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Spatial rainfall estimation by linear and non-linear co-kriging of radar-rainfall and raingage data

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Abstract: The feasibility of linear and nonlinear geostatistical estimation techniques for optimal merging of rainfall data from raingage and radar observations is investigated in this study by use of controlled numerical experiments. Synthetic radar and raingage data are generated with their hypothetical error structures that explicitly account for sampling characteristics of the two sensors. Numerically simulated rainfall fields considered to be ground-truth fields on 4×4 km grids are used in the generation of radar and raingage observations. Ground-truth rainfall fields consist of generated rainfall fields with various climatic characteristics that preserve the space-time covariance function of rainfall events in extratropical cyclonic storms. Optimal mean areal precipitation estimates are obtained based on the minimum variance, unbiased property of kriging techniques under the second order homogeneity assumption of rainfall fields. The evaluation of estimated rainfall fields is done based on the refinement of spatial predictability over what would be provided from each sensor individually. Attention is mainly given to removal of measurement error and bias that are synthetically introduced to radar measurements. The influence of raingage network density on estimated rainfall fields is also examined.

Key words: Spatial rainfall estimation, kriging, ordinary co-kriging, disjunctive co-kriging.

1 Introduction

For real-time operational forecasting of river flows, accurate estimation of the spatial distribution of precipitation rate over river basins is of paramount importance. High spatial variability of precipitation at the river basin scale and sparse raingage network densities are known to be major causes of uncertainty in forecasting streamflows. It is generally recognized that proper prediction of river basin response to rainfall is highly dependent on the accuracy of determining storm system locations within watershed boundaries. Thus, the ability for obtaining higher resolution estimates of spatial variability in the rainfall fields becomes important in the case of identification of locally intense storms which could lead to floods and especially to flash floods.

The accurate estimation of the spatial distribution of rainfall from only raingage data for operational purposes requires a very dense network of automated instruments, which entails large installation and operational costs. In recent years, ground-based radar systems have been implemented to provide continuous estimates of rainfall amounts in space and time over river basins. Since weather-radar can detect precipitation patterns over a large area at short time periods, it could help to fill the space and time gaps which is not in raingage data. Various studies discuss the hydrologic application of weather radar (Byers 1948; Hiatt 1956; Kessler and Wilk 1968; Greene and Flanders 1976;

Crawford 1979). The radar measurements of rainfall are obtained by scanning the precipitation producing clouds via electron radar beam and measuring the backscattered energy of hydrometeors which are then accumulated in time and converted to rainfall rate with appropriate Z-R relationships, the relationship between reflectivity from hydrometeors and the rainfall rate (Battan 1973). However, the process of precipitation reflectivity measurement and the transformation to rainfall rate is associated with very high measurement errors, the sources of which are well documented in the literature. For example, Zawadzki (1984) discussed various sources of random and systematic errors that corrupt radar-rainfall data. These errors create large discrepancies between radar measurements of rainfall rate and the true rainfall rate at the ground surface, which could be as high as 200% (Wilson 1970). Therefore, it is believed that due to high accuracy of point raingage data, the combination of observations from both sensors could improve the areal estimates of rainfall.

Earlier attempts to merge raingage and radar-rainfall measurements included deterministic and statistical techniques. The deterministic approach makes use of raingage measurements for calibration of radar-rainfall data, and was applied by Wilson (1970) and Brandes (1975). Statistical methods include those by Crawford (1979), Eddy (1979), Krajewski and Hudlow (1983), and Ahnert et al. (1986). The works by Krajewski (1987) and Cruetin et al. (1988) used ordinary co-kriging for merging radar-rainfall and raingage data. The latest work by Seo et. al (1989a, and b) evaluated the multisensor rainfall estimation problem by utilizing a comprehensive numerical experiment using ordinary, universal and disjunctive co-kriging techniques.

As an extension of previous work, this paper examines the multisensor precipitation interpolation problem in both linear and nonlinear estimation frameworks. The formulation of the disjunctive co-kriging estimator for spatial rainfall estimation is introduced in more detail and the results are evaluated with the main focus on the advantages and disadvantages of linear versus nonlinear estimators for spatial rainfall estimation. An important consideration in this research is the selection of a suitable estimation technique that produces sufficiently accurate mean areal precipitation estimates for input to hydrologic models, and which may thus improve operational forecasting of river flows and flash floods. The evaluation of estimation techniques is done by generating synthetic radar-rainfall and raingage measurements with known measurement error parameters from hypothesized known ground-truth rainfall fields and merging the two sampled rainfall fields via linear and non-linear co-kriging techniques. Since, in our experiments, the true rainfall field is known, errors in the estimation procedures can be assessed and the adequacy of these techniques for areal rainfall estimation evaluated. In Section 2, the proposed methodologies of linear and non-linear co-kriging techniques for spatial rainfall estimation are described and compared. Section 3 presents the design and implementation of the numerical experiments, generation of ground-truth rainfall fields using a space-time rainfall model, and generation of radar-rainfall and rain gage observations. In Section 4, the results of numerical experiments are analysed and interpreted, and in the last section, the conclusions of the study are presented.

2 Methodology

The rainfall estimation methodology presented in this section makes use of synthetically generated, spatially continuous, radar-rainfall data and synthetically generated point, relatively accurate, measurements provided from rain gages. Our objective was to merge these two measurements in an optimal way to take advantage of the different sampling characteristics each sensor provides to improve areal rainfall estimates. A controlled numerical experiment was designed and carried out to evaluate the relative performance of the proposed estimation procedures under various combinations of sensor measurement error parameters. In this study we resorted to controlled numerical experiments since, due to lack of large enough raingage and radar-rainfall data sets, meaningful

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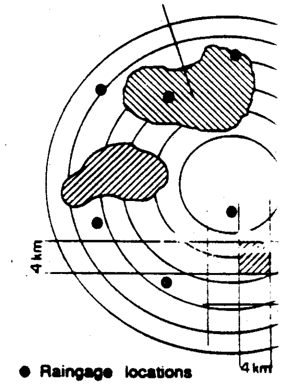

Figure 1. Schematic diagram
signal and the sparse network

evaluation is impossible. Also, by implementation of such a numerical experiment the evaluation of co-kriging techniques could be performed for various rainfall measurement fields with different error statistics.

