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PARAMETER ESTIMATION FOR THE NEXRAD HYDROLOGY SEQUENCE

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1. INTRODUCTION

The goal of the Precipitation Processing System (PPS) of the Next Generation Weather Radar (NEXRAD) system is to produce accurate quantitative precipitation estimates. NEXRAD precipitation estimates will have a wide range of applications, including river forecasting, water management, and engineering design. A detailed analysis of benefits derived from precipitation processing capabilities of NEXRAD is presented in Hudlow et al. [1984].

The precipitation processing algorithms of NEXRAD have been described in Ahnert et al. [1983]. In this paper a statistical model (equation (4)) is formulated for use in estimating site-adaptation parameters of the NEXRAD hydrology sequence and for assessing accuracy of NEXRAD precipitation estimates. The model incorporates the salient statistical features of the NEXRAD hydrology algorithms.

Contents of the sections are as follows. Section 2 contains a description of the reflectivity data which is input to the NEXRAD hydrology sequence. In Section 3 the statistical model relating equivalent reflectivity factor to rainfall rate is presented and state estimators used for precipitation estimation are derived. Range dependence of precipitation processing parameters is emphasized in model development. Parameter estimation is the topic of Section 4. A summary and conclusions are given in Section 5.

2. SAMPLING PROPERTIES OF NEXRAD RADARS

In this section a brief introduction is given to the radar measurements that are used for precipitaton estimation. More detailed accounts can be found in Ahnert et al. [1983] and Doviak and Zrnic [1984].

Equivalent reflectivity factor, which is a measure of the backscattering cross section of meteorological targets in a spatial volume sampled by radar, is the radar observation used for precipitation estimation. The sequence of spatial volumes sampled by radar is specified by range amd azimuth. For NEXRAD the range increment is one kilometer and the azimuth increment is one degree. The land surface beneath the sample volume with azimuth i and range j is denoted D_{ij}.

Equivalent reflectivity measurements are obtained approximately every 5 minutes during precipitation periods. The equivalent reflectivity factor for scan t within hour s at range j and azimuth i is denoted $Z_{s,t}(i,j)$ (hereafter, referred to simply as reflectivity). The double indexing of time, through the hour index s and the scan index t, is used to reflect the two-stage processing in the NEXRAD hydrology sequence. In the first stage, instantaneous precipitation rates are estimated from individual radar scans. In the second stage, hourly rainfall accumulation is estimated from a sequence of radar scans. The hour index s can be viewed as a "storm counter". It represents time, in hours, since the last period of no rainfall. The scan index t ranges from 1 to 12, representing the 12 sequential scans available for precipitation processing within an hour.

The reflectivity data $\{Z_{s,t}(i,j)\}$ are the "raw data" from which precipitation estimates are made. It is important to note that the reflectivity data are highly processed data. An important processing step that has taken place prior to precipitation rate processing is hybrid scan construction. In hybrid scan construction, reflectivity measurements for a series of elevation angles are combined into the single data set represented by $\{Z_{s,t}(i,j)\}$. Three objectives of hybrid scan construction are 1) to maintain, so far as possible, a uniform beam elevation with range from the radar, 2) to minimize ground clutter, and 3) to reduce data contamination during anomalous propagation periods. In addition to hybrid scan construction, a series of quality control steps are carried out to further eliminate or mitigate the effects of nonmeteorological signals (see Ahnert et al. [1983]).

For estimating precipitation rate for scan t from hour s, reflectivity data from the current scan are used. The data set is denoted

$$H_{s,t} = \{Z_{s,t}(i,j); i=1,\ldots,360, j=1,230\} . \qquad (1)$$

To estimate hourly rainfall accumulation, reflectivity data from the current and preceding hours are used as well as rain gage observations for current and preceding hours. The data set for hour s can be represented by

$$\tilde{H}_s = \{H_{u,t}; t=1,\ldots,12; G_u(x_k); k=1,\ldots,N_u; u \leq s\} \quad (2)$$

where $G_u(x_k)$ is the rain gage observation during hour u for the gage at spatial location x_k, and N_u is the number of rain gages available for hour u. As will be seen in the following section, radar and rain gage data from preceding hours enter into the rainfall estimation procedure through the "mean field bias" computation.

