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Rainfall estimation using raingages and radar – A Bayesian approach: 2. An application

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Abstract: In Seo and Smith (this issue), a set of estimators was built in a Bayesian framework to estimate rainfall depth at an ungaged location using raingage measurements and radar rainfall data. The estimators are equivalent to lognormal co-kriging (simple co-kriging in the Gaussian domain) with uncertain mean and variance of gage rainfall. In this paper, the estimators are evaluated via cross-validation using hourly radar rainfall data and simulated hourly raingage data. Generation of raingage data is based on sample statistics of actual raingage measurements and radar rainfall data. The estimators are compared with lognormal co-kriging and nonparametric estimators. The Bayesian estimators are shown to provide some improvement over lognormal co-kriging under the criteria of mean error, root mean square error, and standardized mean square error. It is shown that, if the prior could be assessed more accurately, the margin of improvement in predicting estimation variance could be larger. In updating the uncertain mean and variance of gage rainfall, inclusion of radar rainfall data is seen to provide little improvement over using raingage data only.

Key words: Radar rainfall, gage rainfall, parameter uncertainty, Bayesian estimation.

1 Introduction

This is Part II of the two-part series. In Seo and Smith (this issue), a set of estimators was built in a Bayesian framework to estimate rainfall depth at an ungaged location using raingage measurements and radar rainfall data. The estimators are equivalent to lognormal co-kriging (or, simple co-kriging in the Gaussian domain) with uncertain mean and variance of gate rainfall. In this paper, we evaluate the estimators by performing a simulation experiment.

This work was motivated by earlier findings of Krajewski (1987), Azimi-Zonooz et al. (1989) and Seo et al. (1990) on rainfall estimation by co-kriging of raingage measurements and radar rainfall data. Among their findings, two are of particular interest as they point to the limitations of routinely applying co-kriging in rainfall estimation: 1) performance of ordinary and disjunctive co-kriging falls well below their potential due to large sampling errors in second-order statistics involving gage rainfall, as they are estimated solely form raingage measurements, which are typically few in number, 2) co-kriging variances thus obtained are very often grossly erroneous since co-kriging (or, kriging in general) assumes perfectly known second-order statistics, and 3) disjunctive co-kriging, particularly when homogeneity assumptions are well met, performs significantly better than ordinary co-kriging, but at the expense of about a ten-fold increase in computational requirement.

In building the estimators in Part I, we attempted to remedy the above limitations in that: 1) the estimators can make use of not only the currently available raingage measurements but also a priori information about the second-order statistics involving gage rainfall, available from the past observations of rainfall, 2) the estimators explicitly account for uncertainty in the mean and variance of gage rainfall, and thus more realistic estimation variances may be expected, and 3) disjunctive co-kriging in the Gaussian domain is equivalent to simple co-kriging, and thus, if the assumption that gage rainfall and radar rainfall are jointly second-order homogeneous and multivariate normal is reasonably met, performance of lognormal co-kriging should be comparable to that of disjunctive co-kriging with great savings in computational requirements.

In Azimi-Zonooz et al. (1989) and Seo et al. (1990), raingage measurements and radar rainfall data were generated from high-quality radar rainfall data and a space-time rainfall model. In this work, the estimation environment was made much more realistic in that actual radar rainfall data were used and raingage data were generated based on sample statistics of actual raingage measurements and radar rainfall data.

Evaluation of the estimators was made via cross-validation. For comparison purposes, two types of lognormal co-kriging and two nonparametric estimators were also included. Part II is organized through discussions of the following topics: description of radar and raingage data sets, simulation of raingage data, estimation of correlation functions, assessing prior, results, and conclusions. Frequent reference is made to material in Part I (Seo and Smith, this issue).

2 Description of data sets

One of the main objectives of this work was to evaluate estimators under realistic condi tions, avoiding use of simulated data as far as possible. In this section, we describe th radar rainfall data and the raingage measurements used in this work. The estimators buil in Part I assume that log gage rainfall and log radar rainfall are jointly second-orde homogeneous and multivariate normal. Despite its gently varying terrain, the study are of Oklahoma exhibits orographic effects in annual precipitation . In this work, the homo geneity assumption was considered acceptable since our temporal scale of interest wa only an hour. In this work, we only examined the univariate normality of hourly rad rainfall and hourly gage rainfall (see Discussion Section in Part I for further comments).

2.1 Radar data

Hourly radar rainfall data were obtained from the RAdar DAta Processor, version (RADAP II) data at Oklahoma City covering years 1983, 1985, 1986, and 1987. Th radar was a WSR-57 type with a 2.2 degree beam width and a 10 cm wavelength. Th reflectivity data were given in 15 levels. The data went through several quality contr steps, including attenuation correction, generation of hybrid base-level scan to redu ground-clutter, removal of isolated point targets, and a check on outliers (McDonald ar Saffle 1989).

The data were first converted to radar rainfall according to the conversion table bas on the Marshall-Palmer Z-R relationship. In the conversion step, only the base-level sc data (elevation angle of 0.5 degrees) were used. Radar rainfall data in polar coordinat (180 radials covering a range from 10 to 126 nautical miles) were then converted to sp tially averaged radar rainfall data in Cartesian coordinates by averaging over square bi The size of a bin was set equal to the size of a Hydrologic Rainfall Analysis Proje (HRAP) (Greene and Hudlow 1982) bin, about 4×4 km at Oklahoma City.

