WATER SUPPLY YIELD ANALYSIS FOR THE WASHINGTON METROPOLITAN AREA

KEY WORDS: Water Supply, Statitstical Analysis, Hydrology

ABSTRACT: The Washington D. C. Metropolitan Area (WMA) has experienced rapid population growth in the 1980's. Water supply yield analysis techniques are developed in this paper to assess adequacy of the current WMA water supply system to meet escalating future demands for water. In the statistical model developed for analyzing water supply yield, "annual yield" is a random variable representing the maximum yield the water supply system can provide in a given year. Randomness in annual yield may be attributed solely to randomness in supply or to randomness in both supply and demand. Annual yield random variables are dependent on the operating rules used for the two upstream reservoirs that serve the WMA. The dependence is simple because the operating rules are completely specified by past streamflow and two "operating parameters". The operating parameters are chosen to maximize specified attributes of the annual yield distribution, represented by the "weighted yield". The sum of the historic yield values of the individual components of the WMA water supply system is 482 mgd, a value dangerously close to current mean water use. Historic yield values for the joint system yield models exceed 700 mgd, indicating that the current water supply system is quite reliable.

SUMMARY: Water supply yield analysis techniques are developed to assess adequacy of the current water supply system for the Washington D.C. Metropolitan Area to meet escelating future demands for water.

Water Supply Yield Analysis for the Washington Metropolitan Area

By James A. Smith[1]

1. INTRODUCTION

The Washington Metropolitan Area (WMA), consisting of the District of Columbia, the Maryland suburbs of D.C., and the Virgina suburbs of D.C., has experienced rapid growth in the 1980's. Associated with population growth is a sharp increase in water use (see Figure 1). In this paper yield analysis techniques are developed to assess adequacy of the current WMA water supply system to meet escalating future demands.

Three major water utilities (one for each of the principal geographic subregions) provide drinking water for the WMA. Water supplies for the three utilities are operated jointly to minimize the risk of water supply shortage (institutional arrangements that made joint operation possible are described in Metzger and Doerman [1987] and Sheer and Flynn [1985]). The direct link among the three water utilities is reliance on the Potomac River. Natural flow of the Potomac River can be

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augmented by water supply releases from two upstream reservoirs, a large reservoir located far from the WMA, and a smaller reservoir located in the northwest corner of the WMA.

The statistical model that is developed to analyze water supply yield is nonparametric. The annual yield for year i, Y_i, is a nonnegative random variable representing the maximum yield that the water supply system can provide in year i. Randomness in Y_i may be attributed solely to randomness in supply in year i (as in the yield model of Section 3) or to randomness in both supply and demand (as in the yield model of Section 4). Annual yield random variables are dependent on reservoir operating rules. The dependence is simple because operating rules are completely specified by past streamflow, that is streamflow prior to the release time, and two "operating parameters". The operating parameters are chosen to maximize specified attributes of the annual yield distribution, represented by the "weighted yield".

Techniques developed in this paper borrow from several sources. The work of Palmer et al. [1982], in which a statistical treatment of water supply yield is developed, is a direct antecedent of this paper (see also Vogel and Stedinger [1987]). The search for increased water supply yield from joint reservoir operation is motivated by synergystic gain concepts introduced by Hirsch et al. [1977]. The emphasis on water use variability in the yield model of Section 4 builds upon developments in short-term water use modeling (see, for example, Maidment and Parzen [1984a] and [1984b] and Smith [1988]).

Contents of the sections are as follows. Section 2 contains definitions, notation, and development of the statistical model used for yield analysis. Yield analysis models for the WMA are developed in Section 3. In Section 4 the yield analysis techniques are extended to account for variability in water use. A summary and conclusions are presented in Section 5.

2. DEFINITIONS AND NOTATION

Storage in Jennings Randolph reservoir (the large, upstream reservoir) on day t of year i is denoted $S1_i(t)$; the capacity of water supply storage for Jennings Randolph is denoted C_1 and equals 13.4 billion gallons (SI conversion table is provided in Appendix I.). Storage in Little Seneca reservoir (the smaller, downstream reservoir) is denoted $S2_i(t)$; the capacity of Little Seneca is denoted C_2 and equals 4.0 billion gallons. The release rules are denoted $R1_i(t)$ and $R2_i(t)$, reservoir inflow $Z1_i(t)$ and $Z2_i(t)$. Natural flow of the unregulated portion of the Potomac River is $X_i(t)$. The Potomac River above Washington D.C. has a drainage area of approximately 12,000 square miles. Less than 400 square miles of the basin are controlled by reservoirs. Mean daily flow of the Potomac River is approximately 9,000 million gallons per day (mgd). During the period 1930 - 1981 flow in the Potomac River at Washington D.C. ranged from a minimum of 400 mgd to a maximum of 360,000 mgd.