3. RAINFALL MODEL

Precipitation rate at time τ and spatial location x is denoted $\xi_\tau(x)$. The time index τ represents time, in hours, since the last period of no rainfall. The precipitation rate process corresponding to the temporal and spatial scale of radar measurements is denoted

$$R_{s,t}(i,j) = |D_{ij}|^{-1} \int_{D_{ij}} \xi_\tau(x)dx \qquad (3)$$

where

$|D_{ij}|$ = surface area associated with D_{ij}

and τ is the time of scan t during hour s, that is, τ equals $(s-1)+t/12$.

The statistical model specifies the relationship between the spatially averaged precipitation rate $R_{s,t}(i,j)$ and the radar measurement $Z_{s,t}(i,j)$. Emphasis in model construction is given to range dependence of radar estimates of rainfall. Model parameters are the following:

$\alpha(j)$ = multiplicative parameter for range j;

$\beta(j)$ = exponent for range j;

$r(j)$ = minimum detectable rainfall rate at j;

$\sigma(j)$ = standard deviation of error field at j;

ν = standard deviation of mean field bias.

The model can be expressed as follows.

$$R_{s,t}(i,j)\cdot 1(R_{s,t}(i,j)>r(j)) = [\alpha(j)\cdot Z_{s,t}(i,j)^{\beta(j)}][B_s \varepsilon_{s,t}(i,j)] \qquad (4)$$

where

$$1(R_{s,t}(i,j)>r(j)) = 1 \quad ; \quad \text{if } R_{s,t}(i,j) > r(j)$$

$$1(R_{s,t}(i,j)>r(j)) = 0 \quad ; \quad \text{otherwise,} \qquad (5)$$

the error field ε has, for each s, t, i, and j, a log-normal distribution with median 1 and range-dependent standard deviation $\sigma(j)$, and B_s is a Markov chain with median 1 and standard deviation ν. Natural logarithms of reflectivity (instead of dBZ_e) are used in formulating the model to facilitate computation of statistical quantities required for parameter estimation.

The second term in brackets on the right-hand side of equation (4) represents the error model for radar measurements. The process $\{B_s\}$ represents a uniform mean field bias, which is slowly but randomly varying over the course of a storm. Unlike the mean field bias, the error field ε is spatially varying over the radar field and varies from scan to scan. Both error processes are assumed to be mutually independent and to be independent of the reflectivity process. Spatial correlation structure of the error field ε does not play a direct role in the procedures developed below. Consequently, we will not specify a specific parametric form for spatial correlation of the error field (models of spatial correlation of radar measurement error are discussed in Greene et al. [1980]). Spatial correlation does play an important role in subsequent precipitation processing steps (see Hudlow et al. [1983]).

Simple moment results for the model are given below. These results are useful for parameter estimation, as will be seen in Section 4. The moment results are conditioned on rainfall rate exceeding the minimum detectable level r(j).

$$E[\ln(R_{s,t}(i,j))] = \beta(j)\cdot E[\ln(Z_{s,t}(i,j))] + \ln(\alpha(j)) \qquad (6)$$

$$\mathrm{Var}(\ln(R_{s,t}(i,j)) = \beta(j)^2\cdot \mathrm{Var}(\ln(Z_{s,t}(i,j)) + S(j) \qquad (7)$$

where "E" denotes expected value, "Var" denotes variance, and

$$S(j) = \mathrm{Var}(\ln B(s)) + \mathrm{Var}(\ln\varepsilon_{s,t}(i,j)) \qquad (8)$$

Accumulated rainfall for hour s in bin D_{ij} is denoted $A_s(i,j)$, that is,

$$A_s(i,j) = \int_{s-1}^{s} |D_{ij}|^{-1} \int_{D_{ij}} \xi_\tau(x)\, dx d\tau \,. \qquad (9)$$

The desired rainfall estimates are given by the following conditional expectations:

$$\hat{R}_{s,t}(i,j) = E[R_{s,t}(i,j)|H_{s,t}] \qquad (10)$$

and

$$\hat{A}_s(i,j) = E[A_s(i,j)|\tilde{H}_s] \,. \qquad (11)$$

Equation (10) represents the conditional expectation of rainfall rate given radar observations for the current scan. The second conditional expectation (equation (11)) is for accumulated hourly rainfall given all radar and raingage observations for the present hour and preceding hours.

The following results can be derived from equation (4):

$$\hat{R}_{s,t}(i,j) = \alpha(j)\cdot Z_{s,t}(i,j)^{\beta(j)} \quad (12)$$

$$\hat{A}_s(i,j) = \hat{B}_s\cdot[(1/12)\sum_{t=1}^{12}\hat{R}_{s,t}(i,j)] \quad (13)$$

where

$$\hat{B}_s = E[B_s|\tilde{H}_s] \ . \quad (14)$$

The state estimator for the mean field bias given in equation (14) is obtained from a Kalman filter updating algorithm (see Ahnert et al. [1986]). The Kalman filter procedure also produces a variance estimator

$$\Sigma_s = E[(B_s - \hat{B}_s)^2|\tilde{H}_s] \quad (15)$$

which is used below for assessing accuracy of precipitation estimates.