In many cases, scanning intervals were not regular. As a quality control measu hourly radar rainfall fields were constructed only when there were 5 or more regula spaced base-level scans in an hour. Visual examination of the hourly radar rainfall fie thus obtained showed a consistent presence of ground-clutter near the radar site. A quality control measure, we did not use the radar rainfall data within 60 km radius fr the radar site. No other quality control measures were taken.

Figure 1 shows a scatter-plot of sample skewness and kurtosis (centered fourth-moment) coefficients of log radar rainfall. Each point represents an hourly radar rainfall field which contained at least a thousand non-zero radar bins. There are 1433 data points in Figure 1. Normally distributed data would yield skewness coefficient of 0 and kurtosis coefficient of 3. Figure 1 indicates that the probability density function of log hourly radar rainfall tends to be symmetric, but more peaked than its normal counterpart. Figure 2 shows a histogram of the radar rainfall data and the fitted lognormal probability density function, for which coinciding (in space and time) raingage data were also available with measurement resolution of one-hundredth of an inch. The fit passed the chi-square test at 5% significance level, but failed the Kolmogorov-Smirnov test at 5% significance level. Measurement resolution is very important in testing lognormality. For RADAP II, the minimum detectable rainfall depth is about 0.02 in/hr. In this work, however, we assumed the minimum detectable rainfall depth of 0.01 in/hr, common to both radar rainfall data and raingage measurements. Therefore, the smallest non-zero hourly rainfall was 0.01 in/hr.

2.2 *Raingage data*

Raingage measurements used in this work are hourly gage rainfall data from the National Climatic Data Center (NCDC) at 45 locations under the Oklahoma City radar umbrella (see Figure 3). There were 39 raingage locations in Oklahoma, 2 in Kansas, and 4 in Texas. The observations were taken by observers at principle (primary) stations, secondary stations, cooperative observer stations operated by the National Weather Service (NWS), and the Federal Aviation Agency (FAA) (NCDC 1986).

As a quality check, we first compared the raingage data with coinciding radar rainfall data. The comparison indicated that many raingage data were off by an hour or more. Since the raingage data were reported in local time, whereas the radar data were given in GMT, we suspected that the shifting may be due to lack of correction for daylight savings time. For each gage location, time series of raingage data and coinciding radar rainfall data were then compared and cross-correlogram was computed. It was found that the shifting was rather arbitrary: number of hours shifted varied from one gage location to another, and from one period to another. To make matters worse, there were many missing periods in raingage data. In an effort to use at least portions of the raingage data, we then compared raingage data and coinciding radar rainfall data event by event. The term "event" here is used rather loosely. Because of many missing data, it was not possible to always identify unambiguously the beginning and ending of a storm. For each event, comprising typically of several hours or more, raingage data were synchronized with coinciding radar rainfall data under the criterion of minimum mean square error. In this way, a total of 1405 pairs of radar-gage rainfall data were collected, of which only 690 pairs had the gage resolution of one-hundredth of an inch. Figure 4 shows the scatter-plot, in log-log scale, of radar-gage pair with resolution of one-hundredth of an inch.

Only the univariate lognormality was examined for the raingage measurements. Many raingage data had resolution of one-tenth of an inch (e.g., from Fisher-Porter type gages). This posed a problem in testing lognormality since the exact truncation point could not be known. To examine lognormality, we used only the raingage data with resolution of one-hundredth of an inch. Figure 5 shows the histogram and the fitted lognormal probability density function. The fit passed the chi-square test at 5% significance level, but failed the Kolmogorov-Smirnov test at 5% significance level. In addition, we used the 41-year record of hourly gage rainfall at Oklahoma City, which had a resolution of one-hundredth of an inch. The fit (not shown) is much poorer and failed both tests at 5% significance level.

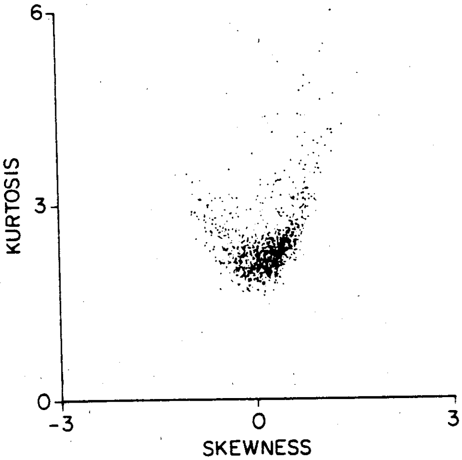

Figure 1

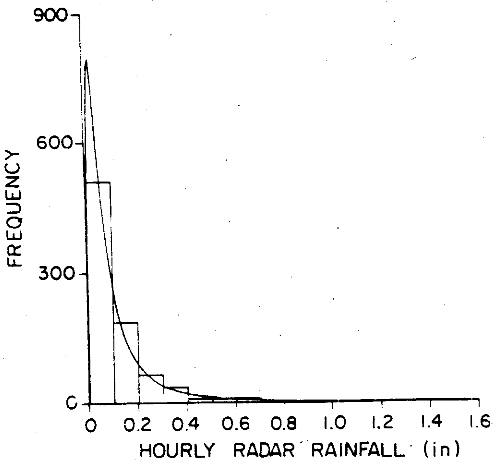

Figure 2

Figure 1. Sample skewness and kurtosis coefficients of log radar rainfall

Figure 2. Histogram of the radar rainfall data for which raingage data were also available with measurement resolution of one-hundredth of an inch. Also shown is the fitted lognormal probability density function