A "water supply year" begins on June 1 and consists of $T = 365$ days. It is assumed that reservoir storage is at capacity at the beginning of each year, that is, $S1_i(0)$ equals C_1 and $S2_i(0)$ equals C_2. Storage in two local water supply reservoirs (one of which drains to the Chesapeake Bay, the other to the tidal Potomac River below Washington D.C.) is not explicitly considered in our yield models. Operation of these reservoirs for water supply is highly constrained. Their combined yield balances an environmental flowby requirement for the Potomac estuary.

We introduce in Sections 3 and 4 two definitions of "annual yield." The following examples illustrate the form annual yield random variables may take in special cases. The annual yield definitions used in Sections 3 and 4 are straightforward extensions of the definitions in Examples 1 - 3 below (we suppress dependence on the reservoir index in the storage, inflow and release notation in the examples).

EXAMPLE 1-

The yield of a single reservoir for year i, Y_i, is the solution to the math programming problem,

$$\text{maximize } y$$

$$\text{such that } R_i(t) = y \tag{1}$$

$$S_i(t) = \min \{S_i(t-1) + Z_i(t) - R_i(t), C\}$$

$$S_i(t) > 0$$

$$\text{for } t = 1, \ldots, T .$$

The reservoir yield Y_i is the largest constant release that can be maintained in year i without at any time completely depleting storage, given that the reservoir begins the year with storage at capacity.

EXAMPLE 2-

The yield of an unregulated river for year i is simply the minimum daily flow, that is,

$$Y_i = \min \{X_i(t),\ t=1,\ldots,T\}. \tag{2}$$

EXAMPLE 3-

The combined yield of a river and upstream reservoir can be defined as the solution to

$$\text{maximize } y$$

$$\text{such that } R_i(t) = \max\{y - X_i(t), 0\} \tag{3}$$

$$S_i(t) = \min\{S_i(t-1) - R_i(t) + Z(t), C\}$$

$$S_i(t) > 0$$

$$\text{for } t = 1, \ldots, T.$$

Releases from the reservoir are made only if natural flow of the river falls below the yield value y we wish to maintain. Implicit in equation (3) is the assumption that reservoir releases are immediately available at the demand point. This model would not be appropriate for examining combined yield of Jennings Randolph reservoir and the Potomac River. Releases from Jennings Randolph reservoir require approximately 5 days to reach Washington, D.C. (Trombley [1982]).

Yield random variables developed in Sections 3 and 4 are defined implicitly by representations like equations (1) - (3). We assume that yield random variables $Y_1, \ldots, Y_n$ are independent and identically distributed (i.i.d.). Their common distribution is denoted by

$$F(y) = P\{Y_i \le y\} \tag{4}$$

Parametric assumptions on yield distributions are not made. The independence assumption rests on critical periods (or minimum annual flow) being separated from year to year by sufficiently long non-drawdown periods (see Loaiciga and Marino [1988] for detailed discussion; see also Matalas [1963]). For reservoir yield samples, the identically distributed assumption is valid and useful only if reservoirs refill annually (we return to this issue in Section 3).

For water supply yield analysis, interest focuses on the lower tail of the distribution F. A useful tool for describing the lower tail of F is the quantile function

$$Q(p) = \inf \{y : F(y) > p\} \qquad p \in [0,1). \tag{5}$$

The quantile function evaluated at p gives the yield that can be maintained in a given year with probability 1-p. Of particular interest is the "safe yield" y_F, which is the lower bound of F, that is

$$y_F = Q(0) \ . \tag{6}$$

In analyzing yield data it is useful to convert the sample $Y_1,\ldots,Y_n$ to their "order statistics" $Y_{(1)}<\ldots<Y_{(n)}$. We term the minimum value $Y_{(1)}$ the "historic yield" (following Palmer et al. [1982]). The "interior" portion of the distribution F can be estimated from order statistics using standard procedures (see, for example, David [1970]). Smith and Weissman [1985] and

Loaiciga and Marino [1988] develop techniques that can be used to estimate "exterior" characteristics of the yield distribution, including the safe yield y_F.

3. YIELD ANALYSIS - CONSTANT DEMAND

The traditional method for assessing yield of a water supply system is to combine the historic yield values for each of the system components. The historic yield values for Jennings Randolph and Little Seneca reservoir, obtained from equation (1), are 63 mgd and 13 mgd, respectively. The historic yield for the Potomac River, obtained from equation (2) is 406 mgd. Combining the historic yield values, we obtain a "system yield" of 482 mgd. Hirsch et al. [1977] show that if components of a water supply system are operated jointly the system yield may be substantially larger than the sum of the individual historic yields. They term the increased yield due to joint operation the synergistic gain. In this section a yield model is formulated for assessing the joint yield of the Potomac River, Jennings Randolph reservoir, and Little Seneca reservoir.

To obtain an analysis of system yield relevant to operational conditions we must explicitly incorporate reservoir operating rules. The basic guidelines for operation of Jennings Randolph and Little Seneca are straightforward. Releases are not made when demand can be met from natural flow of the Potomac River (which is likely to be the case for the period November -

June of any year). When reservoir releases are needed, Jennings Randolph provides the "average" shortfall between demand and natural flow of the Potomac. Little Seneca fills in holes created by "extreme" shortfalls between demand and Potomac flow (releases from Little Seneca reservoir reach the WMA within the day of release). These operating guidelines are based on the different reservoir capacities and travel times of reservoir releases to the WMA.

