A STATISTICAL MODEL OF EXTREME STORM RAINFALL

by

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ABSTRACT

A model of storm rainfall for the Central Appalachian region of the United States is developed. The model represents the temporal occurrence of major storms and for a given storm the spatial distribution of storm rainfall. Spatial inhomogeneities of storm rainfall and temporal inhomogeneities of the storm occurrence process are explicitly represented. The model is used for estimating recurrence intervals of extreme storms. The parameter estimation procedure developed for the model is semi-parametric and requires data from a network of rain gages. The model is applied to a 5000 square mile region in the Valley and Ridge province of Virginia and West Virginia.

1. INTRODUCTION

A model of storm rainfall for the Central Appalachian region of the United States is developed. The model represents the temporal occurrence of major storms and for a given storm the spatial distribution of total storm rainfall. Spatial inhomogeneities of storm rainfall and temporal inhomogeneities of the storm occurrence process are explicitly represented. The model is used for estimating return intervals of extreme storms in the Central Appalachian region. Of particular interest are applications in geomorphology in which frequency information is needed to assess the "geomorphic effectiveness" of extreme storms (see, for example, Wolman and Gerson [1978] and Miller [1988]).

Building blocks for the spatial model of storm rainfall are "marked point processes". The point process modeling framework for rainfall was introduced by LeCam [1961] and has been extensively developed during the past 10 years beginning with the work of Gupta and Waymire [1979]. In the LeCam modeling framework, the underlying point process represents spatial locations of "storm cells". Associated with each cell are a number of attributes, or "marks". These may include maximum rainfall intensity of the cell, velocity of the cell, spatial extent of the cell, among many others. In the storm model developed in Section 2, a single mark, maximum rainfall intensity, is associated with each cell.

The parameter estimation procedure developed for the model requires data from a network of rain gages. The approach to parameter estimation is semi-parametric and follows procedures developed in Karr [1986a] and Smith and Karr [1985]. Foufoula-Georgiou [1988] and Fontaine and Potter [1988] develop storm-transposition techniques for estimating recurrence intervals of extreme

storms. These procedures, which utilize a different class of models and different data sources, are intended primarily for engineering design applications.

Figure 1 shows rain gage totals for a severe storm in the Central Appalachians during June of 1949. The region shown in the figure is the Valley and Ridge portion of the Potomac River basin. This region, which has a drainage area of approximately 5000 square miles, is the study area we will focus on in Section 4. From Hack and Goodlett's [1960] detailed study of geomorphic processes associated with the June 1949 flood, some initial conclusions can be drawn about extreme storms in the study region. A sobering conclusion is that maximum point rainfall totals for a storm may exceed maximum gage totals by quite a large amount. Hack and Goodlett conclude, on the basis of hillslope and valley bottom damage, that rainfall totals in the Little River watershed (marked in Figure 1 by question marks), were much larger than the maximum gaged storm total (14" at a gage 40 miles west of the Little River watershed).

A notable feature of the 1949 storm is the presence of distinct cells of intense rainfall. Extensive hillslope failures were associated with two cells of intense rainfall, one in the Little River watershed, a second in the western Valley and Ridge (near the gage which recorded 14"). A third intense storm cell was located along the Blue Ridge, which forms the eastern boundary of the region shown in Figure 1. No slope failures were reported in the Blue Ridge. The presence of distinct intense storm cells, with length scales less than 40 miles, is characteristic of extreme flood-producing storms of the Central Appalachians (Miller [1988]).

The June 1949 storm is typical of extreme Central Appalachian storms in having heavy rainfall over the Blue Ridge. Rainfall and flood data (see

Section 4) suggest a general decrease in temporal frequency of extreme rainfall with distance from the Atlantic Ocean. This feature is related in part to typical paths of tropical storms (Neumann et al. [1978]). The role of topography is likely important, but less clear. The alternating sequence of valleys and ridges makes it especially difficult to unravel the effects of topography. These qualitative observations play an important role in determining the statistical procedures used to accommodate spatial inhomogeneities of storm rainfall.

To assess the likelihood of extreme storm totals for a given region, we will seek answers to four questions: 1) How often do extreme storms occur?, 2) How large are the point rainfall totals at the centers of extreme cells?, 3) How does rainfall total associated with a cell decay from the center of the cell, and 4) How are cells distributed spatially throughout the region?

Contents of the sections are as follows. Model development is presented in Section 2. Parameter estimation is the topic of Section 3. In Section 4 the storm model is applied to Central Appalachian storms. A summary and conclusions are given in Section 5.