Under realistic operational applications point raingage measurements are accurate but sparse and are corrupted by about 10% measurement error (Larson and Peck, 1979) due to causes such as turbulant wind flow about the gage. Radar measurements are spatially continuous but are affected by various random and systematic errors that are created through the physics of the measurement process and by space-time variations in the properties of precipitation-producing mechanisms. The main sources of errors are variations in the Z-R relationship, nonuniform beam filling, anomalous propagation of radar beam, changes in the rainfall rate at the sub-cloud level detected by radar and the rainfall rate at the ground surface due to causes such as coalescence and evaporation of precipitation particles, and miscalibration of the radar electronics. For a detailed discussion of these error sources see Doviak and Zrnic (1984) and Zawadzki (1984). Synthetic radar and raingage generators implemented in this study attempt to simulate rain gage and radar-rainfall measurements so that their inherent sampling and measurement error characteristics are preserved. Thus, after merging the two data sets using the proposed estimation methodologies, comparisons with ground-truth rainfall fields can be used in order to evaluate and compare each estimation technique.

2.1 *Estimation procedures*

We begin the problem of estimating the spatial distribution of rainfall by assuming that the rainfall field $Z(u)$ constitutes a second order homogeneous and isotropic random process in 2-D space and that rainfall observations include: (1) point raingage measurements of rainfall uniformly scattered over a 200×200 km area, and (2) radar measurements of rainfall which consist of areally averaged observations on 4×4 km regular grids over the same area. Figure 1 is a schematic diagram of the problem domain showing sub-cloud precipitation echoes detected by a weather-radar electron beam and a network of sparse raingages scattered under the radar umbrella. Both measurements are accumulated over time intervals ΔT which could range from hourly to daily periods. Our goal is to further improve the areal estimates of rainfall, which are traditionally provided from raingage data alone, by including radar-rainfall measurements in the estimation, with the goal of providing more accurate areal rainfall estimates. Consequently, the problem is formulated as finding a function, $f(G(u_i),R(v_j))$, $i=1,\ldots,n_g$; $j=1,\ldots,n_r$, which is an unbiased

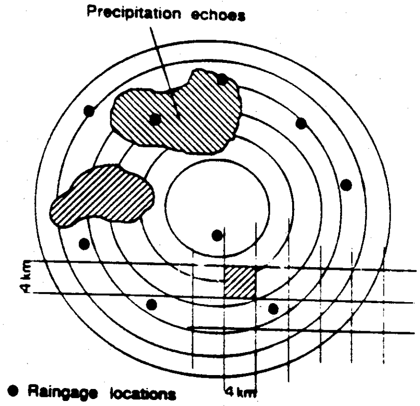

Figure 1. Schematic diagram of the problem domain consisting of precipitation echoes from radar returned signal and the sparse network of raingages

minimum variance estimator of $Z(u_0)$, where G and R are used to identify gages and radar measured fields respectively, and n_g and n_r are the number of observations used in the estimation. $Z(u_0)$ is theoretically obtained from averaging the rainfall rate $Z(u)$ over the area A (u_0 denotes the areal support over which the spatially averaged rainfall estimates are obtained) as follows:

$$Z(u_0)=\frac{1}{A}\int_A Z(u)du. \tag{1}$$

Here A is the area (4×4 km) where both rainfall estimates or radar-rainfall observations of rainfall are obtained. The best approximation of $Z(u_0)$ by the measurable function $f(G(u_i),R(v_j))$ is the conditional expectation of $Z^*(u_0)$ (Matheron 1976) given the raingage and radar-rainfall observations.

$$Z^*(u_0)=f(G(u_i),R(v_j))=E[Z(u_0)|G(u_i),R(v_j)],\ i=i,...,n_g;\ j=1,...,n_r. \tag{2}$$

Computation of the above conditional expectation requires knowledge of the joint probability density function of (n_g+n_r+1) variables $Z(u_0)$, $G(u_i)$, and $R(v_j)$, which is difficult to obtain under operational conditions. This is due to the lack of sufficient rainfall field realizations needed to construct the joint distribution of the rainfall observations and the high computational costs of obtaining such an estimate. However, if the assumption could be made that the rainfall process is multivariate Gaussian, which is seldom a good assumption, then the above conditional expectation would become a linear operator, which requires nothing more than knowledge of the covariance function which could be estimated from observations. It is only in this case that the assumption of second order homogeneity of the rainfall process leads to the full definition of the multivariate density function. Therefore, to avoid this unrealistic assumption of normality in the framework of rainfall estimation from raingage and radar-rainfall observations, the disjunctive co-kriging (DCK) estimator is proposed, which requires no prior assumptions about the distribution of rainfall fields or knowledge of the covariance function. However, for the DCK estimator, the assumption is made that the rainfall process can be obtained as a transformation of a second order homogeneous field which has a bivariate standard Gaussian distribution.

In the ordinary co-kriging of raingage and radar-rainfall data, proposed by Krajewski (1987), the estimator $Z^*(u_0)$ was obtained as a linear combination of $G(u_i)$'s and $R(u_j)$'s given by the ordinary co-kriging (OCK) estimator

$$Z^*_{OCK}(u_0)=\sum_{i=1}^{n_g}\lambda_i G(u_i)+\sum_{j=1}^{n_r}\gamma_j R(v_j) \tag{3}$$

in which the coefficients λ_i's and γ_j's are obtained as Lagrange multipliers for a constrained optimization problem. The unbiasedness condition for the above estimator ensures that $E[Z^*_{OCK}(u_0)]=E[Z(u_0)]$ and results in the following conditions on the weighting coefficients for the raingage and the radar-rainfall measurements

$$\sum_{i=1}^{n_g}\lambda_i=1,\ \text{and}\ \sum_{j=1}^{r}\gamma_j=0. \tag{4}$$

The DCK estimator is obtained by forming the estimator (Yates 1986)

$$Z^*_{DCK}(u_0)=\sum_{i=1}^{n_g}f_i[g(u_i)]+\sum_{j=1}^{n_r}h_j[r(v_j)] \tag{5}$$