The operating rules that are used for Jennings Randolph and Little Seneca are of the following form. To meet a constant demand of y for the WMA, the Jennings Randolph release rule is

$$R1_i(t) = \max\ \{a_1(y - X_i(t)) + a_2,\ 0\} \tag{7}$$

where a_1 is a nonnegative constant and a_2 is a real constant. The release rule for Little Seneca is

$$R2_i(t) = \max\ \{y - \tilde{X}_i(t),\ 0\} \tag{8}$$

where

$$\tilde{X}_i(t) = X_i(t) + R1_i(t-5) \tag{9}$$

is natural flow of the Potomac plus the routed releases from Jennings Randolph (recall from Section 2 that releases from Jennings Randolph reservoir require 5 days to reach the WMA).

With the preceding operating rules, the system yield for year i is defined by

$$\begin{aligned}
&\text{maximize} \quad y \\
&\text{such that} \\
&R1_i(t) = \min \{ \max \{a_1(y-X_i(t))+a_2,0\} \ , \ S1_i(t-1)+Z1_i(t)\} \\
&R2_i(t) = \max \{y - \tilde{X}_i(t),0\} \\
&S1_i(t) = \min \{S1_i(t-1) + Z1_i(t) - R1_i(t),C_1\} \\
&S2_i(t) = \min \{S2_i(t-1) + Z2_i(t) - R2_i(t),C_2\} \\
&S2_i(t) > 0 \\
&\qquad \text{for } t = 1,\ldots,T \ .
\end{aligned} \tag{10}$$

If the Jennings Randolph release rule specified by equation (7) is greater than the available storage, $S1_i(t-1) + Z1_i(t)$, the actual release, $R1_i(t)$, is reduced to the available storage. In this case, storage in Jennings Randolph is completely depleted; the system does not, however, necessarily fail. The system fails when Little Seneca runs out of water.

The optimization problem defined by equation (10) is solved n times, where n is the number of years of historical streamflow observations. The solution for year i gives the value of system yield for year i, which is denoted Y_i. The yield distribution of random variables $Y_1,...,Y_n$ obtained from equation (10) is dependent on reservoir operating rules. The dependence is simple because the operating rules are completely specified by streamflow observations and the two parameters a_1 and a_2. We will denote by $Q_a(p)$ the quantile function of the yield distribution indexed by the parameters $a = (a_1,a_2)$. In other words, $Q_a(p)$ is the quantile function of yield random variables Y_1, Y_2,... obtained from equation (10), given that the operating parameters of equation (7) are (a_1,a_2).

The parameters a_1 and a_2 are chosen to optimize specific characteristics of the yield distribution. The optimization problem can be formulated as follows: choose the parameters (a_1,a_2) to maximize the "weighted yield"

$$I(a) = \int_0^1 w(p)Q_a(p)dp \qquad (11)$$

where w(p) is a nonnegative weight function satisfying

$$\int_0^1 w(p)dp = 1 \ .$$

The weight function w(p) determines attributes of the yield distribution that are to be optimized. The following examples illustrate flexibility of (11).

EXAMPLE 4. If the weight function is the Dirac delta function

$$w(p) = \delta_{p_0} \tag{12}$$

then

$$I(a) = Q_a(p_0) \tag{13}$$

In particular if p_0 equals 0, the objective is to maximize the safe yield y_F.

EXAMPLE 5. If the weight function is constant and equal to 1,

$$I(a) = E_a[Y_i] \tag{14}$$

The objective in this case is to maximize the expected yield.

EXAMPLE 6. If

$$\begin{aligned} w(p) &= 10 \qquad p \in [0,.1] \\ w(p) &= \ \ 0 \qquad \text{otherwise,} \end{aligned} \tag{15}$$

then the objective is to maximize the "average yield" over the lower 10% of the yield distribution, that is,

$$I(a) = 10 \cdot \int_{0}^{.1} Q_a(p)dp \tag{16}$$

Because the yield distribution is treated in a nonparametric framework, there is little hope of evaluating the weighted yield I(a) analytically. The weighted yield can, however, be estimated by

$$\hat{I}(a) = \sum_{i=1}^{n} w_i Y_{(i)} \tag{17}$$

where

$$w_i = \int_{(i-1)n^{-1}}^{i \cdot n^{-1}} w(p)dp \tag{18}$$

and $Y_{(i)}$ is the ith order statistic obtained from equation (10).