2. MODEL DESCRIPTION

Let $\{N(t)\}$ be a Poisson process on the interval $[0,1]$ with intensity function $\lambda(t)$. Denote the arrival times by $T_1,\ldots,T_{N(1)}$. Associated with each arrival time is a random field on R^2 specified by

$$Z_i(x) = \int_{R^2\times R_+} z \exp(-2c^2\|x-y\|^2)d\eta_i(y,z) \quad , \quad i \leq N(1) \quad , \qquad (1)$$

where η_i is a Poisson process on $R^2 x R_+$ with intensity function $a(y)b(y)\exp\{-b(y)z\}$. Model components have the following interpretation. The Poisson process N represents arrival times of extreme rainfall events during a year (represented by the time interval [0,1]). The random field Z_i represents total storm rainfall for the ith storm during the year, so $Z_i(x)$ is total storm rainfall at spatial location x for the ith storm. The storm field Z_i is assumed to be large relative to the study area, so the index set of Z_i is R^2. The random fields $Z_1,\ldots,Z_{N(1)}$ are mutually independent; furthermore they do not depend on the occurrence times $T_1,\ldots,T_{N(1)}$. We will denote mean areal precipitation over the region A in R^2 during the ith storm as follows:

$$X_i(A) = |A|^{-1} \int_A Z_i(x)dx \ . \qquad (2)$$

Structure of an individual storm field can be described as follows. Storm rainfall at a given location is the sum of contributions from individual "rain cells". Locations of rain cell centers are determined by a spatial Poisson process with intensity function a(x). Spatial variability of the intensity function may result from topographic inhomogeneities. Physical lifting of moist air masses over mountain areas may trigger instabilities which in turn produce intense storm cells. Maximum rainfall for a given cell is specified by an exponentially distributed random variable with spatially varying function b(x). Again topographic inhomogeneities are a possible source of spatial variability. For the function b(x) the physical mechanism is one of augmentation rather than triggering.

The storm contribution of an individual cell decreases with distance according to a quadratic exponential function. The decay parameter c is not allowed to vary spatially, primarily for reasons of computational

tractability. Note that if a(x) and b(x) do not vary spatially, structure of a storm field is determined by three parameters: the spatial rate of occurrence of cells, a, the average cell center amount b^{-1}, and the cell decay parameter c. In this case the storm field model is the same as the model used by Rodriguez-Iturbe, Cox, and Eagleson [1987] (and also quite similar to the storm model of Smith and Karr [1985]).

Frequency results for the storm model are based on the counting process

$$N_u(t) = \sum_{i=1}^{N(t)} 1(X_i(A) > u) \tag{3}$$

where

$$1(X_i(A) > u) = \begin{cases} 1 & ; \text{ if } X_i(A) > u \\ 0 & ; \text{ otherwise.} \end{cases} \tag{4}$$

is the indicator function for the event that mean areal rainfall over the region A for the ith storm exceeds u. The counting process N_u is a "p-thinning" of the Poisson process N (see Karr [1986a]). It follows that N_u is a Poisson process with intensity function

$$\lambda_u(t) = \lambda(t)(1 - H(u)) \tag{5}$$

where H is the distribution function of mean areal rainfall over the region A, that is,

$$H(y) = P\{X_i(A) \leq y\} \quad . \tag{6}$$

Our principal interest is estimating recurrence intervals for extreme storm totals. Spatial representation of annual maximum rainfall is obtained through the set-indexed process M defined by

$$\begin{aligned} M(A) &= \max\{ X_i(A) \; ; \; i \leq N(1) \} \quad ; \quad N(1) > 0 \\ M(A) &= 0 \quad ; \quad N(1) = 0 \end{aligned} \tag{7}$$

The distribution function of M(A) is given by

$$F(y) = P\{M(A) \leq y\} \; . \tag{8}$$

$$= P\{N_y(1) = 0\} \tag{9}$$

$$= \exp\{-\Lambda(1 - H(y))\} \; , \tag{10}$$

where

$$\Lambda = \int_0^1 \lambda(u)du \quad . \tag{11}$$

The quantile function of M(a) is given by

$$Q(p) = F^{-1}(p)$$

$$= H^{-1}(1 + \Lambda^{-1}\log(p)) \quad ; \quad p > \exp(-\Lambda) \quad . \tag{12}$$

Often it is of interest to assess seasonal variation in the frequency of extreme storm totals or floods (see Smith and Karr [1986] and Smith [1987]). A seasonally varying quantile function for our model can be defined as

follows:

$$Q_t(p) = \inf\{u:\lambda_u(t) \leq 1-p\} \tag{13}$$

$$= H^{-1}(1 + \lambda(t)^{-1}\log(p)) \quad p > \exp(-\lambda(t)) \ . \tag{14}$$

The quantile function $Q_t(p)$ can be interpreted as the annual storm quantile associated with a storm rate of occurrence of $\lambda(t)$. The seasonally varying quantile is useful in assessing the "uniqueness" of storms, such as the June 1949 storm in the Central Appalachians, which do not occur during the period when storms are most common. Seasonally varying estimates of storm frequency are particularly useful in geomorphic and hydrologic applications due to the seasonally varying nature of the processes being studied.