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where f_i's and h_j's are a sequence of nonlinear functions, which makes DCK a nonlinear estimator and is more general than the OCK estimator. As explained above the DCK method makes use of the transformed variables which are assumed to be bivariate normally distributed. These transformations are defined as

$$G(u_i)=\Phi_g[g(u_i)]=\sum_{k=0}^{K_g} C_k H_k[g(u_i)] \tag{6}$$

$$R(v_j)=\Phi_r[r(v_j)]=\sum_{k=0}^{K_r} D_k H_k[r(v_j)] \tag{7}$$

where H_k is a Hermite polynomial of order k, and $g(u_i)$ and $r(v_j)$ are the standard normal random variables which are obtained from the transforms

$$g(u_i)=\Phi_g^{-1}[G(u_i)] \tag{8}$$

$$r(v_j)=\Phi_r^{-1}[R(v_j)]. \tag{9}$$

A Hermite polynomial of order k may be evaluated by the recursive relationship

$$H_{k+1}(y)=yH_k(y)-kH_{k-1}(y) \tag{10}$$

where $H_0=1$ and $H_1=y$. Making use of the orthogonality of Hermite polynomials with Gaussian density the coefficients C_k can be determined by

$$C_k=(k!)^{-1}(2\pi)^{-1/2}\int_{-\infty}^{\infty}\Phi(y)H_k(y)e^{-1/2y^2}dy. \tag{11}$$

The coefficients C_k can be obtained in different ways. The most widely used method in geostatistics is by numerical integration by the Gauss-Hermite quadrature (Abramowitz and Stegum 1970)

$$C_k=(k!)^{-1}(2\pi)^{-1/2}\sum_{j=1}^{J}\Phi(Y_j)W_jH_k(Y_j)e^{Y_j^2/2} \tag{12}$$

where J is the total number of terms used for abscissas Y_j and weight factors W_j. The above numerical integration is employed if Φ_g and Φ_r in Eqs. (6) and (7) are nonlinearly approximated. However, for piecewise linear interpolation, an analytical solution for Hermite integration is possible as shown by Puente and Bras (1982) and Krajewski and Azimi-Zonooz (1987). For piecewise linear interpolation, the following approximation of the anamorphosis function was proposed

$$\Phi(y)=\begin{cases} z_1 & y\le y_1 \\ a_k y+b_k & y_k\le y\le y_{k+1} \quad k=1,\ldots,n-1 \\ z_n & y\ge y_n \end{cases} \tag{13}$$

where $z_1,z_2,\ldots,z_n$ are ordered data observations and $y_1,y_2,\ldots,y_n$ are corresponding values of standard Gaussian distribution function. From Eq. (13) it follows that,

$$a_k=\frac{z_{k+1}-z_k}{y_{k+1}-y_k} \tag{14}$$

$$b_k=\frac{z_k y_{k+1}-z_{k+1}y_k}{y_{k+1}-y_k} \tag{15}$$

Incorporating Eqs. (13), (14), and (15) into Eq. (11) and integrating results in an analytic solution. The final expressions are

$$C_0=z_1G(y_1)+\sum_{i=1}^{n-1}\{b_iG(y)|_{y_i}^{y_{i+1}}-a_ig(y)|_{y_i}^{y_{i+1}}\}+z_n[1-G(y_n)] \tag{16}$$

$$C_1=-z_1g(y_1)+\sum_{i=1}^{n-1}\{-b_ig(y)|_{y_i}^{y_{i+1}}-a_i[G(y)-yg(y)]|_{y_i}^{y_{i+1}}\}+z_n[g(y_n)] \tag{17}$$

and for $k \geq 2$

$$C_k=[-z_1H_{k-1}(y_1)g(y_1)+z_nH_{k-1}(y_n)g(y_n) + \sum_{i=1}^{n-1}\{g(y)H_{k-1}(y)b_i|_{y_i}^{y_{i+1}}+a_i[yg(y)H_{k-1}(y)+g(y)H_{k-2}(y)|_{y_i}^{y_{i+1}}]\}](k!)^{-1} \tag{18}$$

where $g(y)$ is standard normal density

$$g(y)=(2\pi)^{-1/2}e^{-1/2y^2} \tag{19}$$

and

$$G(y)=(2\pi)^{-1/2}\int_{-\infty}^{\infty}g(y)dy. \tag{20}$$

Expanding the DCK estimator, Eq. (5), in a series of Hermite polynomials gives

$$Z^*_{DCK}(u_0)=\sum_{k=0}^{K_g}\sum_{i=1}^{n_g}f_{ik}H_k[g(u_i)]+\sum_{k=0}^{K_r}\sum_{j=1}^{n_r}h_{jk}H_k[r(v_j)] \tag{21}$$

where f_{ik} and h_{jk} are coefficients of Hermite expansion. The problem is to determine the weights f_{ik} and h_{jk} such that the following holds:

(1) The estimator $Z^*(u_0)$ is a minimum variance estimator, i.e.

$$\mathrm{Var}[Z^*(u_0)-Z(u_0)]=E[\{Z^*(u_0)-Z(u_0)\}^2]\rightarrow \min \tag{22}$$

(2) The estimator $Z^*(u_0)$ is an unbiased estimator of $Z(u_0)$, i.e.

$$E[Z^*(u_0)-Z(u_0)]=0 \tag{23}$$

The uniqueness of above unbiasedness condition is that the summation of f_{ik}'s for k=0 in Eq. (21) preserves the mean of the raingage observations given by C_0 in Eq. (6) through the following equation:

$$\sum_{k=0}^{K_g}f_{ik}=C_0. \tag{24}$$

Following the approaches given by Journel and Huijbregts (1978) and Yates (1986) we obtain the following system of equations

$$E[\Phi_g[g(u_0)]|g(u_\alpha)]=\sum_{i=1}^{n_g}E[f_i[g(u_i)]|g(u_\alpha)]+\sum_{j=1}^{n_r}E[h_j[r(v_j)]|g(u_\alpha)] \tag{25}$$

$$E[\Phi_g[g(u_0)]|r(v_\beta)]=\sum_{i=1}^{n_g}E[f_i[g(u_i)]|r(v_\beta)]+\sum_{j=1}^{n_r}E[h_j[r(v_j)]|r(v_\beta)] \tag{26}$$

where $\alpha=1,\cdots,n_g$ and $\beta=1,\cdots,n_r$. Expanding the bivariate standard Gaussian densities in terms of Hermite polynomials gives (Matheron 1976)

$$f(x,y)=(k!)^{-1}\sum_{k=0}^{\infty}(\rho_{xy})^k H_k(x)H_k(y)g(x)g(y). \tag{27}$$