EXAMPLE 4a. If

$$I(a) = Q_a(p_0) \tag{19}$$

then

$$w_i = 1 \qquad p_0 \in ((i-1)n^{-1}, i \cdot n^{-1}] \tag{20}$$
$$w_i = 0 \qquad \text{otherwise}$$

EXAMPLE 5a. If

$$I(a) = E_a[Y_i] \tag{21}$$

then

$$w_i = n^{-1} \qquad \forall \; i \tag{22}$$

EXAMPLE 6a. If

$$I(a) = 10 \cdot \int_0^{.1} Q_a(p)dp \tag{23}$$

then

$$w_i = 10n^{-1} \qquad i < 10n^{-1} \tag{24}$$
$$w_i = 0 \qquad i > 10n^{-1}$$

Table 1 contains parameter estimates and weighted yield values for four choices of weight function, given in terms of the discretized weights $(w_1, \ldots, w_n)$. The total number of years, n, is 50. To maximize the historic yield we take w_1 equal to 1 and all other weights equal to 0. The optimal parameters are: $a_1 = .4$, $a_2 = 70$. The optimal historic yield is 719 mgd, an increase of 237 mgd above the sum of the component historic yields!

By placing all weight on the historic yield year, operations are closely tied to a particular sequence of hydrologic events. The optimal operating parameters change markedly (from (.4,70.) to (1.1,0.)) if the weight function is changed to (.7,.3,0.,...,0.). Note, however, that the historic yield for the parameters (1.1,0) drops only 9 mgd from the optimal value of 719 mgd. The operating parameters are clearly very sensitive to this relatively small change in the weight function. The optimal yield, however, is not. For the weight function (.7,.3,0.,...,0.), the two years that receive positive weight are 1930 and 1966. Minimum flow in 1930 (the historic yield year) was 480 mgd, approximately 60 mgd more than in 1966. In 1930, however, Potomac flow remained below 700 mgd until the end of November. In 1966 the Potomac reached (and remained above) 1000 mgd by the end of September. The two years pose sharp contrasts between severity and duration of drought.

Figure 2, which shows the reservoir drawdown and refill cycle for Jennings Randolph during the historic yield year using (1.1,0.) as operating parameters, supports the refill assumption that underlies the modeling framework developed in Section 2. Drawdown lasts until the end of November. By early Spring the reservoir has filled to capacity. Even in the most severe of years, refill will be achieved prior to June 1.

4. YIELD ANALYSIS - VARIABLE DEMAND

The yield results of the preceding section are not directly comparable to the information available concerning long-term trend in water use (see Figure 1). In this section annual yield random variables will be represented in terms of mean daily water use so that yield and demand are directly comparable. A simple model of daily water use, termed a conditional autoregressive process (see Smith [1988]), is used to assess the role of water use variability in determining water supply yield.

Water use for day t of year i is denoted $D_i(t)$. If mean daily water use for year i is y, the conditional autoregressive model is specified by

$$D_i(t) = m(t) \cdot y + b \cdot [D_i(t-1) - m(t-1) \cdot y] + y^{1/2} A_i(t) \tag{25}$$

where b is a real-valued parameter, m(t) is the "unit demand function"

$$m(t) = \frac{E[D_i(t)]}{\sum_{j=1}^{T} E[D_i(j)]} \tag{26}$$

and $\{A_i(t)\}$ is an i.i.d. sequence of Gaussian random variables with mean 0 and variance s^2.

The unit demand function on day t, m(t), is the ratio of average water use on day t of the year to average daily water use over the course of the year. The unit demand does not vary from year to year even if long-term trend in mean water use is present. This assumption implies that, although mean water use may exhibit long-term trends over time, seasonal and day-of-week structure of water use do not.

The reservoir operating rules of equations (7) and (8) are easily modified to account for daily variability in water use. If mean daily water use for the year is y, the Jennings Randolph release is

$$R1_i(t) = \max \{a_1[m(t+5)\cdot y - X_i(t)] + a_2, 0\} \qquad (27)$$

The Little Seneca release is

$$R2_i(t) = \max \{D_i(t) - \tilde{X}_i(t), 0\} \qquad (28)$$

The system yield for year i, Y_i, is the solution to,

$$\text{maximize } y \tag{29}$$

such that

$$D_i(t) = y \cdot m(t) + b[D_i(t-1) - y \cdot m(t-1)] + y^{1/2} A_i(t)$$

$$R1_i(t) = \min \{ \max \{a_1[m(t)y - X_i(t)] + a_2, 0\} , S1_i(t-1) + Z1_i(t)\}$$

$$R2_i(t) = \max \{D_i(t) - \tilde{X}_i(t), 0\}$$

$$S1_i(t) = \min \{S1_i(t-1) + Z1_i(t) - R1_i(t), C_1\}$$

$$S2_i(t) = \min \{S2_i(t-1) + Z2_i(t) - R2_i(t), C_2\}$$

$$S2_i(t) > 0$$

$$\text{for } t = 1,\ldots,T .$$

The annual yield Y_i in this formulation is the largest mean daily water use that the water supply system can provide in year i.

In previous formulations (Section 3), randomness in annual yield is attributed solely to randomness in streamflow. In the current formulation randomness is additionally attributed to random fluctuations in daily water use. Because water use exhibits marked trend over time, it is not possible to use water use observations directly. Instead we use historical water use data to estimate parameters of the water use model (see Smith

[1988]). A "data set" of error variables $\{A_i(t);t=1,...,T;i=1,...,n\}$ for use in equation (29) is then obtained by simulation. The parameters that must be estimated are the unit demand function m(t), the autoregressive parameter b, and the standard error coefficient s.