It follows from equations (12) and (14) that for estimating recurrence intervals of extreme storms we can decompose the problem into two parts: estimating the rate of occurrence of storms and estimating the distribution of extreme rainfall for an individual storm.

It is difficult to explicitly calculate the distribution H of storm rainfall. It is possible, however, to compute the Laplace transform of H, which is denoted $L_H(s)$. The following result is obtained as a special case of equation (A3) in the Appendix, which gives the Laplace functional of the storm field Z_i.

$$L_H(s) = E[\exp(-sX_i(A))]$$

$$= \exp\{-\int_{R^2} a(y)g(y)/[b(y) + g(y)]\, dy\} \tag{15}$$

where

$$g(y) = s|A|^{-1}\int_A \exp(-2c^2|x-y|^2)\, dx \quad . \tag{16}$$

Recurrence intervals for extreme storm totals can be obtained directly from the Laplace transform using the Tauberian theorem (Feller [1971]), which implies that

$$\frac{H(y)}{L_H(1/y)} \to 1 \qquad y \to \infty \tag{17}$$

Unfortunately, the Tauberian approximation, for practical values of y, is poor.

From the Laplace transform of $Z_i(x)$ we obtain the following first and second order moments.

$$\begin{aligned}\mu(x) &= E[Z_i(x)] \\ &= \int_{R^2} a(y)\, \exp\{-2c^2|x-y|^2\}\, b(y)^{-1}\, dy \end{aligned} \tag{18}$$

$$\begin{aligned}\sigma^2(x) &= Var(Z_i(x)) \\ &= 2\int_{R^2} a(y)\, \exp(-2c^2|x-y|^2)\, b(y)^{-2}\, dy \end{aligned} \tag{19}$$

From equation (A3) we can also obtain the joint Laplace transform of $Z_i(x_1)$ and $Z_i(x_2)$, which provides the following representation for the covariance function:

$$\begin{aligned}c(x_1,x_2) &= Cov(Z_i(x_1),Z_i(x_2)) \\ &= 2\int_{R^2} a(y)\, \exp(-c^2(|x_1-y|^2+|x_2-y|^2)\, b(y)^{-2} dy \end{aligned} \tag{20}$$

In the stationary case, that is, when $a(y) = a$, and $b(y) = b$, moment results simplify as follows:

$$\mu = 2^{-1}\pi a c^{-2} b^{-1} \tag{21}$$

$$\sigma^2 = 2^{-1}\pi a c^{-2} b^{-2} \tag{22}$$

$$c(x_1, x_2) = \exp(-c^2 |x_1 - x_2|^2)\ \sigma^2 \tag{23}$$

In the stationary case we also obtain a simplified expression for the distribution of storm rainfall at a point x. The result follows from the fact that $Z_i(x)$ has gamma-distributed marginals in the stationary case. The distribution H of $Z_i(x)$ is given by

$$H(y) = \int_0^y \Gamma(\alpha)^{-1} z^{\alpha-1} b^{\alpha} \exp\{-bz\}\, dz \tag{24}$$

where

$$\alpha = (1/2)\pi a c^{-2}. \tag{25}$$

If the parameter functions $a(x)$ and $b(x)$ are slowly varying relative to the correlation distance c^{-1}, equation (24) can be used to approximate the frequency distribution of maximum annual rainfall at the point x. This approach will be used in Section 4. To evaluate the annual frequency distribution of storm rainfall for a general region A, Monte Carlo methods must be used.

3. PARAMETER ESTIMATION

In this section estimators are developed for the functions $a(x)$, $b(x)$, and $\lambda(t)$ and for the cell decay parameter c. As noted in Section 2 topographic inhomogeneities are likely causes of spatial inhomogeneities in the functions $a(x)$ and $b(x)$. We will not attempt to parameterize the functions $a(x)$ and $b(x)$ in terms of topographic data. As shown in Section 4 it is especially difficult to quantitatively relate topographic information to spatial inhomogeneities in rainfall. An implicit assumption underlying the estimation procedure is that the functions $a(x)$ and $b(x)$ are slowly varying in space relative to the correlation distance c^{-1}.

It is assumed that the data set Ψ_n available for parameter estimation consists of observations from ν rain gages over a period of n years, that is,

$$\Psi_n = \{T_{ij}, Z_{ij}(x_k);\ i=1,\ldots,n; j=1,\ldots,N_i; k=1,\ldots,\nu\} \tag{26}$$

The occurrence time of the jth storm in year i is T_{ij}. The rain gage observation at the kth rain gage for the jth storm in year i is $Z_{ij}(x_k)$. Rain gage observations are at the fixed locations $x_1,\ldots,x_\nu$ throughout the period of record. The notation for the number of storms has changed slightly. Dependence on time of year has been suppressed; we have added dependence on the year index i. Dependence on the year index i has also been added to the storm field $Z_{ij}(x)$ and occurrence time T_{ij}.