Then the conditional expectation of function $\Phi(x)$ in Eqs. (25) and (26) can be written as

$$E[\Phi(X)|Y]=\sum_{k=0}^{\infty}(\rho_{xy})^k C_k H_k(Y). \tag{28}$$

Incorporating Eqs. (28), (25) and (26) into Eq. (21) leads to the DCK estimate

$$Z^*_{DCK}(u_0)=\sum_{k=0}^{K_g}C_k H^*_k[g(u_0)]. \tag{29}$$

$H^*_k[g(u_0)]$ is given by

$$H^*_k[g(u_0)]=\sum_{i=1}^{n_g}a_{ik}H_k[g(u_i)]+\sum_{j=1}^{n_r}b_{ik}H_k[r(v_j)] \tag{30}$$

where f_{ik} and h_{jk} in Eq. (21) are written as $C_k a_{ik}$ and $C_k b_{jk}$. The a_{ik}'s and b_{jk}'s are determined from

$$\sum_{i=1}^{n_g}a_{ik}\rho^k_{g\alpha i}+\sum_{j=1}^{n_r}b_{jk}\rho^k_{gr\alpha j}=\rho^k_{go\alpha} \qquad i;,\ \alpha=1,\ldots,n_g \tag{31}$$

$$\sum_{i=1}^{n_g}b_{ik}\rho^k_{rg\beta i}+\sum_{j=1}^{n_r}a_{jk}\rho^k_{r\beta j}=\rho^k_{rgo\beta} \qquad j;,\ \beta=1,\ldots,n_r \tag{32}$$

where $\rho_{gr\alpha i}$ and $\rho_{rg\beta j}$ are cross correlation functions between radar-rainfall and gage fields, $\rho_{go\alpha}$ is the correlation function of point to be estimated and the gage field, and $\rho_{rgo\beta}$ is the cross correlation function between point to be estimated on the gage field and the radar-rainfall field.

2.2 *Approximation of the anamorphosis function*

The computations for obtaining a disjunctive co-kriging estimate begin by obtaining the cumulative distribution of each data point in the radar-rainfall and raingage observations. The estimates of cumulative probability levels are then used to obtain the corresponding standard Gaussian random variable. The next step is to approximate the function given by Eqs. (6) and (7) by an expansion of Hermite polynomials referred to as the anamorphosis function approximation in this study. A fully controlled numerical experiment was designed and carried out to compare the two approaches for approximation of the anamorphosis function. The approaches are the numerical solution for computation of Hermite coefficients, given by Eq. (12), and the analytic solution given by Eqs. (16), (17), and (18). To test and compare these two approaches, samples of various sizes (50, 100, 200, 400, 800) were drawn from two-dimensional random, homogeneous, and isotropic fields with exponential covariance function. The correlation length of the fields varied from 5 to 40 units and the sampling domain was 100 by 100 units. The data were point observations at randomly generated locations. The locations were kept constant for all realizations of the generated fields. The smaller samples were always included in the larger samples, so that we could simulate the expansion of our observational network.

The fields were generated using the Turning Band Method (TBM), described by Mantoglou and Wilson (1982). Ten realizations of the fields were used for each set of parameters. All the fields were $N(0,1)$ (normally distributed with zero mean and unit variance). The observations were generated for various levels of measurement error: 0, 10, and 50 percent. The measurement error is expressed as having a standard deviation equal to the specified percentage of the field value at a given point. In this way the higher the absolute field values, the higher are the observational errors. However, since the mean of the error term is always zero, the generated noise does not introduce a bias into the samples.

Normally distributed samples were generated and transformed to obtain lognormally distributed data and the performance of the two approaches was compared. Results show that for large samples the fit of the Hermite approximation is poor in the upper tail of the distribution for the numerical solution method (see Fig. 2a, for example). This behavior of the numerical solution approach is attributed to the initial step in the algorithm, where fitting a polynomial regression to the sample to obtain points necessary for the numerical integration scheme is made. The fit is dominated by a large number of points in the middle region of the distribution. The problem could be alleviated by careful selection of the polynomial degree or a weighted least squares fit. Such a procedure, however, would increase computational costs, and make the algorithm even more complicated. Another problem, common to both algorithms is that under certain sampling conditions, there is no unique solution to inverse problems, Eqs. (8) or (9). An example of such a situation is shown in Fig. 2b. The approach taken was to search for the roots of Eqs. (8) or (9) in the interval

$$y_i-2u\le y_i\le y_i+2u \tag{33}$$

where y_i corresponds to z_i for which the solution is needed and u is the $\max\{y_i-y_{i-1};\ i=1,\dots,n\}$. If two or more solutions were present in the specified range then the minimum solution was selected in the upper tail and maximum in the lower tail. The investigated sampling conditions, such as correlation distance and measurement error, did not appear to affect the relative performance of the two algorithms. The overall

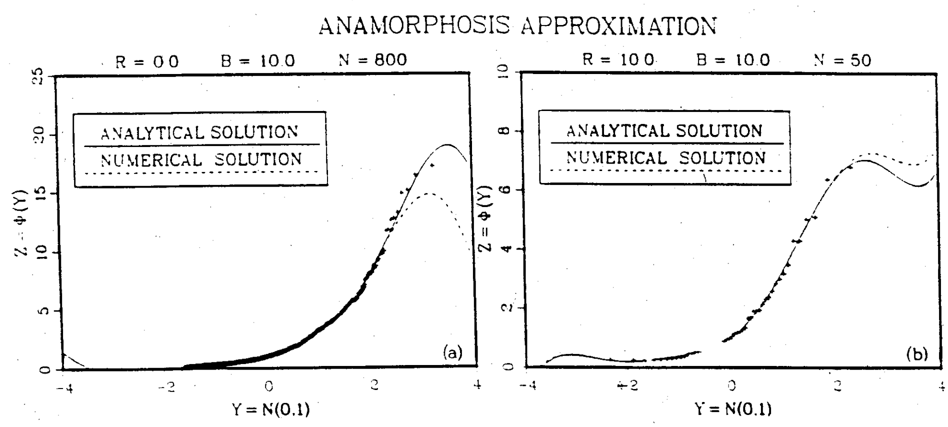

Figure 2. Examples of anamorphosis function approximation (R is measurement error, B is correlation length of simulated random field, and N is the sample density)