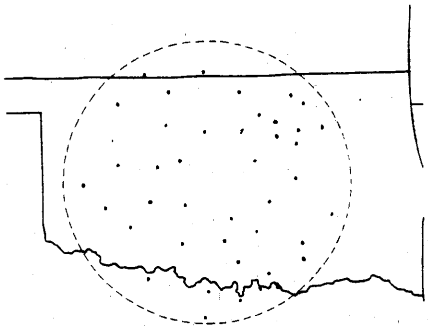

Figure 3

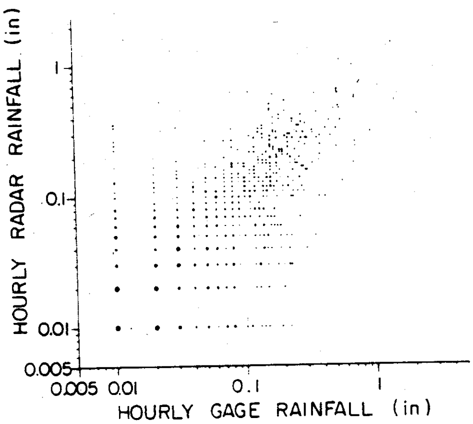

Figure 4

Figure 3. Raingage network under the Oklahoma City radar umbrella

Figure 4. Radar rainfall versus observed gage rainfall with measurement resolution of one-hundredth of an inch

3 Simulation of raingage measurements

Our original intention was to use observed data only. The number of high-resolution raingage measurements, however, was too small to serve our purpose, and we had to resort to simulation to generate a large number of raingage data. In generating raingage data, we assumed the following linear relationship between log gage rainfall and coinciding log radar rainfall:

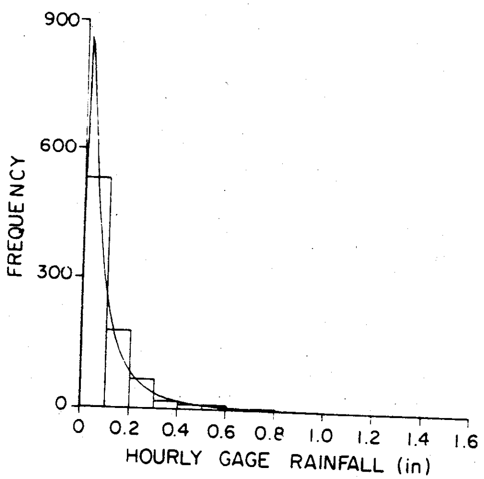

Figure 5. Histogram of the gage rainfall data with measurement resolution of one-hundredth of an inch. Also shown is the fitted lognormal probability density function

$$Y_g = cY_r + d + \varepsilon \tag{1}$$

where Y_g is the log gage rainfall, Y_r is the log radar rainfall, c and d are constants, and ε is the random error with $E[\varepsilon]=m_\varepsilon=0$. Probably the simplest way to generate gage rainfall would have been to assume that the random error, ε, is a white-noise process. Figure 4, however, suggests that ε is negatively correlated with Y_r. Incorporating the correlation between ε and Y_r, we assumed that Y_r and ε are bivariate normal. Then, it can be easily shown that Y_g and Y_r are bivariate normal, and conditional probability density function of Y_g given Y_r is normal with the following mean and variance:

$$E[Y_g|Y_r] = cY_r + d + r_{r\varepsilon}(0)(\sigma_\varepsilon/\sigma_r)(Y_r - m_r) \tag{2}$$

$$Var[Y_g|Y_r] = \sigma_\varepsilon^2\{1 - r_{r\varepsilon}^2(0)\} \tag{3}$$

where $r_{r\varepsilon}(0)$ is the lag-zero cross-correlation coefficient between Y_r and ε, σ_ε is the standard deviation of ε , σ_r is the standard deviation of Y_r, and m_r is the mean of Y_r. The parameters c, d, and σ_ε^2 were estimated via trial-and-error. From Eq. (1), we have:

$$m_g = cm_r + d \tag{4}$$

$$\sigma_g^2 = c(c+2\alpha)\sigma_r^2 + \sigma_\varepsilon^2 \tag{5}$$

$$Cov_{gr}(0) = (c+\alpha)\sigma_r^2 \tag{6}$$

where m_g is the mean of Y_g, σ_g is the standard deviation of Y_g, and $Cov_{gr}(0)$ is the lag-zero cross-covariance between log gage rainfall and log radar rainfall, and $\alpha = r_{r\varepsilon}(0)\sigma_\varepsilon/\sigma_r$. The trial-and-error then consisted of the following steps: assume σ_ε^2, solve for c and α using Eqs. (5) and (6), solve for d using Eq. (4), compute σ_ε^2 using Eq. (1), and repeat the steps until the sample σ_ε^2 is the same as the assumed σ_ε^2.

