and $\mathbf{Y}_{j|j-1}$ using the Taylor expansion and retaining only the first-order approximation. Eq. (9) is thus transformed into the following linear stochastic system:

$$\mathbf{Y}_j^* = \mathbf{\Phi}_{j,j-1} \mathbf{Y}_{j-1}^* + \mathbf{W}_{j-1} \tag{13}$$

$$\mathbf{Y}_j^* = \mathbf{Y}_j - \mathbf{Y}_{j|j-1}; \quad \mathbf{Y}_{j-1}^* = \mathbf{Y}_{j-1} - \mathbf{Y}_{j-1|j-1} \tag{14}$$

$$\mathbf{\Phi}_{j,j-1} = -\left[\left(\frac{\partial \mathbf{F}}{\partial \mathbf{Y}_j}\right)^{-1} \left(\frac{\partial \mathbf{F}}{\partial \mathbf{Y}_{j-1}}\right) \right]_{(\mathbf{Y}_{j|j-1}, \mathbf{Y}_{j-1|j-1})} \tag{15}$$

The covariances of $\mathbf{Y}_j^*$ and $\mathbf{Y}_{j-1}^*$ are equal to those of $\mathbf{Y}_j$ and $\mathbf{Y}_{j-1}$, respectively. The real-time estimator is thus available within FLDWAV in this particular application of the linear Kalman filtering algorithm. The filtering algorithm can be summarized via the following steps: (1) based on the optimal estimation of $\mathbf{Y}$ at t_{j-1} ($\mathbf{Y}_{j-1|j-1}$), a predictive estimation of $\mathbf{Y}$ for the new time t_j ($\mathbf{Y}_{j|j-1}$) is computed from the FLDWAV model; (2) the covariance of this predictive estimation ($\mathbf{P}_{j|j-1}$) is computed by the following equation:

$$\mathbf{P}_{j|j-1} = \mathbf{\Phi}_{j,j-1} \mathbf{P}_{j-1|j-1} \mathbf{\Phi}_{j,j-1}^T + \mathbf{Q}_{j-1} \tag{16}$$

in which $\mathbf{P}_{j-1|j-1}$ is the covariance of $\mathbf{Y}_{j-1|j-1}$; (3) the Kalman gain matrix for time t_j is determined by the following equation:

$$\mathbf{K}_j = \mathbf{P}_{j|j-1} \mathbf{H}_j^T (\mathbf{H}_j \mathbf{P}_{j|j-1} \mathbf{H}_j^T + \mathbf{R}_j)^{-1} \tag{17}$$

(4) when the new measurement (Z_j) is available, the predictive estimation is updated to produce the optimal state estimation of Y for time t_j, ($Y_{j|j}$), by applying the following equation:

$$Y_{j|j} = Y_{j|j-1} + K_j[Z_j - H_j Y_{j|j-1}] \tag{18}$$

(5) the covariance of $Y_{j|j}$ is computed by the following equation:

$$P_{j|j} = [I - K_j H_j] P_{j|j-1} \tag{19}$$

in which **I** is the identity matrix; and (6) steps 1 through 5 are repeated, incrementing the time step.

Applications

The first application of the model is a 291.7 mile reach of the Lower Mississippi River (LM) from Red River Landing to Venice. A total of 25 cross-sections located at unequal intervals ranging from 5 to 20 miles are used to describe the reach. The average channel bottom slope is a very flat (0.0000064). Typical rising time of the flood waves is about 30 days. The discharge hydrograph and rating curve are used as upstream and downstream boundary conditions in the simulation. The Manning's n vs. discharge relation for each reach bounded by gauging stations is calibrated within the FLDWAV model using the 1969 spring flood. In this example, a historical flood (the 1963 flood) is used to check the performance of the real-time estimator. The accuracy of predictions of any flood routing model depends on the accuracy of the specified boundary conditions. The performance of the real-time estimator used in a case where the boundary conditions are not correct is presented in Figures 1-2. Figure 1 shows the observed upstream boundary discharge hydrograph and three simulated boundary conditions with errors, and Figure 2 presents the average RMS error of eight gauging stations vs. the forecasting time with and without the Kalman filter updating. In all three cases, significant improvement in the predictions is achieved when using the Kalman filter; the improvement increases as the forecasting time is reduced from the 5-day to lesser lead-times.