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3 Experimental design

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where

$f_s^{(1)}(\tau)=\beta e^{-\beta(\tau-s)}e^{-\beta T}$

$f^{(2)}(\mathbf{X})=\frac{1}{2\pi\sigma_1\sigma_2}\exp\{-\frac{x}{2c}$

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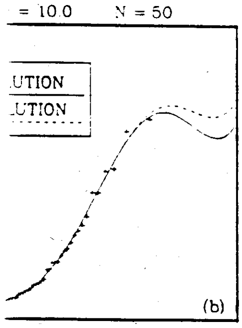

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results of the experiment for transformation of non-Gaussian data, to solve the problem $y=\Phi^{-1}(z)$, in general showed a very similar performance for the two investigated procedures. However, since the fit of the piecewise linear approximation is much better in the upper tail of the distribution for large samples, this procedure was adopted in the solution of disjunctive co-kriging equations. This feature of piecewise linear approximation is mainly attributed to the first step in the algorithm that effectively forces the fit to the data to more or less pass through all observed points.

3 Experimental design

The evaluation of ordinary and disjunctive co-kriging algorithms is performed in a framework similar to that used in the numerical experiments by Krajewski (1987) and Seo et al. (1989a, and b). The following describes the steps involved in the generation of synthetic ground-truth rainfall fields, and radar-rainfall and raingage observation fields.

3.1 Multidimensional model of ground-truth rainfall field

For generation of ground-truth rainfall fields, the space-time rainfall model of Waymire et al. (1984) was utilized in this study to obtain simulated rainfall fields with various climatic characteristics. The model generates rainfall fields using a spatial stochastic process which has qualitative features similar to rainfall events produced by extratropical cyclonic storms. To simulate the rainfall patterns observed in this storm type, the rainfall fields are assumed to consist of a system of cellular patterns contained within regions of cluster potential centers which themselves are located within larger scale rainbands (Waymire et al. 1984). The rainfall process is generated from a fixed time and space origin with prespecified input parameters. The cluster potential regions are first generated as a spatial Poisson process, and the expected number of rain cells with a Poisson distribution are fixed within each cluster potential region. The formation and dissipation of rainfall events are then simulated according to the space-time mathematical structure of the model. The rain cells within a rainband are generated according to their underlying space-time probability density function given by (Waymire et al. 1984)

$$f_{s,\mathbf{X}}(\tau,\mathbf{y})=0 \qquad \tau\leq s-T \tag{34}$$

$$f_{s,\mathbf{X}}(\tau,\mathbf{y})=f_s^{(1)}(\tau)f^{(2)}(\mathbf{y}-(\mathbf{X}+\mathbf{U}_b(\tau-s))) \qquad \tau>s-T \tag{35}$$

where

$$f_s^{(1)}(\tau)=\beta e^{-\beta(\tau-s)}e^{-\beta T} \qquad \tau>s-T \tag{36}$$

$$f^{(2)}(\mathbf{X})=\frac{1}{2\pi\sigma_1\sigma_2}\exp\{-\frac{x_1^2}{2\sigma_1^2}-\frac{x_2^2}{2\sigma_2^2}\}, \qquad \mathbf{X}=(x_1,x_2)\in\mathbf{R}^2 \tag{37}$$

In Eqs. (36) and (37), $f_s^{(1)}(\tau)$ and $f^{(2)}(\mathbf{X})$ are the corresponding density functions (assumed to be independent of each other), for the occurrence of cell centers in time and space due to a cluster potential at support $\mathbf{X}$. s is the time of arrival of the rainbands. T is the mean cell duration. $\mathbf{U}_b$ is the rainband velocity relative to a fixed coordinate system on the ground. β is the cellular birth rate, and σ_1 and σ_2 are the standard deviations describing the degree of dispersion in occurrence of rain cells in space on support $\mathbf{X}$. The spatial and temporal dissipation of cellular rainfall intensity are assumed to be given by the following geometrical forms

$$g(a,r)=i_0 g_1(a)g_2(r) \tag{38}$$

where

$$g_1(a)=e^{-\alpha a} \qquad a\geq 0$$

$$g_1(a)=0 \qquad a<0 \qquad (39\text{-}40)$$

$$g_2(r)=\exp\{-\frac{r^2}{2D^2}\} \qquad r\geq 0 \qquad (41)$$

and i_0 is the rainfall intensity at the cell center at t=0, a is the mean cell age, r is the radius from the cell center, and D and α are the parameters that determine the spatial and temporal extent of the cell geometry respectively. The mathematical model of space-time rainfall introduced by Waymire et al. (1984) was used to generate ground-truth rainfall fields with hourly integration times over a 200×200 km area with raingages located in 4×4 km bins. Rainfall fields were generated using a two dimensional stochastic point process at gage locations. A sampling time interval of 0.1 hour was used over the hourly simulation periods. Closed form expressions for the covariance function of the space-time rainfall model are presented by Waymire et al. (1984). The theoretical derivations of covariance function conform with the empirical descriptions of extratropical cyclonic storms by Zawadzki (1973).
The three simulated climates (examples of which are shown in Fig. 3) range from frequent storms, with high intensities and large number of cells (climate 1) to climates with less frequent storms and lower intensities (climate 2). Details of the characteristics of each climate are given by Valdes et al. (1985).

3.2 Radar-rainfall field generator

The statistical error structure of radar-rainfall data is not fully understood, therefore a conceptual statistical method of Corrupting the ground-truth rainfall field was used. The radar-rainfall field generation model of Krajewski and Georgakakos (1985) was utilized to superimpose measurement noise with known second order statistics at each point of the hypothetical ground-truth rainfall field. The proposed methodology derives the statistical parameters of the noise field given some prespecified conditions. First of all, it is assumed that the error field $e(\mathbf{u})$ can be expressed as the logarithmic ratio of radar-rainfall field to the ground-truth rainfall field (Hudlow et al. 1979)

$$e(\mathbf{u})=\log_{10}[\frac{R(\mathbf{u})}{O(\mathbf{u})}] \qquad (42)$$

where $\mathbf{u}$ is the vector of coordinates in 2-D space, $R(\mathbf{u})$ is the generated radar data, and $O(\mathbf{u})$ is the ground-truth rainfall data from the multidimensional rainfall model's hourly simulations. The error field $e(\mathbf{u})$ is assumed to be the product of a random component and a deterministic component given by