Table 1 shows sample statistics of Y_g and Y_r and parameters estimated from the above procedure using the raingage data set of resolution of one-hundredth of an inch and coinciding radar rainfall data. Closeness between m_g and m_r indicates little bias in radar

Table 1. Sample statistics of log gage rainfall and log radar rainfall

	m_g	m_r	σ_g^2	σ_r^2	$r_{gr}(0)$	c	α	d	σ_ε^2	$r_{r\varepsilon}(0)$
1	-2.75	-2.68	1.23	1.18	0.61	0.97	-0.34	-0.16	0.91	-0.39
2	-2.90	-2.86	1.19	1.25	0.62					
3	-2.62	-2.68	0.96	1.18	0.82	1.00	-0.26	0.05	0.39	-0.45

1. Statistics of observed log gage rainfall and coinciding log radar rainfall. Also shown are the estimated parameters values. Number of gage-radar rainfall pairs is 690.
2. Statistics of simulated log gage rainfall and coinciding log radar rainfall. Number of gage-radar rainfall pairs is 3410.
3. Same as 1, but the gage rainfall data are scaled so that |log gage rainfall - log radar rainfall | is reduced by 33 percent for each pair.

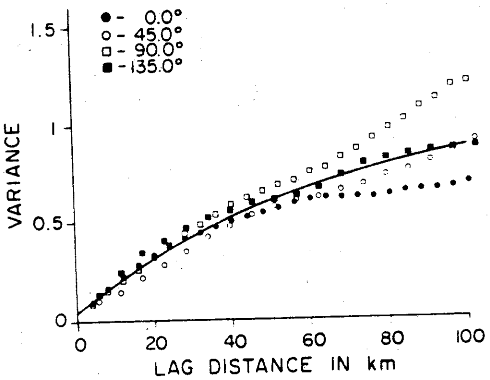

Figure 6

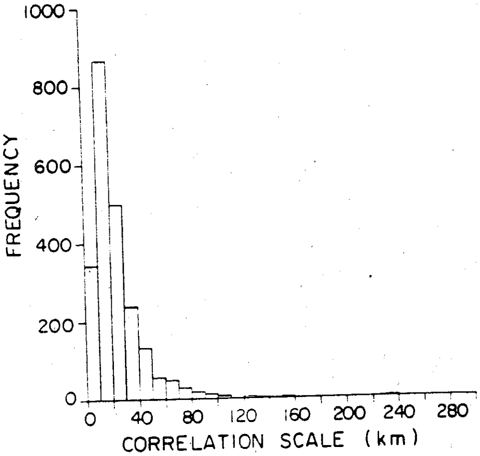

Figure 7

Figure 6. An example of experimental semi-variogram of log radar rainfall and the best fitting model

Figure 7. Histogram of the correlation scale of log radar rainfall

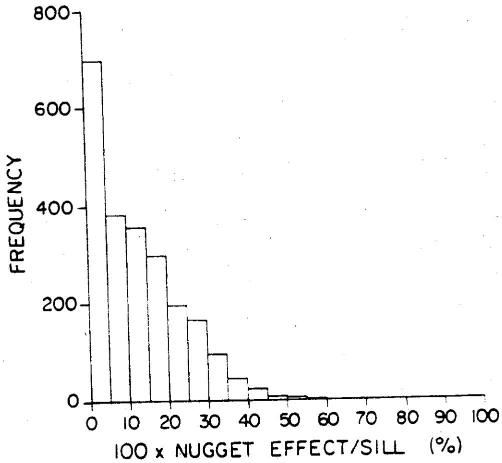

Figure 8. Histogram of the nugget effect, expressed as percentage of the sill, in fitted semi-variogram (
log radar rainfall

rainfall data. Closeness between σ_g^2 and σ_r^2 suggests little reduction in variance of log radar rainfall in spite of smoothing due to post-processing and spatial averaging.

4 Estimation of covariance and cross-covariance functions

To obtain the covariance function of log radar rainfall, the experimental semi-variogram was first computed along eight different directions over each hourly radar rainfall field. Spherical, Gaussian, and exponential covariance models were then fitted to the experimental semi-variogram. Among the three, the model that yielded the smallest sum of squared residuals was chosen and used as the perfectly known covariance function of log radar rainfall. Figure 6 shows an example experimental semi-variogram (only 4, out of 8, directional semi-variograms are shown) and the best fitting semi-variogram model. Figure 7 shows the histogram of correlation scale of log hourly radar rainfall obtained from a total of 1433 hourly radar rainfall fields. Correlation scale is defined as the integral scale of the best fitting correlation function. In many cases, the semi-variogram of log radar rainfall data exhibited the nugget effect (see Figure 8), and thus the covariance function of log radar rainfall was structured as follows:

$$Cov(Y_{ri},Y_{rj}) = \begin{cases} \sigma_r^2 & \text{if } d=0 \\ (\sigma_r^2-C_{or})r_{rr}(|d|) & \textit{otherwise} \end{cases} \tag{7}$$

where $Cov(Y_{ri},Y_{rj})$ is the covariance between log radar rainfall at i and log radar rainfall at j, σ_r^2 is the variance of radar rainfall, d is the distance between Y_{ri} and Y_{rj}, C_{or} is the nugget effect, $r_{rr}(|d|)$ is the correlation function of log radar rainfall, assumed to be translation- and rotation-invariant.