The estimator for the mean number of storms during a year Λ is simply

$$\hat{\Lambda} = n^{-1} \sum_{i=1}^{n} N_i \tag{27}$$

Because the random variables N_i have a Poisson distribution with mean Λ these estimators are consistent and asymptotically normal, as n goes to infinity.

The estimators for the spatial parameter functions a(x) and b(x) are based on estimators of the moment functions $\mu(x)$ and $\sigma^2(x)$. Moment estimators are obtained using a nonparametric kernel estimator which weights stations by distance (see Karr [1986a] for details on nonparametric kernel estimators). They are given as follows.

$$\hat{\mu}(x) = (n\ N_i)^{-1} \sum_{i=1}^{n} \sum_{j=1}^{N_i} \sum_{k=1}^{\nu} w_k\ Z_{ij}(x_k) \tag{28}$$

$$\hat{\sigma}^2(x) = \hat{s}^2(x) - \hat{\mu}^2(x) \tag{29}$$

where

$$\hat{s}^2(x) = (n\ N_i)^{-1} \sum_{i=1}^{n} \sum_{j=1}^{N_i} \sum_{k=1}^{\nu} w_k\ Z_{ij}(x_k)^2 \tag{30}$$

$$w_k = \frac{K(|x-x_k|/\gamma)}{\sum_{j=1}^{\nu} K(|x-x_j|/\gamma)} \tag{31}$$

and

$$K(x) = \exp(-2x^2) \tag{32}$$

The parameter γ is the bandwidth parameter of the kernel estimator. Note that

the kernel, K, is of the same functional form as the "spread function" that distributes rainfall about a rain cell in equation (1).

The correlation distance c^{-1} is estimated from sample values of spatial correlation,

$$\rho_{kl} = (nN_i)^{-1}\sum_{i=1}^{n}\sum_{j=1}^{N_i}[(Z_{ij}(x_k)-\hat{\mu}(x_k))/\hat{\sigma}^2(x_k)][Z_{ij}(x_l)-\hat{\mu}(x_l))/\hat{\sigma}^2(x_l)] \tag{33}$$

In the stationary case ρ_{kl} is a sample estimator of correlation of storm rainfall for two sites separated by distance $\|x_k-x_l\|$. Note that the sample estimator ρ_{kl} is scaled for spatial variability in the mean and variance. Our estimate of c is based on the assumption that the stationary result for covariance can be applied to the scaled correlation estimators. We do not, however, attempt to directly fit the covariance function of equation (23). We solve equation (23) for each pair or rain gages obtaining:

$$c_{kl} = [-\ln(\rho_{kl})]\|x_k-x_l\|^{-1} \tag{34}$$

The estimator we use for c is the median of values obtained from equation (34),

$$\hat{c} = \text{median}\{c_{kl}: k=1,\ldots,\nu; l=1,\ldots,\nu; k\neq l\} \tag{35}$$

In this way we minimize the effects of gage pairs for which the stationary approximation to the covariance function is poor.

The strategy for kernel estimation of the moment functions in our problem is different from normal applications. In normal applications the bandwidth is chosen as a decreasing function of sample size. We want to choose the

bandwidth to reflect the spread function for cell rainfall. To implement the nonparametric kernel estimators we choose the bandwidth parameter γ equal to the estimated correlation distance $\hat{c}^{-1}$.

The estimators of b(x) and a(x) are based on the stationary moment equations (21) and (22). Applying these equations locally we obtain the following parameter estimators

$$\hat{b}(x) = \hat{\mu}(x)/\hat{\sigma}^2(x) \tag{36}$$

$$\hat{a}(x) = 2\pi^{-1}\hat{\mu}(x)\hat{b}(x)\hat{c}^2 \tag{37}$$

The estimators $\hat{a}(x)$, $\hat{b}(x)$, and $\hat{c}$ are, in general, biased. In the stationary case a consistency argument can be made for increasing gage density using results in Karr [1986a] for Poisson samples of random fields. From a practical standpoint, spatial inhomogeneities and sparse gage networks preclude the useful application of large sample results.

4. CENTRAL APPALACHIAN STORMS

In this section the storm rainfall model is applied to extreme storms of the Central Appalachian region. We focus on the 5000 square mile section of the Valley and Ridge province shown in Figure 1. The region is bounded on the west by the Appalachian Plateau and to the east by the Blue Ridge and Piedmont. Daily rain gage observations are available from 19 gages over the period 1950 - 1980. A storm is treated as a three day period with rainfall accumulation greater than 5 inches at one or more gages. Using results of

Section 2 we derive spatially varying estimates of the 100 year storm. We begin the section with a qualitative description of Central Appalachian storms.