$$e(\mathbf{u})=\varepsilon(\mathbf{u})S(\mathbf{u}) \qquad (43)$$

where $\varepsilon(\mathbf{u})$ is a homogeneous Gaussian random field and $S(\mathbf{u})$ is the deterministic component which is a function of the local ground-truth rainfall field magnitude and gradient (Greene et al. 1980)

$$S(\mathbf{u})=\frac{<|\nabla O(\mathbf{u})|>O_{max}(\mathbf{u})+O(\mathbf{u})<|\nabla O(\mathbf{u})|_{max}>}{2<|\nabla O(\mathbf{u})|_{max}>O_{max}(\mathbf{u})} \qquad (44)$$

where $<|\nabla O(\mathbf{u})|>$ is the average absolute value of ground-truth rainfall field gradient in four directions around point $\mathbf{u}$, $<|\nabla O(\mathbf{u})|_{max}>$ is the maximum absolute gradient, and $O_{max}(\mathbf{u})$ is the maximum value in the ground-truth rainfall field. The deterministic com-

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3.3 Point rainfall data ge.

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3.4 Numerical experimen

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ponent $S(\mathbf{u})$ influences the generated radar field such that the errors are higher where magnitude and gradient of the original field are higher. Note that the radar generator attempts to simulate the random and systematic errors that occur in the radar-reflectivity rainfall rate conversion process as described by Zawadzki (1984). Incorporation of Eq. (42) in Eq. (43) leads to radar field magnitude as a function of ground-truth rainfall field expressed as

$$R(\mathbf{u})=O(\mathbf{u})10^{\varepsilon(\mathbf{u})S(\mathbf{u})} \tag{45}$$

The next step in the radar-rainfall field generation is to specify the first and second order moments of the noise component $\varepsilon(\mathbf{u})$ and generate it as a Gaussian random field using the turning bands method (TBM) (Mantoglou and Wilson 1982). The parameters for generation of the radar-rainfall field are derived by specifying (1) the bias of radar field in the mean expressed as $E(R)/E(O)$, allowing reproduction of the mean of $\varepsilon(\mathbf{u})$, (2) the variance of logarithmic ratio (R/O) given by Eq. (42), from which the variance of random component $\varepsilon(\mathbf{u})$ is derived, and (3) the correlation function of $\varepsilon(\mathbf{u})$, assumed to be isotropic and exponential.

3.3 Point rainfall data generator

Raingage measurements are generated by first selecting the locations of the point raingage data using random sampling from a uniform distribution over a 200×200 km area. Then the hypothetical point values of rainfall are assigned to those locations. The raingage values are generated from the ground-truth rainfall field so that the sampling and measurement error characteristics of the point measurements are preserved. Point precipitation sampling errors are assumed to be lognormally distributed variables with the mean equal to the value of ground-truth field at the grid containing the gage location, and the standard deviation proportional to the mean (Krajewski 1987)

$$z_g(u_i)\propto LN\{z_p(u_i),\sigma_p^2\}+LN\{0,a^2 z_p^2(u_i)\} \tag{46}$$

where $LN\{\cdot\}$ denotes lognormal distribution with mean $z_p(u_i)$ and variance σ_p^2. The mean, $z_p(u_i)$ is assumed to be the ground-truth rainfall value at grid point u_i, and a is the measurement error expressed as a percentage of the mean.

3.4 Numerical experiments

A controlled numerical experiment was carried out to analyze the accuracy of ordinary block kriging, ordinary co-kriging, and disjunctive co-kriging for rainfall estimation. Rainfall fields having three different climatic characteristics were generated as simulated original fields by using the space-time rainfall model developed by Waymire et al. (1984). The experiment was carried out by repeating the analysis for 10 realizations from each climate type, while keeping the noise parameters and raingage configuration unchanged. The results were then averaged across the ensemble of realizations for examining the effects of noise parameters on the estimated fields. The radar-rainfall and raingage generators developed and described by Krajewski (1987) were used to simulate radar-rainfall and raingage fields with their prespecified sampling characteristics. The radar-rainfall noise parameters chosen were bias, $(E[R]/E[O])$=1 and 2, noise variance, $\mathrm{var}\{\log_{10}(R/O)\}$=.005 and .02, and the noise (parameter ε in Eq. (45)) correlation distance=8.0 and 16.0 km. Raingage densities were set to 32, 160, and 286 gages over the 200×200 km area. The first gage density represents approximately the raingage density over the continental USA (one gage/1000_2000 km^2, Wilson and Brandes, 1979). The networks of 160 and 286 gages were also selected in order to examine the effect of higher raingage densities on spatial estimates of rainfall. Climate 1, with the highest number of

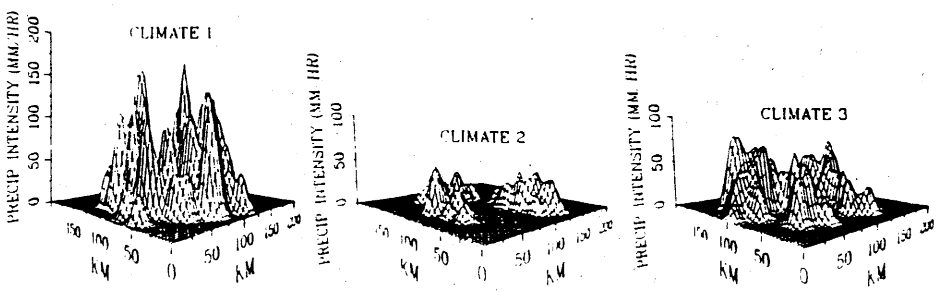

Figure 3. Simulated hourly rainfall fields for various climates

cluster potential centers and the highest cell intensity (Fig. 3a) provides a very rigorous evaluation of estimation techniques for assessing the noise parameters which have the highest influence on the outcome of the estimated rainfall fields.

4 Results of numerical experiments

Comparison among various estimators were made in terms of their effectiveness to filter out measurement noise and bias from the radar-rainfall field, and their ability to estimate spatial variability in the ground-truth rainfall fields for various climates. Since we had known true rainfall fields and sampled raingage and radar-rainfall fields with known measurement error parameters, measures such as how well the merged field describes the spatial variability in the true rainfall field after removing noise and bias introduced by measurement error where obtainable.