The statistical model can be used to assess accuracy of the rainfall estimates of equations (12) and (13). We will focus on the conditional mean-squared errors defined as follows:

$$\hat{V}_{s,t}(i,j) = E[(\hat{R}_{s,t}(i,j)-R_{s,t}(i,j))^2|H_{s,t}] \quad (16)$$

$$\hat{W}_s(i,j) = E[\hat{A}_s(i,j)-A_s(i,j))^2|\tilde{H}_s] \ . \quad (17)$$

Using equation (4) it can be shown that

$$\hat{V}_{s,t}(i,j) = (\nu\cdot\sigma(j))^2\cdot\hat{R}_{s,t}(i,j))^2 \quad (18)$$

and

$$\hat{W}_s(i,j) =$$

$$(1/12)^2\sum_{t=1}^{12}\{\sigma(j)^2\cdot\hat{B}_s^{\ 2}+(\sigma(j)^2+1)\cdot\Sigma_s\}\{\hat{R}_{s,t}(i,j)\}^2$$

$$+ E[\{(1/12)\sum_{t=1}^{12}R_{s,t}(i,j) - A_s(i,j)\}^2|\tilde{H}_s] \ . \quad (19)$$

The second term in equation (19), which is termed the "sampling error", depends on properties of the radar and precipitation processing algorithm only through the sampling frequency (12 scans per hour). The sampling error can be evaluated by a simulation study such as Bell et al. [1988] have done for the Tropical Rainfall Measurement Mission sensors.

4. PARAMETER ESTIMATION

To implement the precipitation estimation procedures of equations (12) and (13) and the variance estimators of equations (18) and (19) it is necessary to specify the model parameters $\alpha(j)$, $\beta(j)$, $r(j)$, $\sigma(j)$, and ν. In this section, three procedures for estimating site-adaptation parameters of the NEXRAD hydrology sequence are discussed; they are 1) least squares, 2) a distributional approach, and 3) an iterative approach.

Given the formulation of the statistical model in equation (4), least squares is a natural candidate for parameter estimation. Implementation of a least squares procedure requires simultaneous radar and rain gage data and procedures to account for the different sampling properties of the two sensors (see Zawadzki [1982] and Austin [1987]). Calheiros and Zawadzki [1987] note that simultaneous measurements by radar and a large number of rain gages located at various distances within the radar range are seldom available for parameter estimation. They propose that the range-dependent parameters, $\alpha(j)$ and $\beta(j)$, be chosen so that the distribution of the estimated rainfall rate, given by equation (12), match the distribution of rainfall rate estimated from rain gages with long records. A similar "distributional approach" for parameter estimation is proposed by Atlas et al. [1988].

The distributional approach does not provide all of the parameters needed for implementing the variance estimators. The distributional approach also does not directly account for error terms in equation (4). Outlined below is an iterative approach to parameter estimation which combines components of the least squares and distributional approaches.

The iterative approach follows the distributional approach in using non-simultaneous radar and rain gage data. The unconditional distribution of rainfall rate at range j is denoted

$$F_j(r) = P\{R_{s,t}(i,j) \le r\} \ . \quad (20)$$

The distribution of spatially averaged rainfall rate depends on range because the area of the sample volume over which rainfall rates are averaged depends on range. From rain gage data a sample estimator of spatially averaged rainfall rate is obtained. The estimator is denoted:

$$\hat{F}_j(r) = \text{sample estimator of } F_j(r).$$

Procedures for calulating sample distributions of rainfall rate from rain gage data are discussed in Calheiros and Zawadzki [1987]. The unconditional distribution of reflectivity at range j is denoted as follows:

$$G_j(z) = P\{Z_{s,t}(i,j) \le z\} \ . \quad (21)$$

From radar reflectivity data a sample estimator of reflectivity is obtained. The estimator is denoted as follows:

$$\hat{G}_j(z) = \text{sample estimator of } G_j(z).$$

As Calheiros and Zawadzki [1987] note, estimates of rainfall rate and reflectivity distributions may need stratification to account for seasonality and diurnal effects.