The estimators built in Part I assume that the correlation function of log gage rainfall and the cross-correlation function between log gage rainfall and log radar rainfall are perfectly known. In this work, we assumed that both the correlation function and the cross-correlation function are the same as the correlation function of log radar rainfall. If ε and Y_r were independent, the assumption would be unnecessary given the linearity in Eq. (1) (Creutin et al. 1987). when ε and Y_r are not independent, the assumption is equivalent to assuming that the cross-correlation function between ε and Y_r and the correlation function of ε are the same as the correlation function of log radar rainfall. The covariance function of log gage rainfall was then specified as follows:

$$Cov(Y_{gi},Y_{gj}) = \begin{cases} \Theta^{-1}s_g^2 & \text{if } d=0 \\ \Theta^{-1}(s_g^2-\alpha_g s_g^2)r_{rr}(|d|) & \textit{otherwise} \end{cases} \tag{8}$$

where s_g^2 is the sample variance of log gage rainfall, Θ is the scaling constant defined as $\Theta=s_g^2/\sigma_g^2$, assumed to be a gamma 2 random variable when σ_g^2 is the true variance of log gage rainfall (see Part I), $\alpha_g s_g^2$ is the sample nugget effect with α_g assumed to be equal to C_{or}/σ_r^2, $0<\alpha_g<1$, and d is the distance between Y_{gi} and Y_{gj}. The cross-covariance function was similarly specified as follows:

$$Cov(Y_{gi},Y_{rj}) = \begin{cases} \Theta^{-1/2}r_{gr}(0)s_g\sigma_r & \text{if } d=0 \\ \Theta^{-1/2}\{r_{gr}(0)s_g\sigma_r-\alpha_c(\alpha_g s_g^2 C_{or})^{1/2}\}r_{rr}(|d|) & \textit{otherwise} \end{cases} \tag{9}$$

where $r_{gr}(0)$ is the lag-zero cross-correlation coefficient between log gage rainfall and log radar rainfall, α_c is the constant specifying the degree of coregionalization between the white-noise components of log radar rainfall and log gage rainfall, $0<\alpha_c<1$, assumed to be equal to $r_{gr}(0)$, and d is the distance between Y_{gi} and Y_{rj}.

Correlation function and cross-correlation function thus defined always constitute a valid linear model for coregionalization (Journel and Huijbregts 1978) in that the covariance matrix $\begin{bmatrix} Q_{gg} & Q_{gr} \\ Q_{rg} & Q_{rr} \end{bmatrix}$ is always positive definite.

5 Assessing prior

There were four parameters to be specified in the prior, i.e., q', v', b', and H' (see Eq. (16) in Part I). In this work, the parameters were determined via method of moments using prior knowledge about $E[\beta]$, $Var[\beta]$, $E[\sigma_g]$ and $Var[\sigma_g]$. Due to lack of past raingage data, we had to resort exclusively to radar rainfall data in specifying the four moments. When estimation was desired at hour i, $E[\beta]$ was set equal to $m_r(i-1)$, where $m_r(i)$ denotes the mean of log radar rainfall at hour i. The assumption is that, over a long run, the mean of log radar rainfall is unbiased relative to mean of log gage rainfall. $Var[\beta]$ was set equal to $Var[m_r(i)-m_r(i-1)]$ estimated from the RADAP II data of 1983. Obviously, there are other ways to obtain a potentially more informative prior for β. One such example would be to use a physically-based rainfall model (e.g., Georgakakos and Lee 1987), which predicts mean ground-level rainfall at hour i using all the available observations up to and including hour $i-1$. $E[\sigma_g]$ was set equal to $\sigma_r(i-1)$, where $\sigma_r(i)$ denotes the standard deviation of log radar rainfall at hour i. The assumption is that, over a long run, the variance of radar rainfall is unbiased relative to variance of gage rainfall. $Var[\sigma_g]$ was set equal to $Var[\sigma_r(i)-\sigma_r(i-1)]$ estimated from the RADAP II data of 1983. There was a total of 32 pairs of consecutive hourly radar rainfall fields in the RADAP II data of 1983, and estimates of $Var[\beta]$ and $Var[\sigma_g]$ were 0.082 and 0.025, respectively.