The first check on the performance of the estimators was made to verify whether or not the methods effectively remove bias from the radar field. Figs. 4 and 5 show the average power spectrum for residual of estimated rainfall fields across all realizations for bias (E [R]/ E [O])=1, and bias=2 cases for Climate 1. Residuals of the estimated rainfall fields are obtained by subtracting the estimated rainfall fields from the ground-truth rainfall field at each 4×4 km grid. The estimation approach referred to as Co-kriging I was per-

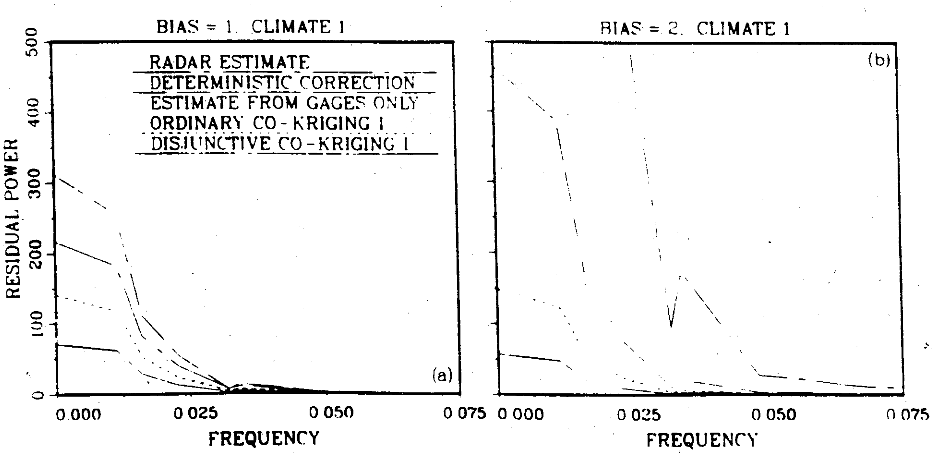

Figure 4. Comparison of Co-kriging techniques - Average spectral density of residual fields for semivariograms estimated from sampled gage field with (a) *bias*=1 and (b) *bias*=2

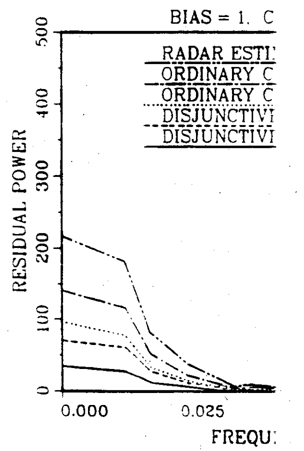

Figure 5. Comparison of C
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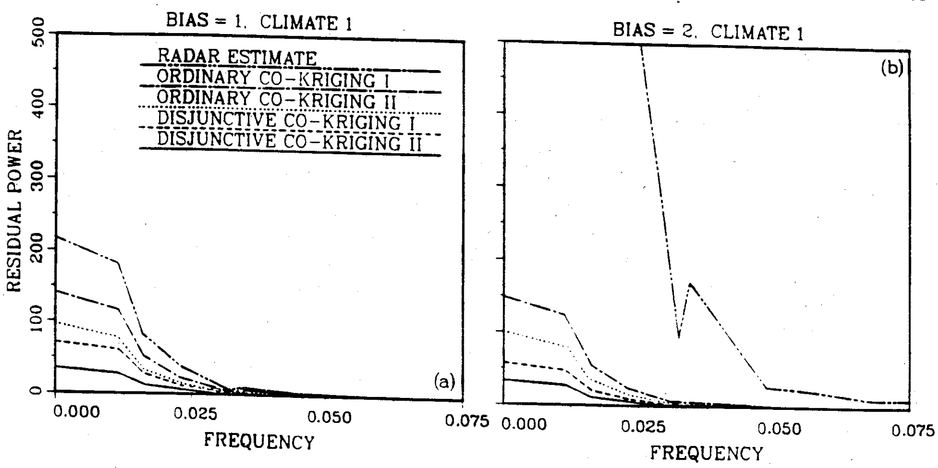

Figure 5. Comparison of Co-kriging techniques - Average spectral density of residual fields for semivariograms estimated from ground-truth rainfall field with (a) *bias*=1 and (b) *bias*=2

formed using variograms estimated from sampled gage fields and compared with estimations performed with full knowledge of second order statistics (variograms estimated from the ground-truth rainfall field), referred to as Co-kriging II. In the framework of static estimation, Co-kriging II provides an upper limit on how well the linear and nonlinear Co-kriging techniques could estimate the spatial variability in the ground-truth rainfall field. As is evident from the results in Figs. 4a and 4b, low and high frequency components of the radar-rainfall-residual field are effectively attenuated by Co-kriging techniques. In Fig. 4b for the bias=2 case, Disjunctive Co-kriging I provides the best performance for filtering out the superimposed bias, where the residual spectrum is very close to an uncorrelated random process over low and high frequency intervals.

Spatial rainfall estimates from raingages only using Ordinary Block-kriging and Ordinary Co-kriging I (linear Co-kriging of radar-rainfall and raingage data) provide almost the same performance for estimating spatial variability in the ground-truth rainfall field. This demonstrates that the inclusion of radar-rainfall data in the Ordinary Co-kriging I case does not lead to significant improvements over estimation from raingage observations alone using ordinary block kriging. The deterministic correction of radar-rainfall data using raingages (Brandes, 1975) for the bias=1 increases the noise correlation distance in the residual of estimated rainfall field, and shows the highest residual power in low frequency range for bias=2 case. This feature of deterministic correction is attributed to the fact that sampling characteristics of rainfall measurements do not enter the estimation procedure. In contrast, since Co-kriging techniques are performed in the framework of minimum variance unbiased estimation, they posses a better bias removal capability for the radar-rainfall field.

In Figs. 5a and 5b Disjunctive Co-kriging II shows the least amount of correlated noise in the low and high frequency ranges, where the residual power spectrum is effectively flat and is not significantly different from a white noise random process. These plots also demonstrate that nonlinear Co-kriging of raingage and radar-rainfall data with variograms estimated from sampled raingage data, using Disjunctive Co-kriging I, provides better bias removal over linear co-kriging with full knowledge of second order statistics (Ordinary Co-kriging II).