The second example is the 130-mile reach of the lower Columbia River (C) below Bonneville Dam, including the 25-mile tributary reach of the lower Willamette River. This reach of the Columbia has a very flat slope (0.000011), and the flows are affected by the tide from the Pacific Ocean. The tidal effect extends as far upstream as the tailwater of Bonneville Dam during periods of low flow. Reversals in discharge due to tidal effects during low flow are possible as far as 110 miles upstream. Typical rising time in tidal fluctuations is about 6 hours. A total of 27 cross sections with unequal distance intervals ranging from 0.6 to 12 miles are selected to describe the river system. Upstream and downstream boundaries are observed discharges and stages, respectively. The Manning n vs. water elevations relations were calibrated using FLDWAV for a 4-day period in 1974. The real-time estimator is applied to a 2-day period in 1979. Since the flow in this reach is significantly affected by the backwater of the tide, the model response to the accuracy of the downstream boundary condition is presented in Figures 3-4. Figure 3 shows a period of observed stage hydrograph and two simulated boundary conditions with different types of error. Figure 4 compares the average RMS error of four intermediate gauging stations vs. the forecast lead-time. It shows that

significant improvement in predictions can be achieved only when forecast lead-time is less than about 4 hours in this rapidly varied wave situation.

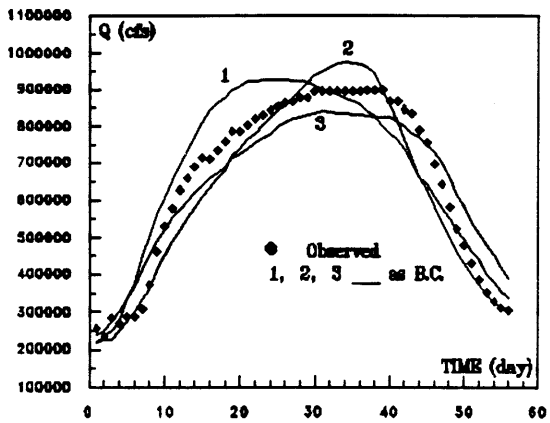

Figure 1 Upstream Boundary (LM)

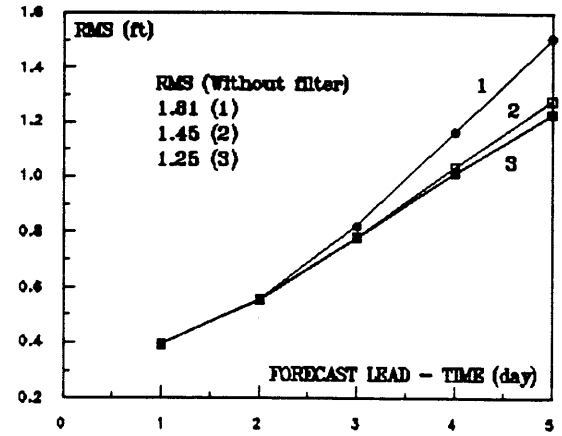

Figure 2 Averaged RMS's (LM)

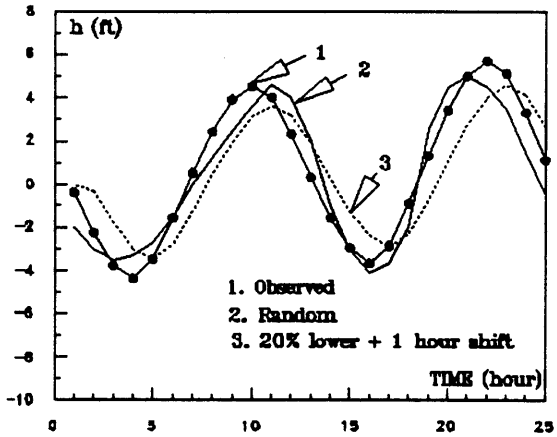

Figure 3 Downstream Boundary (C)

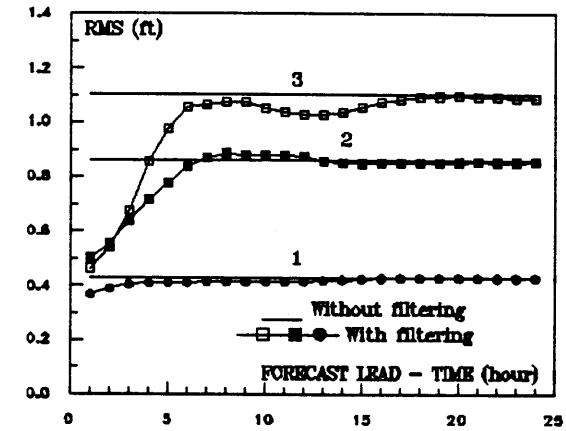

Figure 4 Averaged RMS's (C)

Conclusions

The Kalman filter type real-time estimator based on one-dimensional hydrodynamic equations is very useful when it is combined with the flood routing model FLDWAV. It uses the well-developed algorithms of FLDWAV and provides the generalized channel routing model with the stochastic analysis capability and the function of updating by optimal use of the real-time on-line observations. Computationally efficient improvements in real-time flood forecasting can be achieved using this enhanced flood routing model for typical river floodwaves; however, negligible improvements are obtained for tidal generated waves except for lead-times less than 4 hours.

Reference

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