Given the estimates of $E[\beta]$, $Var[\beta]$, $E[\sigma_g]$, and $Var[\sigma_g]$, parameters q', v', b', and H' were obtained as follows. Since the distribution of Θ is gamma 2 with mean $1/q'$ and variance $2/(v'q'^2)$, distribution of σ_g is inverted gamma 2 (Raiffa and Schlaifer 1961) with the following mean, variance, and coefficient of variation:

$$E[\sigma_g] = q'^{1/2} s_g (v'/2)^{1/2} \frac{\Gamma(v'/2-1/2)}{\Gamma(v'/2)} \tag{10}$$

$$Var[\sigma_g] = q' s_g^2 v'/(v'-2) - E^2[\sigma_g] \tag{11}$$

$$CV[\sigma_g] = \left(\frac{2}{v'-2} \frac{\Gamma^2(v'/2-1/2)}{\Gamma^2(v'/2)} - 1 \right)^{1/2} \tag{12}$$

where $\Gamma(\,)$ denotes the Gamma function, and s_g^2 is the sample variance of log gage rainfall. Given the estimates of $E[\sigma_g]$ and $Var[\sigma_g]$, v' was obtained by solving Eq. (12), and q' was obtained from Eq. (11). It may seem odd that s_g^2 has to be known before q' can be obtained. It is because we adopted $\sigma_g^2 = \Theta^{-1} s_g^2$ (Eq. (3) in Part I) rather than $\tilde{\sigma}_g^2 = \Theta^{-1}$. s_g^2 acts only as a scaling constant, and estimate and estimation variance are independent of the actual value of s_g^2. Once q' and v' were obtained, H' and b' were specified by making use of the fact that the marginal distribution of β is Student t with $E[\beta] = b'$ and $Var[\beta] = H'^{-1} q' v'/(v'-2)$.

In the simplified Bayesian estimation, where only raingage data were used in parameter updating, we could have used the mean and variance of log radar rainfall at the current hour, rather than at the preceding hour, since, unlike in the exact Bayesian estimation, the likelihood function did not use currently available radar rainfall data. In this work, however, we used mean and variance of log radar rainfall at the preceding hour so that a comparison between the two estimators could be made.

6 Simulation experiment

Due to sparsity of the raingage network, cross-validation was unavoidable in evaluating the estimators. Given an hourly radar rainfall field, the following steps were involved in a single simulation run:

a) Compute the second-order statistics of log radar rainfall, including experimental semi-variogram.

b) Fit the experimental sime-variogram with spherical, Gaussian, and exponential models, and take the best fitting model as the perfectly known covariance function of log radar rainfall.

c) Generate raingage measurements at gage locations.

d) Eliminate a raingage measurement.

e) Compute sufficient statistics for the three different likelihood functions given in Part I of the paper.

f) Update the uncertain parameters β and Θ in three different ways as described in Part I of the paper.

g) Compute estimates and estimation variances.

h) Go to Step d until the raingage measurements are exhausted.

In Step c, the number of raingage measurements ranged from 6 to 30. In Step e, for each raingage measurement, the 5 nearest radar rainfall data for each raingage measurement were used to construct the covariance matrix of log radar rainfall and cross-covariance matrix between log radar rainfall and log gage rainfall (see Discussion Section in Part I for further comments). In Step g, the 5 nearest raingage measurements and the 5 nearest radar rainfall data for each raingage measurement were used to compute estimates and estimation variances.

For comparison purposes, we included two lognormal co-kriging estimators and two nonparametric estimators. In the first lognormal co-kriging, mean and variance of log gage rainfall were set equal to the mean and variance of log radar rainfall at the preceding hour, respectively. This was equivalent to assuming that prior knowledge was the dominant source of information about β and Θ. In the other, mean and variance of log gage rainfall were estimated solely from the raingage measurements available at the current hour. This was equivalent to assuming that the sample on hand was the dominant source of information.

6.1 Nonparametric estimation procedures

Two estimation procedures, that are nonparametric in spirit, were also compared with the Bayesian procedures. The nonparametric procedures are currently used with the Next Generation Weather Radar System (NEXRAD) off-site precipitation processing procedures. One procedure relies heavily on raingage data, although radar data are used to delineate the areas receiving positive rainfall. The procedure is termed the gage-only analysis. The second procedure uses both radar data and gage data. It is termed the non-parametric gage-radar procedure.

The premise of the gage-only analysis is that radar data can be used to delineate rainy areas but that quantitative rainfall estimates can not be used due to gross errors in magnitudes. This procedure is intended for use in situations where specific types of radar data contamination, such as anomalous propagation or bright band, are detected. The gage-only rainfall estimate at an arbitrary location can be represented as follows:

$$\hat{Z}_{go} = \sum_{i=1}^{n_g} Z_g(i) \frac{m(go)}{m(i)} 1(Z_r(go)>0) \tag{13}$$

where $Z_g(i)$ is the gage rainfall at location i, n_g is the number of surrounding raingage data used, $m(i)$ is the mean annual rainfall at location i. $Z_r(go)$ is the radar rainfall at the arbitrary location, and $1(Z_r(go)>0)$ is equal to 1 if $Z_r(go)>0$, and 0 otherwise. It should be noted that the gage-only analysis accommodates long-term heterogeneity of rainfall associated with orographic enhancement. This feature is not used in the simulation runs.

The multisensor rainfall estimate for a given bin can be represented as a linear combination of the radar rainfall estimate and the gage-only estimate for the given bin, The estimate can be represented as follows:

$$\tilde{Z}_{go} = a\hat{Z}_{go} + (1-a)Z_r(go) \tag{14}$$

where

$$a = \exp(-\gamma D/\rho) \tag{15}$$

In Eq. (15), γ is a constant to be determined, D is the distance to the nearest gage, and ρ is the correlation scale of the radar rainfall field. The gage-radar estimator depends on gage data only through the gage-only estimator. Weights depend on two principal factors, distance to a raingage and spatial correlation of the radar rainfall field. The first is a surrogate for variance of the gage-only estimator. The second is a surrogate for convective activity. In general, the radar estimate will receive large weight when raingages are distant and convective activity is high. Note that the multisensor analysis also accommodates heterogeneity of rainfall associated with orographic enhancement. Both the gage-only analysis and nonparametric gage-radar procedure are used without transformation of values. This avoids difficulties associated with transformation of 0 values.