Figure 3 shows weekly-averaged values of the estimated unit demand function. The estimate of the autoregressive parameter, b, obtained in Smith [1988] is .76; the estimate of the standard error coefficient, s, is .89.

Table 2 shows optimal operating parameters for the set of weights used in Table 1. Surprisingly, the optimal weighted yield for the weights (1.,0.,...,0.) is unchanged from the previous formulation despite the additional variability introduced in daily water use (and despite the different interpretation of annual yield random variables). Note, however, that the optimal weighted yield for other sets of weights are less than the comparable values in Table 1. The estimated unit demand function m(t) (Figure 3) provides an explanation. Mean water use peaks in July. During the extended reservoir drawdown period in fall and early winter, mean water use is at its minimum. Operations for the historic yield year (1930) extend into the period of decreasing water use in late fall and early winter. For other years (and especially 1966, the second drought of record) water supply operations are critical during the period of peak water use and terminate before the trough in water use. The results of Table 2 show how seasonality in mean daily water use affects water supply yield.

The effect of variability in mean daily water use is that operation of Jennings Randolph shifts toward a more constant release pattern. For each set of weights the optimal parameter a_1 in Table 2 is less than the corresponding value in Table 1 and the optimal value of a_2 is greater than the value in Table 1. Clearly, the operating parameters of Table 1 are not optimal if variability in water use is taken into account. The paramters used for actual WMA water supply operations are (1.0,10.).

5. CONCLUSIONS

In this paper water supply yield models have been developed to assess adequacy of the current WMA water supply system to meet escalating water demands. The following features of the yield models are noteworthy.

1) In each of the models the fundamental concept is "annual yield". Annual yield for a given year is a nonnegative random variable which represents the maximum yield that the water supply system can provide in that year. Randomness in annual yield may be attributed solely to randomness in supply (as in the yield model of Section 3) or to randomness in both supply and demand (as in the yield model of Section 4).

2) The statistical model that is used to analyze yield random variables is nonparametric. It is assumed that annual yield

random variables are i.i.d. It is noted in Section 2 that justification for the yield model rests in part on the fact that reservoirs refill annually. For application to water supply systems with multi-year reservoir drawdown (as in much of the western U.S.), the yield models would require modification.

3) Annual yield random variables are dependent on reservoir operating rules. For the WMA water supply system, dependence is simple because operating rules are completely specified by streamflow observations and two real-valued parameters (a_1, a_2). The form of reservoir operating rules for the WMA water supply system is dictated by reservoir size and travel time. The large reservoir, which is located far from the WMA, provides "average" shortfalls between demand and natural flow of the Potomac River. The small reservoir, located close to the WMA, covers shortfalls arising from extreme demands. The operating parameters (a_1, a_2) determine the boundary between "average" and "extreme".

4) Operating parameters (a_1, a_2) are chosen to optimize a specified attribute of the yield distribution. The "weighted yield", $I(a)$, is introduced as the criterion for selecting the operating parameters. It is shown that the sample estimator $\hat{I}(a)$ can be used to obtain solutions to the optimization problem.

5) The statistical approach used for developing reservoir operating rules contrasts sharply with stochastic dynamic programming. In stochastic dynamic programming, structure of operating rules is general and strict parametric assumptions are

made on the random component of the problem. We take the opposite approach. We severely restrict the structure of operating rules and treat the random component of the problem in a nonparametric framework.

6) The yield model is extended in Section 4 to accomodate variability in water use. In the yield model of Section 4, annual yield is interpreted as the largest mean daily water use that the system can provide without experiencing supply shortfalls on any day. This formulation allows direct comparison of water supply yield with trends in mean water use.

7) The historic yield values of Jennings Randolph reservoir, Little Seneca reservoir, and the Potomac River, are, respectively, 63 mgd, 13 mgd, and 406 mgd. Combining the yield values of the three components of the WMA water supply system produces a yield of 482 mgd. The historic yield values for joint system yield models of both Sections 4 and 5 exceed 700 mgd. The synergistic gain attributed to the system operating rules is in excess of 200 mgd. Most importantly, synergistic gain places water supply yield well above the current mean water use.

The yield analysis results are encouraging. For the present, the WMA water supply system is clearly quite reliable. If water use continues to grow at the rate of the past 5 years, mean daily water use will equal the historic yield of the water supply system in approximately 30 years. Additional water supply will ultimately be needed for the WMA. The techniques used in this paper have relevance to future design problems. If water supply storage is to be added to the WMA system, it should be added, as Little Seneca reservoir was, to functionally augment the current water supply system.