To estimate the minimum detectable rainfall rate at range j, a threshold reflectivity z_0, slightly larger than the noise threshold, is chosen. The range dependent frequency associated with the reflectivity threshold is

$$p(j) = \hat{G}_j(z_0). \tag{22}$$

The minimum detectable rainfall rate at range j is estimated to be the rainfall rate with frequency p(j), that is,

$$\hat{r}(j) = \hat{F}_j^{-1}(p(j)) . \tag{23}$$

Given estimates of the minimum detectable rainfall rate, sample moments can be calculated corresponding to the theoretical moments of equations (6) and (7). From rain gage data, estimates of the mean and variance of log rainfall rate are obtained. From reflectivity data estimates of the mean and variance of log reflectivity at range j are obtained. These estimates are denoted as follows:

$$\hat{m}(j) = \text{sample mean of } \log_e \text{ rainfall rate at range } j;$$
$$= \int_{\hat{r}(j)}^{\infty} \ln(r)\,d\hat{F}_j(r) . \tag{24}$$

$$\hat{v}(j) = \text{sample variance of } \log_e \text{ rainfall rate at range } j;$$
$$= \int_{\hat{r}(j)}^{\infty} (\ln(r)-\hat{m})^2 d\hat{F}_j(r) . \tag{25}$$

$$\hat{\mu}(j) = \text{sample mean of } \log_e \text{ reflectivity at range } j;$$
$$= \int_{z_0}^{z_{max}} \ln(z)\,d\hat{G}_j(z) . \tag{26}$$

$$\hat{n}(j) = \text{sample variance of } \log_e \text{ reflectivity at range } j;$$
$$= \int_{z_0}^{z_{max}} (\ln(z)-\hat{\mu}(j))^2 d\hat{G}_j(z) , \tag{27}$$

where z_{max} is the reflectivity threshold.

Substituting sample moments into the moment equations (6) and (7) we obtain the following:

$$\hat{m}(j) = \ln(\alpha(j)) + \beta(j)\cdot\hat{\mu}(j) \tag{28}$$

and

$$\hat{v}(j) = \beta(j)^2\cdot n(j) + S(j) . \tag{29}$$

Given an estimate, $\hat{S}(j)$, of the error term S(j), the following moment estimators would be obtained

$$\hat{\beta}(j) = [(\hat{v}(j) - \hat{S}(j))/\hat{n}(j)]^{1/2} \tag{30}$$

$$\hat{\alpha}(j) = \exp\{\hat{m}(j) - \hat{\beta}(j)\cdot\hat{\mu}(j)\} . \tag{31}$$

Estimation of the error term can be based on a limited data set of simultaneous radar and rain gage data and a simple parametric model for S(j). A power law model of the form

$$S(j) = \phi + \delta j^{\gamma} \tag{32}$$

is suggested. Given estimates of $\alpha(j)$, $\beta(j)$, and r(j), an estimate of S(j) can be obtained from equations (4) and (32). The joint estimation procedure for $\alpha(j)$, $\beta(j)$, r(j), and S(j) is then carried out iteratively.

If simultaneous radar and rain gage data sets are not available at a site, estimates of S(j) from other sites may be used in equation (30). In general, it may be useful to composite simultaneous radar and rain gage data sets from similar sites for estimation of the error terms.

To obtain useful variance estimators from equations (18) and (19) the error term S(j) must be decomposed into its two components. The standard deviation of the mean field bias model, ν, can be estimated using standard estimation procedures associated with the Kalman Filter (see, for example, Harrison and Stevens [1976]). Implementing the procedure requires a simultaneous radar and raingage data set. Given estimates of ν and S(j) the standard deviation $\sigma(j)$ can be estimated.

The procedure, or mix of procedures, used for estimating NEXRAD parameters will depend heavily on data availability. It is expected that parameter estimation will be carried out in three phases. In the first phase, radar data from similar sites will be used to obtain initial parameter estimates. Following installation of a new system, radar data for the site will be collected until accurate site-specific parameter estimates can be obtained. In the third phase, radar data from the site are analyzed periodically to determine whether changes to parameter values are needed. Each phase of parameter estimation will use a mix of simultaneous and non-simultaneous radar rain gage data sets.

5. SUMMARY AND CONCLUSIONS

A statistical model relating radar reflectivity measurements to rainfall rate has been described. The model is developed for the purpose of estimating site-adaptation parameters of the NEXRAD hydrology sequence. Emphasis is placed on range-dependence of precipitation processing parameters. Procedures for assessing accuracy of preciptation estimates are also developed based on the statistical model.

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