Several important features of extreme storms can be deduced from observations of big floods. Figure 2 shows maximum unit discharge (largest flood during the period of record divided by drainage area) for 70 stations in the Central Appalachian region (the period of record is 1930-1980). The general decrease of maximum unit discharge with drainage area and high variability of the maximum unit discharge plot at small drainage areas can be related to spatial variability of storm rainfall at the storm cell scale. In Figure 3, Piedmont basins, at the eastern edge of the Central Appalachians are distinguished from Appalachian Plateau basins at the western end of the region. Extreme Piedmont floods are much larger than extreme Appalachian Plateau floods. This feature reflects one form of spatial inhomogeneity of Central Appalachian storms, a general decline in extreme storm totals with distance from the Atlantic. Figures 2 and 3 suggest that the relative importance of spatial inhomogeneity is at least as great as spatial variability for assessing frequency of extreme storm totals.

Orographic effects have an important influence on storm rainfall in the Central Appalachians. Elevation ranges from less than 1000 feet in valleys to greater than 4000 feet at hill crests. The role of topography, however, is highly dependent on location within the Central Appalachians. During the period 1949 - 1981 the Blue Ridge gage, denoted by "1" in Figure 1, recorded storm totals of 5 inches or more for 13 storms. Of the 13 events, 6 have storm totals greater than 10 inches. For the western Valley and Ridge gage, denoted by "2" in Figure 1, a single event received a storm total greater than 5 inches during the same period. Both gages are crest gages at elevations

near 3500 feet.

Big storms differ from other storms in the Central Appalachians in that they only occur during summer and fall. During the period 1950 - 1981, 15 storms occurred in the study region. All are contained in the six month period extending from the end of June until the end of November. It follows from equation (27) that the estimate of the mean storm frequency Λ is .45 (15/33) storms per year.

The estimate of the correlation distance c^{-1} obtained from equation (35) is 32 miles. The estimates of the spatial rate of occurrence function range from a minimum of .0008 cells per square mile in the western portion of the region to a maximum of .001 cells per square mile along the eastern edge of the region. The estimate of mean rainfall at the center of a cell $b(x)^{-1}$ ranges from a minimum of 2.5 inches in the western portion of the region to a maximum of 4.0 inches in the eastern portion of the region. The regional parameter functions are used in the stationary distribution (equation (24)) to obtain spatially varying estimates of the 100 year storm total. The 100 year storm ranges from a maximum of 19 inches at the eastern edge of the region to a minimum of 12 inches at the western edge.

Hack and Goodlett [1960] note that hillslope failures similar in magnitude to those associated with the 1949 flood have occurred during the past several thousand years throughout the Valley and Ridge province. Jacobson [1988] has estimated the frequency of slope failure for several geologic formations in the Valley and Ridge province. A fundamental question in geomorphology is whether differences in slope activity between two regions are due to geologic controls or to contrasts in the frequency of extreme rainfall totals. In at least one special case the answer can clearly be obtained from the rainfall frequency information obtained above. A 14" storm

is three times as likely to occur in the Blue Ridge as in the western Valley and Ridge province. Miller [1988] notes, on the other hand, that slope activity is less common in the Blue Ridge than in the Valley and Ridge. The relative frequency information provides a rough quantitative measure of the relative geomorphic effectiveness of an extreme storm (14" or larger) on the crystalline rocks of the Blue Ridge versus the sandstones and shales of the Valley and Ridge.

5. SUMMARY AND CONCLUSIONS

In this paper we have developed a modeling framework for evaluating the frequency of occurrence of extreme storm totals. The temporal occurrence of extreme storms is modeled by a Poisson process with seasonally varying rate. The spatial model of storm rainfall is based on a marked point process representation in which a spatial point process represents locations of storm cells and the associated marks represent maximum rainfall intensity at the center of a cell. Cell rainfall decays according to a quadratic exponential function with distance from the center of the cell. The spatial rate of occurrence of cells $a(x)$ and the mean cell center rainfall, $b(x)^{-1}$, are allowed to vary spatially.

The statistical procedure that is used to estimate the functions $a(x)$ and $b(x)$ is based on nonparametric kernel estimators of the the expected value function $\mu(x)$ and variance function $\sigma^2(x)$. The model is applied to storm rainfall in a 5000 square mile region of the Central Appalachians. It is shown that spatial inhomogeneities of extreme storm rainfall are pronounced in the region. Spatial inhomogeneities are attributed to orographic effects and

coastal preference of tropical storm tracks. The estimated 100 year storm total ranges from a maximum of 19 inches at the eastern boundary of the region to 12 inches in the western end of the region. The storm frequency model is used to assess the geomorphic effectiveness of extreme storms in the Central Appalachians.

Several areas for further research can be noted. Storm definition is based on a rainfall threshold. Threshold methods have been developed for flood frequency analysis (Smith [1987]) in which the flood threshold is explicitly incorporated into the model. It would be desirable to incorporate the rainfall threshold into the storm model. It would also be useful to augment the information from rain gages, where possible, with radar and satellite measurements of rainfall. A framework for multisensor models of rainfall is presented in Smith and Krajewski [1987].