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APPENDIX I. CONVERSION TO SI UNITS

To Convert	To	Multiply by
million gallons	cubic meters	3785.
square miles	sqare kilometers	2.592

APPENDIX II.-REFERENCES

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APPENDIX III. NOTATION

$a = (a_1, a_2)$ reservoir operating parameters

b = conditonal autoregressive parameter in water use model

$S1_i(t)$ = storage in Jennings Randolph on day t of year i

$S2_i(t)$ = storage in Little Seneca on day t of year i

$R1_i(t)$ = release rule for Jennings Randolph

$R2_i(t)$ = release rule for Litlle Seneca

$Z1_i(t)$ = inflow to Jennings Randolph

$Z2_i(t)$ = inflow to Little Seneca

$X_i(t)$ = natural flow of the Potomac River

$\tilde{X}_i(t)$ = $X_i(t)$ plus routed Jennings Randolph releases

$D_i(t)$ = WMA water use

$A_i(t)$ = error sequence in water use model

Y_i = annual yield for year i

$Y_{(i)}$ = ith order statistic of annual yield

$F(y)$ = annual yield distribution functin

$Q(p)$ = annual yield quantile function

y_F = safe yield

$I(a)$ = weighted yield using operating parameter a

$\hat{I}(a)$ = sample estimator of $I(a)$

E_a = Expectation given operating parameter a

$w(p)$ = weight function

$m(t)$ = unit demand function

C1 = capacity of Jennings Randolph water supply storage

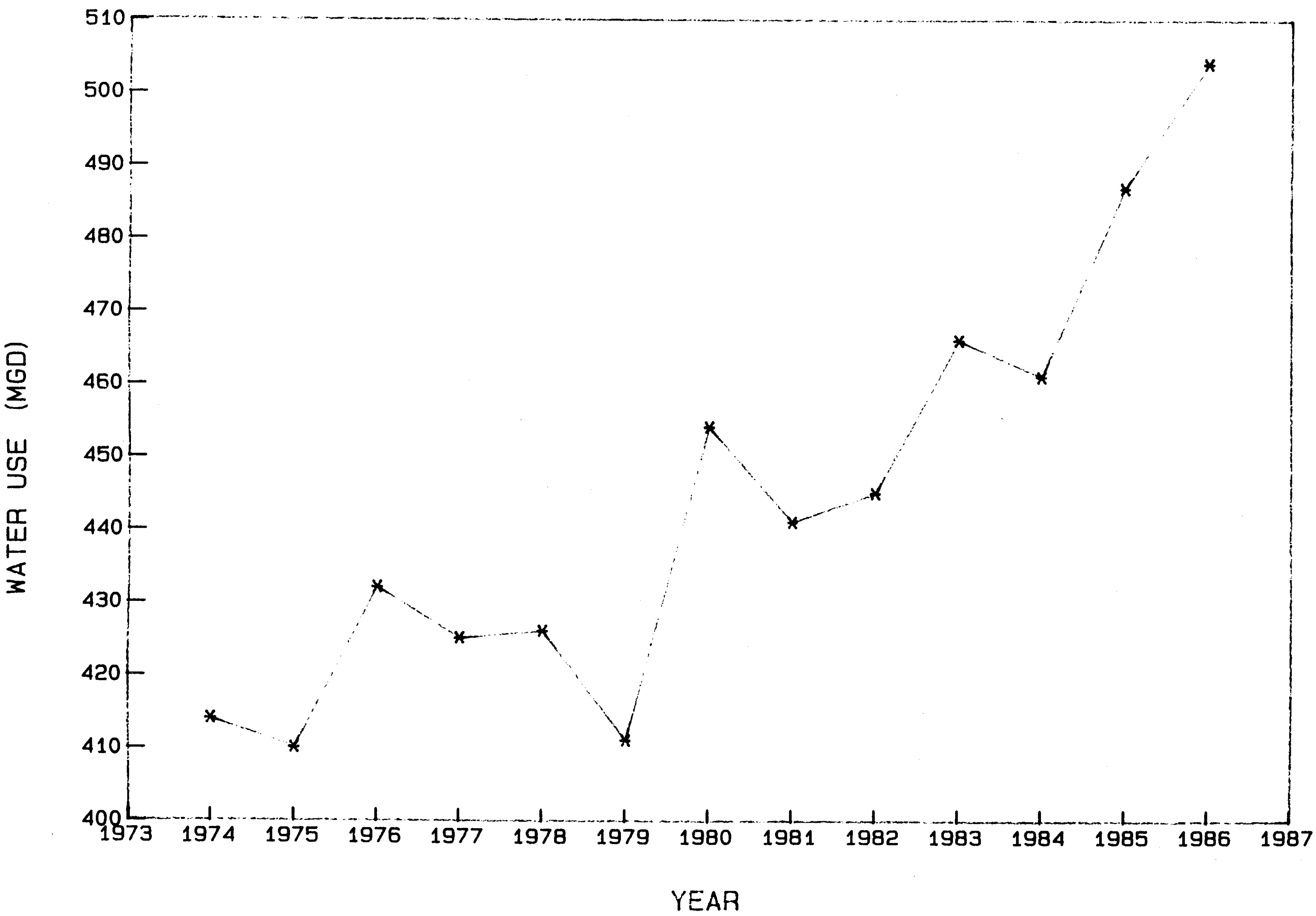
C2 = capacity of Little Seneca

T = number of days in a "water supply" year

n = number of years of historical data.