In the next section, we present the results. To designate the estimators, the following abbreviations are used:

L1 - lognormal co-kriging, prior information dominating

B1 - Bayesian estimation using both radar and raingage data in updating the uncertain parameters

B2 - Bayesian estimation using only raingage in updating the uncertain parameters

L2 - lognormal co-kriging II, sample information dominating

RO - radar-only estimation

GO - nonparametric gage-only estimation

RG - nonparametric gage-radar estimation.

In B1, there were two ways of updating the uncertain parameters, exact and approximate (see Part I). Comparison between the two shows little or no difference, and thus we retained only the approximate version.

7 Results

The total number of simulation runs made (or, hourly radar rainfall fields used) was 325 resulting in 3410 data points. Table 1 shows the sample statistics of log radar rainfall and log gage rainfall. Because of the nature of the simulation, there were unrealistically high values of gage rainfall. The upper bound for hourly gage rainfall depth was set as 3.03 inches, the maximum hourly radar rainfall depth observed in the radar rainfall data used in the simulation experiment.

Figures 9 and 10 show scatter-plots of mean error (ME) and root mean square error (RMSE) of estimates at each hour from L1, B1, B2, L2, RO, GO, and RG, respectively.

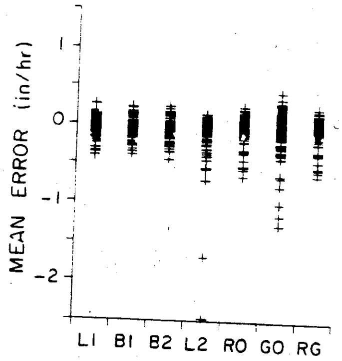

Figure 9

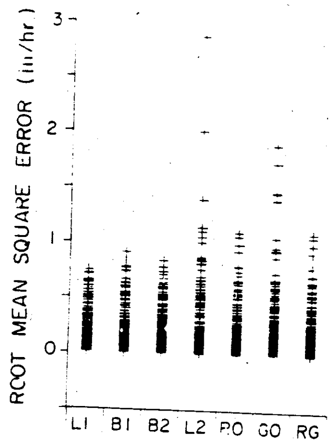

Figure 10

Figure 9. Mean error (ME) of estimates at each hour from L1, B1, B2, L2, RO, GO, and RG

Figure 10. Root mean square error (RMSE) of estimates at each hour from L1, B1, B2, L2, RO, GO, and RG

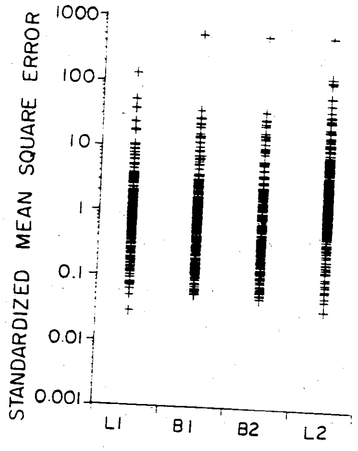

Figure 11

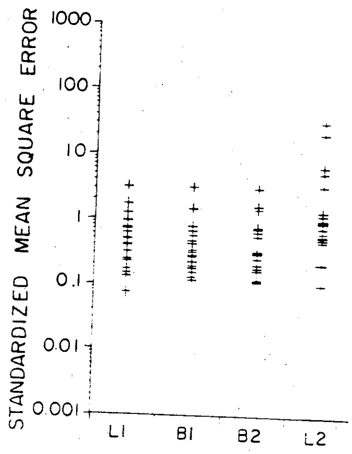

Figure 12

Figure 11. Standardized mean square error (SMSE) of estimates and estimation variances at each hour from L1, B1, B2, and L2

Figure 12. Same as Figure 11, but only a subset of 1987 RADAP II data was used.

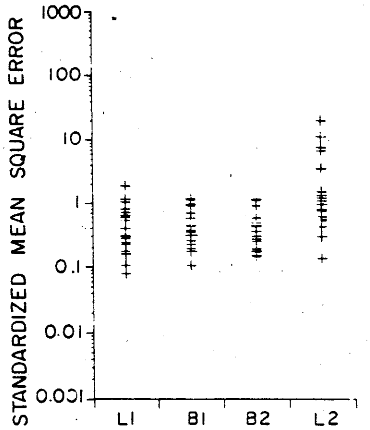

Figure 13. Same as Figure 12, but with reduced variabilities

ME and RMSE measure unbiasedness and error variance, respectively. As expected, performance of GO was poorer. Outlier-like ME's and RMSE's from L2 are due, in part, to lack of robustness in exponential back-transformation used in lognormal co-kriging (Journel and Huijbregts 1978). Both ME's and RMSE's from RG are virtually identical to those from RO. B1 and B2 are seen to provide some improvement over L2, but not over L1. It indicates that prior was the dominant source of information over data at hand in updating mean and variance of gage rainfall. Little difference in performance between B1 and B2 indicates that, as far as parameter updating was concerned, radar data did not add much to the information available from raingage data alone.