APPENDIX

The Laplace functional of a random field Z is defined by

$$L_Z(m) = E[\exp\{-\int_{R^2} Z(x)\, m(dx)\}] \tag{A1}$$

where m is a positive measure on the Borel sets of R^2. The Laplace functional of the random field Z_i defined by equation (1) is given below.

$$L_{Z_i}(m) = E[\exp\{-\int_{R^2} Z_i(x)\, m(dx)\}] \tag{A2}$$

$$= \exp\{-\int_{R^2} a(y)\, g(y)/[b(y) + g(y)]\, dy\} \tag{A3}$$

where

$$g(y) = \int_{R^2} \exp\{-2c^2 \|x-y\|^2\}\, m(dx) \ . \tag{A4}$$

Proof.

$$L_{Z_i}(m) = E[\exp\{-\int_{R^2} Z_i(x)\, m(dx)\}] \tag{A5}$$

$$= E[\exp\{-\int_{R^2} \int_{R^2 \times R_+} z \exp\{-c^2 \|x-y\|^2\} d\eta_i(y,z) m(dx)\}] \tag{A6}$$

$$= E[\exp\{-\int_{R^2 \times R_+} z\, [\int_{R^2} \exp\{-2c^2 \|x-y\|^2\, m(dx)\ ]\, d\eta_i(y,z)\}] \tag{A7}$$

$$= E[\exp\{-\int_{R^2 \times R_+} z\, g(y)\, d\eta_i(y,z)\}] \tag{A8}$$

$$= \exp\{-\int_{R^2\times R_+} z(1-\exp\{-zg(y)\})a(y)b(y)\exp\{-b(y)z\}dydz\} \qquad (A9)$$

$$= \exp\{-\int_{R^2} a(y)[\int_{R_+} b(y)\exp\{-b(y)z\}-b(y)\exp\{-(b(y)+g(y))z\}dz]dy\} \qquad (A10)$$

$$= \exp\{-\int_{R^2} a(y)[1 - b(y)/(b(y)+g(y))]\, dy\} \qquad (A11)$$

$$= \exp\{-\int_{R^2} a(y)\, g(y)\, /[b(y) + g(y)]\, dy\} \qquad (A12)$$

The key step in the proof is (A9) in which we evaluate the Laplace functional of the Poisson process η_i for the function $zg(y)$. Representation for the Laplace functional of a Poisson process is given in Karr [1986a].

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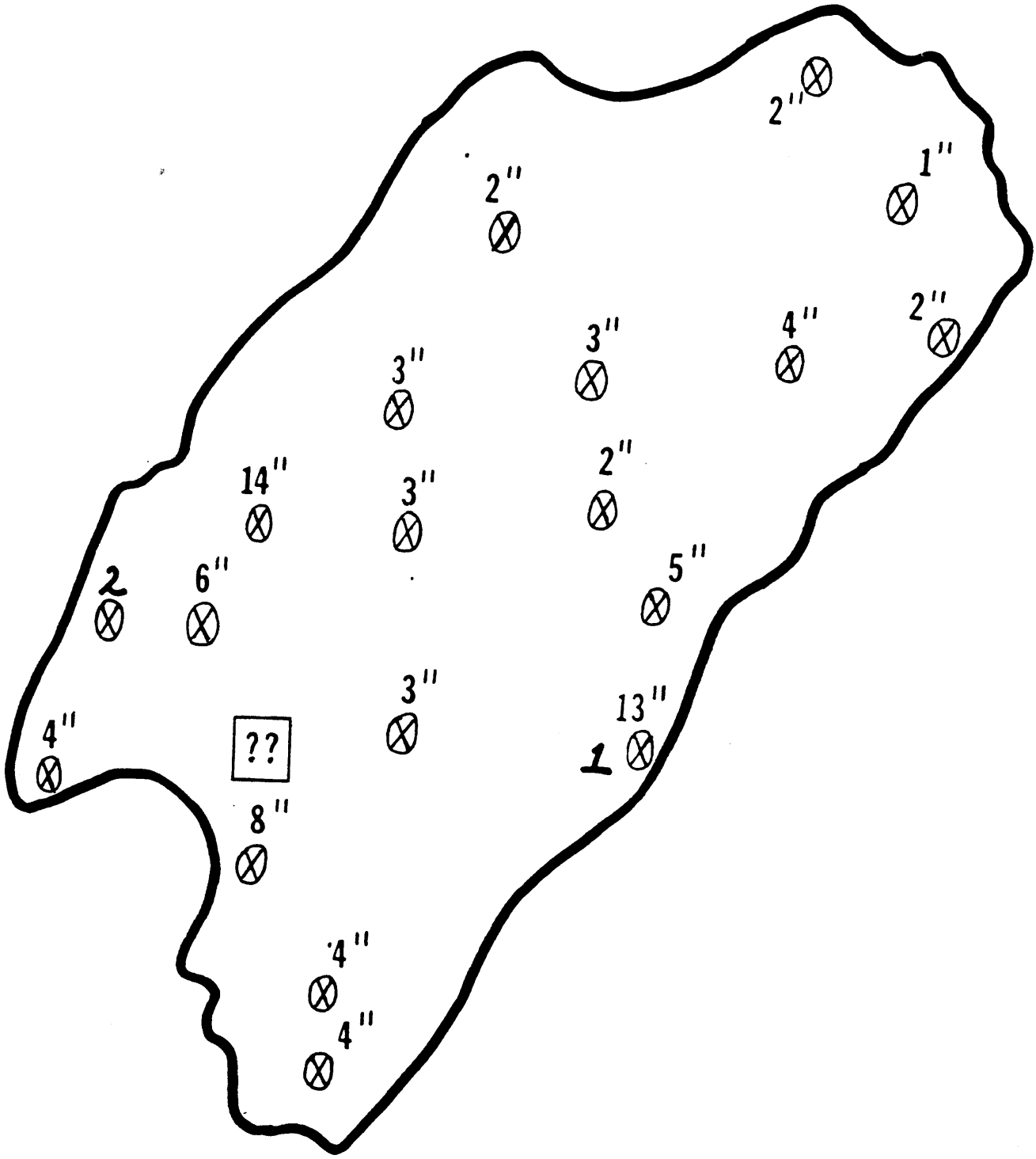