List of Figures

FIGURE 1.



WATER USE (MGD)
510
500
490
480
470
460
450
440
430
420
410
400
1973
1974
1975
1976
1977
1978
1979
1980
1981
1982
1983
1984
1985
1986
1987
YEAR

FIGURE 2.

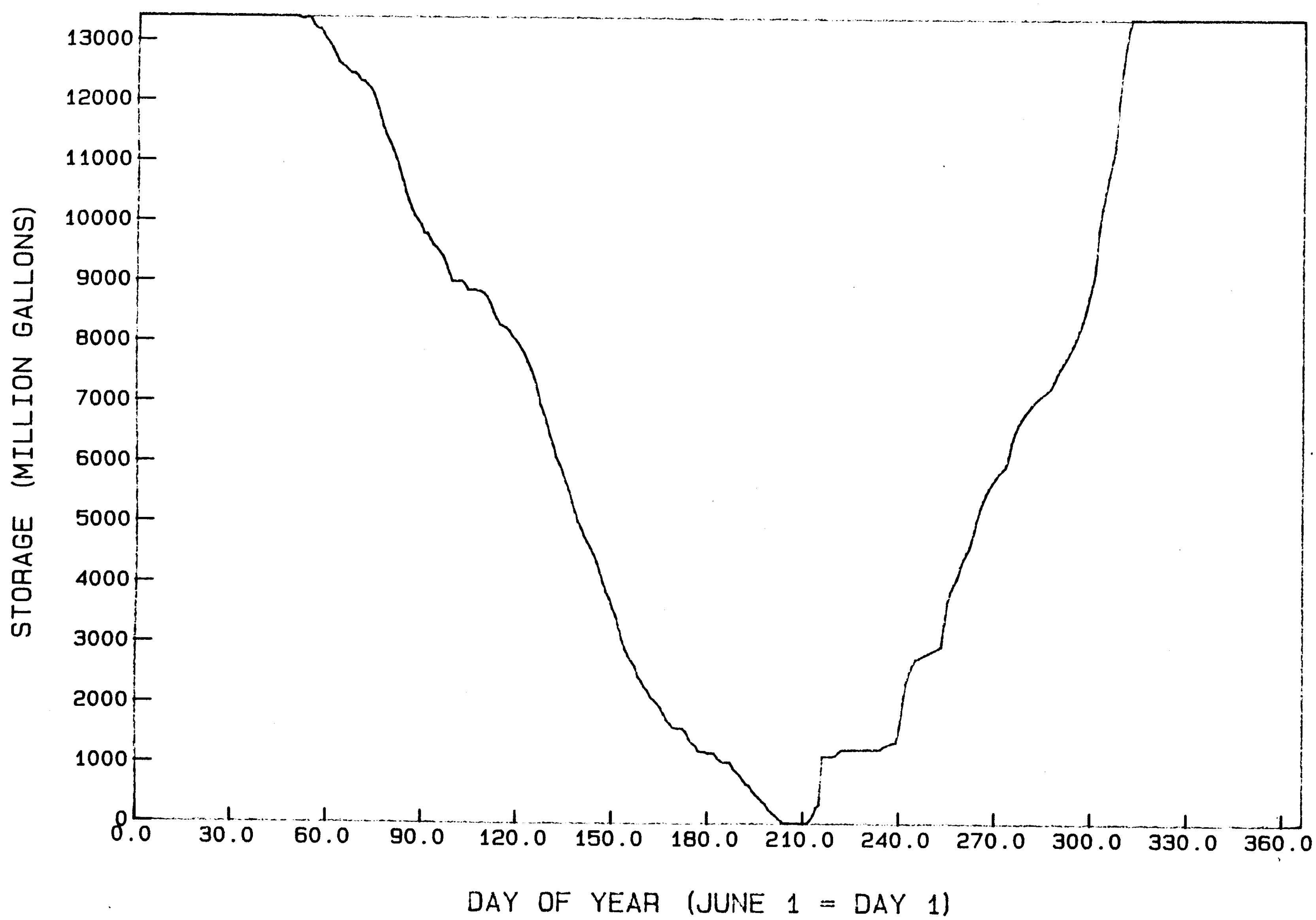
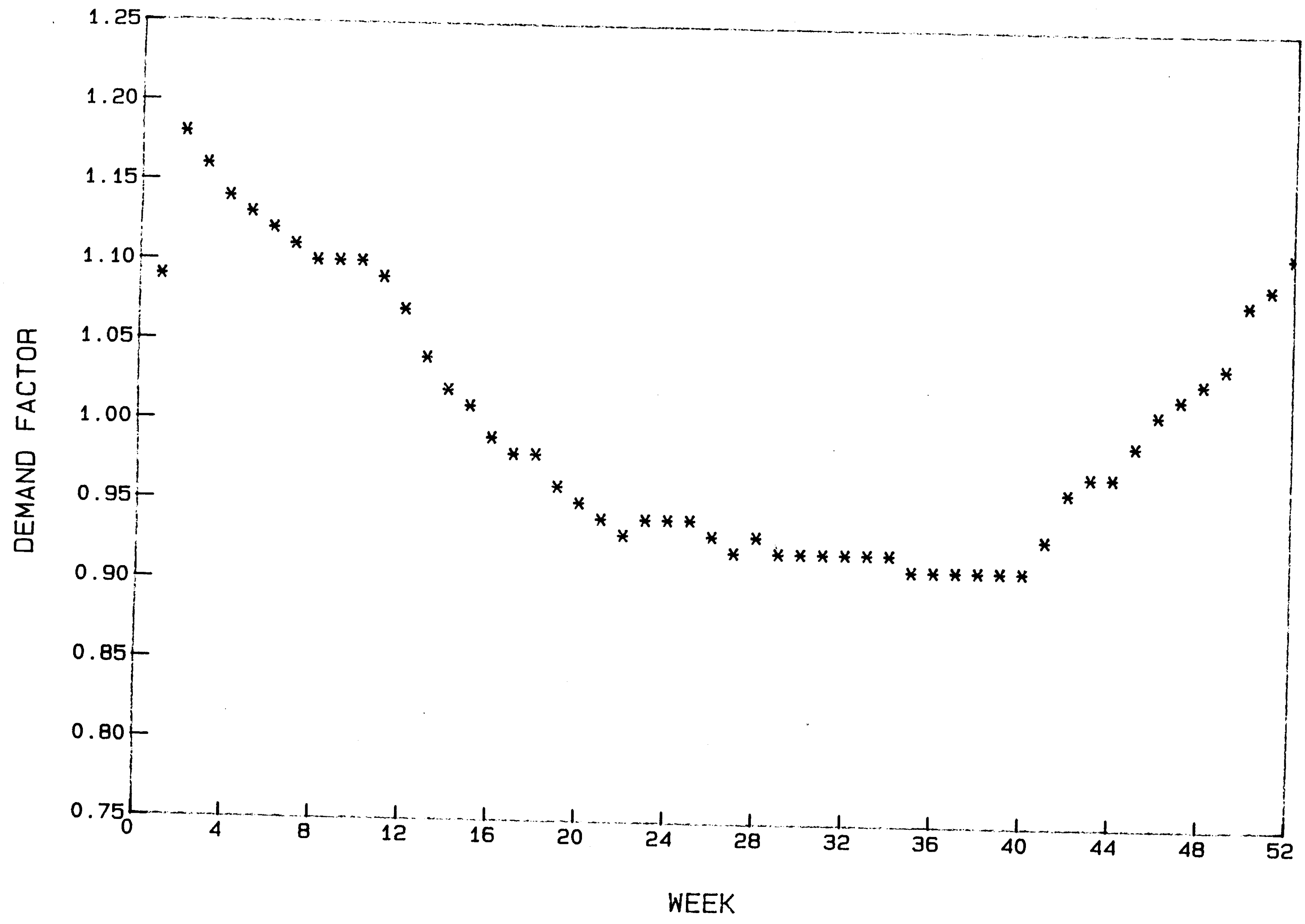