Neither B1 nor B2 performed significantly better than RO. It is remined that radar rainfall data had little bias against raingage data. It is well known that co-kriging is very effective in removing the adverse effect of mean field bias in radar rainfall data (Krajewski 1987; Azimi-Zonooz et al. 1989; Seo et al. 1990). Comparison against RO, then, was a very strict test on L1, B1, B2, and L2 since their performance depended largely on residual prediction alone. We also note that cross-validation was disadvantageous to residual prediction since, due to sparsity of the gage network, surrounding gage data very often lay outside of correlation scale of gage rainfall.

Now we examine the accuracy of estimation variances from L1, B1, B2, and L2. An advantage of using co-kriging type estimators over radar-only estimation is that estimation variances are provided. As noted in the Introduction Section, one of the main objectives of developing the Bayesian estimation procedures was to obtain more accurate estimation variances. Figure 11 shows a scatter-plot of standardized mean square error (SMSE, for definition, see, for example, Chua and Bras 1982) of estimates and estimation variances at each hour from L1, B1, B2, and L2. SMSE measures accuracy of estimation variance. A perfect estimator would yield SMSE's of 1. In interpreting scatter-plots of SMSE, we do not overly concern ourselves with exact unity of SMSE's. Given the host of simplifying assumptions used, some degree of consistent bias in estimation variance is not unexpected. Also, estimation variance being a second-order property, SMSE is more prone to sampling error than ME or RMSE given the same sample size, and thus a wider scatter is expected. For these reasons, we pay particular attention to relative dispersiveness of SMSE's. B1 and B2 are seen to provide some improvement over L2, but their SMSE's remain very dispersive. B2 is seen to provide only a very small improvement over B1.

The dispersiveness of SMSE's from B1 and B2 could have been reduced if prior had been assessed following some type of stratification or classification scheme with respect to seasonality, storm type, storm development, etc. Figure 12 shows a scatter-plot of SMSE at each hour using a subset of RADAP II data of 1987. A post analysis revealed that estimates of $Var[m_r(i)-m_r(i-1)]$ and $Var[\sigma_r(i)-\sigma_r(i-1)]$ from that subset were close to the actual values used in assessing prior. Improvement by B1 and B2 over L2 is more noticeable, at least in the mini-max sense. B1 and B2 are also seen to provide a small improvement over L1.

In Figures 11 and 12, dispersiveness of SMSE's was due, in large part, to large natural variabilities in rainfall process itself and error process in radar observation of rainfall. To examine performance under reduced variabilities, we uniformly reduced the difference between gage rainfall and radar rainfall, with radar rainfall fixed, by 33 percent in the 690 pairs of raingage data and radar rainfall data. Table 1 shows the resulting sample statistics and estimated parameter values. Using the new parameter values in generating raingage data, we performed cross-validation using the same subset of RADAP II data of 1987. Figure 13 shows the resulting scatter-plot of SMSE. Improvement by B1 and B2 over L2 is more pronounced, and clearly reveals the adverse effect of uncertainty in mean and variance of gage rainfall. Also, improvement by B1 and B2 over L1 is more noticeable as information from data at hand began taking effect owing to reduced variabilities. B1, however, shows little or no improvement over B2, indicating that additional information from radar data was still insignificant in updating mean and variance of gage rainfall.

8 Conclusions

In Part I of the paper (Seo and Smith, this issue), Bayesian estimation procedures were developed to estimate rainfall depth at an ungaged location using raingage measurements and radar rainfall data. The estimation procedures are equivalent to lognormal co-kriging (simple co-kriging in the Gaussian domain) with uncertain mean and variance of gage rainfall. In Part II, the estimation procedures were evaluated via cross-validation using hourly radar rainfall data from RADAP II at Oklahoma City and simulated hourly raingage data. Simulation of raingage data was based on hourly raingage measurements.

The Bayesian estimation procedures were found to provide some improvement over lognormal co-kriging, under the criteria of mean error, root mean square error, and standardized mean square error. It is shown that, if prior could be assessed more accurately, the margin of improvement in predicting estimation variance could be larger. Performance of the Bayesian estimation procedures is found to be limited in predicting estimation variance by large natural variabilities in rainfall process itself and error process in radar observation of rainfall. If the variabilities are smaller, the Bayesian estimation procedures are shown to provide much more accurate estimation variances than lognormal co-kriging. In updating the mean and variance of gage rainfall, inclusion of radar rainfall data is found to provide only a marginal improvement over using raingage data alone.

Given that the margin of improvement over radar-only estimation is small when radar data are bias-free, estimation procedures based on bias removal (e.g., Smith and Krajewski 1990) are seen as an alternative approach. Also, given that the prior information outweighs the information from currently available data, estimation procedures based on climatological statistics (for gage-only estimation, see, e.g., Bastin et al. 1984) are seen as an alternative approach. Finally, efforts must be made to account for orographic effect. Physically-based models of orographic rainfall (e.g., Barrera and Schaake 1990), which may be coupled with the estimation procedures developed in this work, deserve much attention.

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