Figure 1. Rain gage totals for June 1949 storm in the Valley and Ridge Portion of the Potomac River basin.

FIGURE 2.
MAXIMUM UNIT DISCHARGE
CENTRAL APPALACHIAN REGION

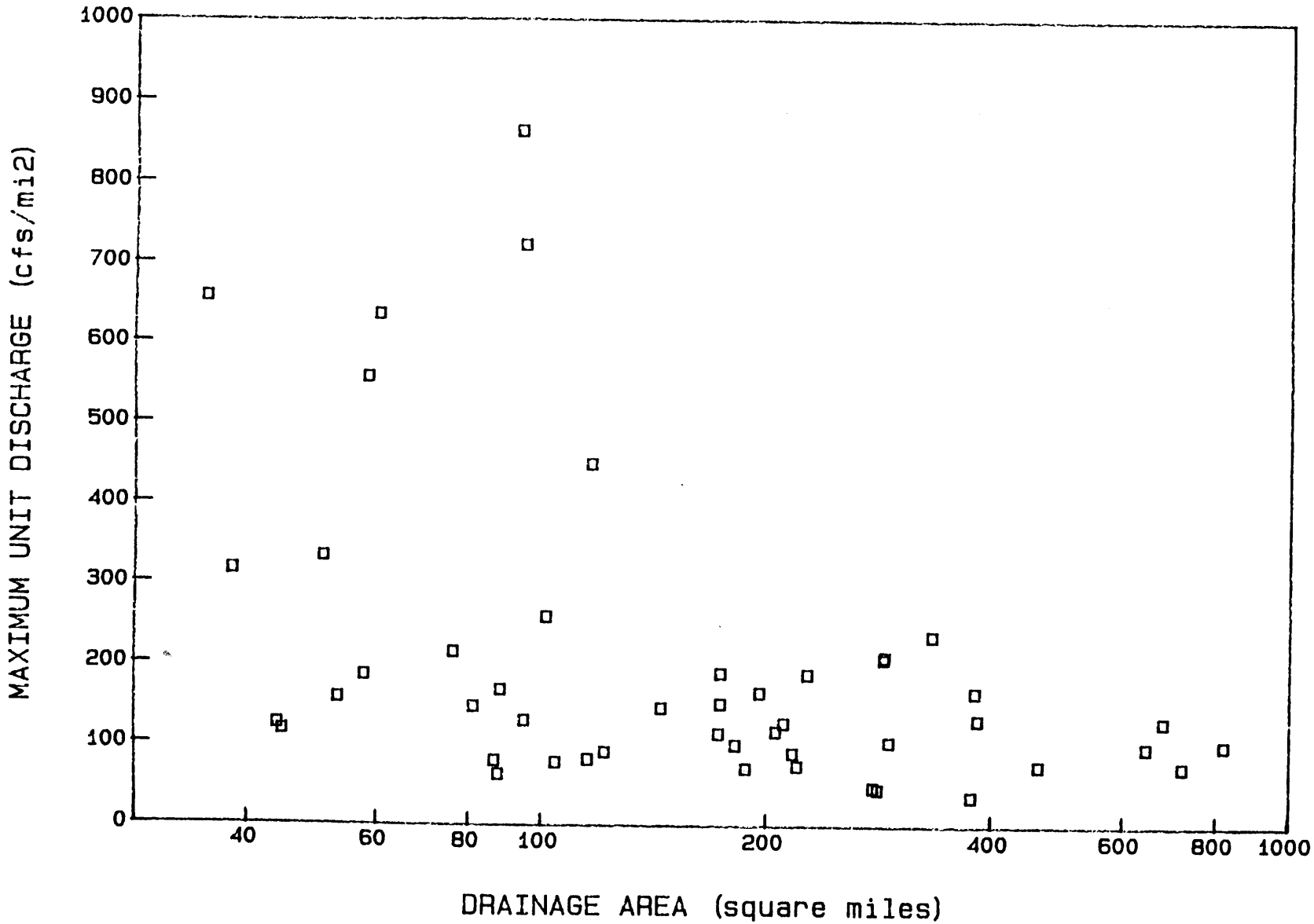
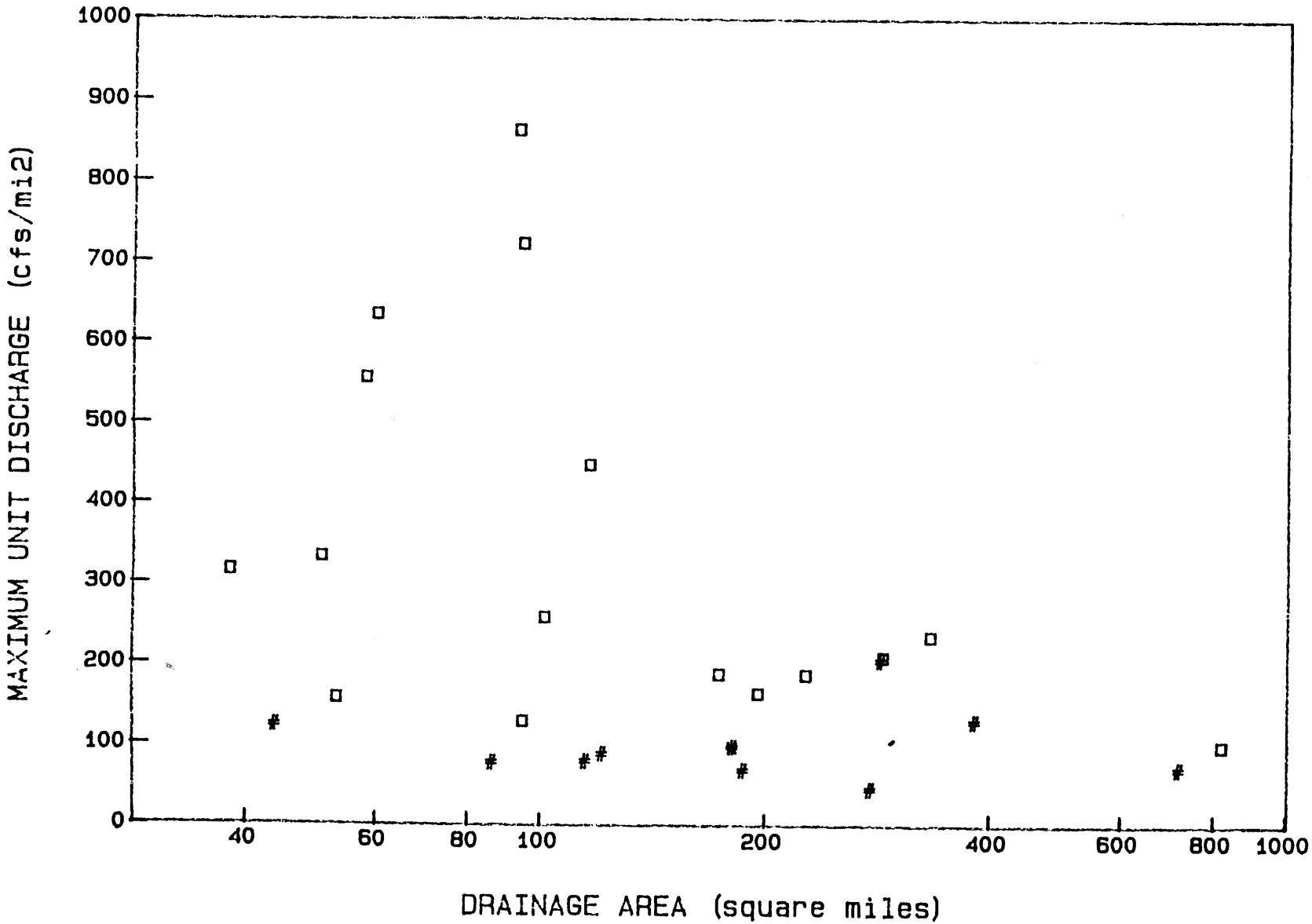
FIGURE 3
MAXIMUM UNIT DISCHARGE
APPALACHIAN PLATEAU-#; PIEDMONT-BOX