STORAGE (MILLION GALLONS)
13000
12000
11000
10000
9000
8000
7000
6000
5000
4000
3000
2000
1000
0
0.0
30.0
60.0
90.0
120.0
150.0
180.0
210.0
240.0
270.0
300.0
330.0
360.0
DAY OF YEAR (JUNE 1 = DAY 1)

FIGURE 3



DEMAND FACTOR
1.25
1.20
1.15
1.10
1.05
1.00
0.95
0.90
0.85
0.80
0.75
0
4
8
12
16
20
24
28
32
36
40
44
48
52
WEEK

WEIGHTS	OPTIMAL PARAMETERS a_1	a_2	WEIGHTED YIELD I(a)
(1.,0,...........,0)	.4	70.	719 (719)
(.7,.3,0,.........,0)	1.1	0.	755 (710)
(.6,.3,.1,0,......,0)	.9	40.	786 (707)
(.5,.2,.2,.1,0,...,0)	.8	70.	820 (700)

Table 1. Optimal operating parameters and weighted yield values (in mgd) for four sets of weight functions. Under "Weighted Yield", historic yield values for the optimal parameters are given in parenthesis.

WEIGHTS	OPTIMAL PARAMETERS a_1	a_2	WEIGHTED YIELD I(a)
(1.,0,............,0)	.3	90.	719 (719)
(.7,.3,0,.........,0)	1.0	10.	733 (705)
(.6,.3,.1,0,......,0)	.7	90.	765 (699)
(.5,.2,.2,.1,0,...,0)	.7	90.	805 (699)

Table 2. Optimal operating parameters and weighted yield values (in mgd) for four sets of weight functions with variable demand. Under "Weighted Yield", historic yield values for the optimal parameters are given in parenthesis.