The radar bias and noise removal capability of estimators are also examined in terms of root mean square error (RMSE) between the estimated and ground-truth rainfall fields.

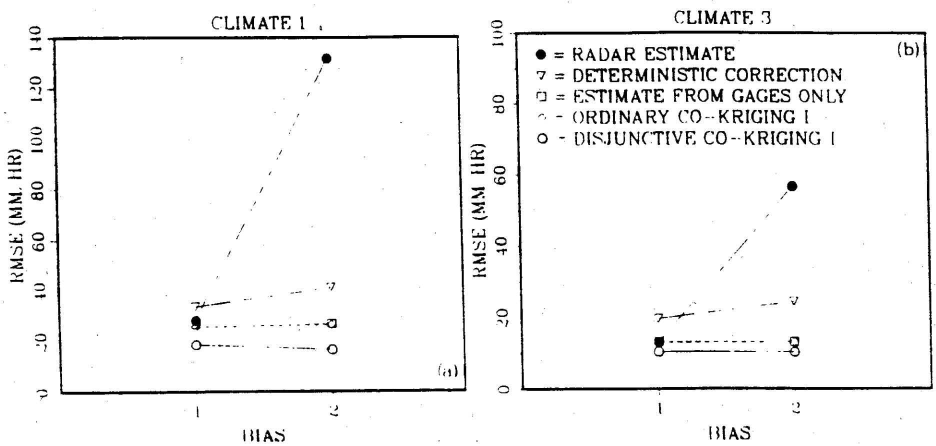

Figure 6. Effects of radar-rainfall bias on the RMSE of estimated fields

These results are plotted and shown in Fig. 6 for Climates 1 and 3. Disjunctive co-kriging I again shows the lowest RMSE in the estimated rainfall fields and other estimators show the same type of performance as observed before. Figure 7 displays the influence of two levels of random noise components in the radar-rainfall field, var$\{\log_{10}(R/O)\}$=.005 and .02, on the residual power of estimated fields. The radar-rainfall residual field for higher noise level shows slightly higher power in the low frequency range. The estimated fields obtained using co-kriging techniques are relatively insensitive to the variations of radar-rainfall field random noise parameters in the investigated range.

Figure 8 summarizes the effect of raingage network density on the estimated rainfall fields. Higher raingage network densities lead to improved areal rainfall estimates for Climate 1. However, increasing the gage density for Climate 3 does not lead to noticeable improvements in the spatial estimates of rainfall. Estimated rainfall fields for the two noise correlation distances were similar.

Table 1 is a summary of ME (mean error) and RMSE statistics for various spatial rainfall estimators that were examined in this study. These results were obtained by averaging each error statistic across entire estimated rainfall fields. As was evident from previ-

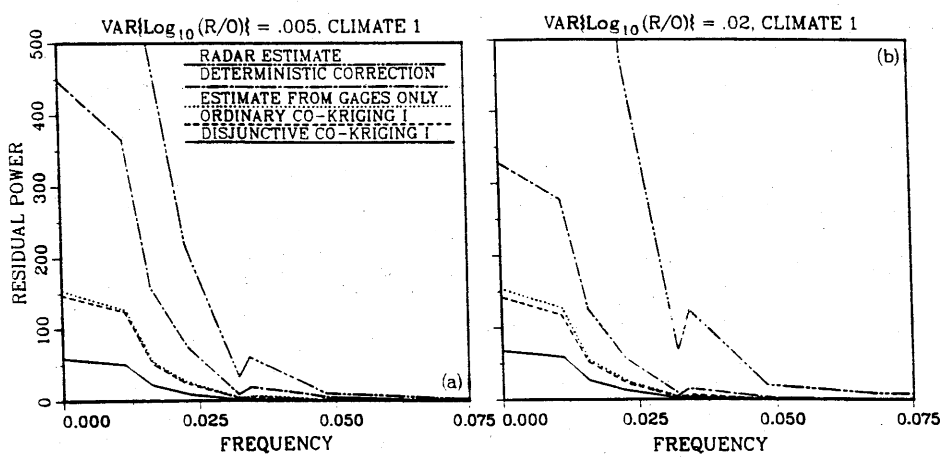

Figure 7. Average spectral density of estimated rainfall residual field for two noise variance levels

Table 1. Comparison of ME

Climate	Error Type (mm/hr)
1	ME
	RMSE
2	ME
	RMSE
3	ME
	RMSE

Ord. Blk.: Ordinary Block Kr
Ord. Co-I: Ordinary Co-krigi
Ord. Co-II: Ordinary Co-krig

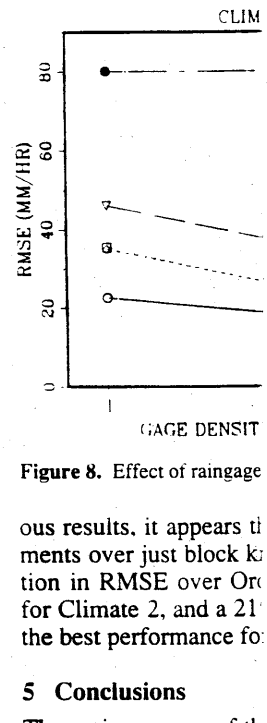

Figure 8. Effect of raingage

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5 Conclusions

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Table 1. Comparison of ME and RMSE statistics for various climates and estimators

Climate	Error Type (mm/hr)	Radar Field	Brandes Method	Ord. Blk.	Ord. Co-I	Ord. Co-II	Dis. Co-I	Dis. Co-II
1	ME	19.58	3.75	1.86	1.91	1.60	0.91	0.33
	RMSE	79.91	37.36	26.74	25.94	20.69	17.42	12.45
2	ME	4.22	1.91	1.76	1.56	0.97	0.83	0.54
	RMSE	17.79	8.86	8.68	8.25	6.43	4.92	3.49
3	ME	8.36	2.68	0.79	1.13	0.34	1.33	0.00
	RMSE	34.86	22.04	13.19	13.09	9.81	10.31	5.70

Ord. Blk.: Ordinary Block Kriging
Ord. Co-I: Ordinary Co-kriging I
Ord. Co-II: Ordinary Co-kriging II
Dis. Co-I: Disjunctive Co-kriging I
Dis. Co-II: Disjunctive Co-kriging II

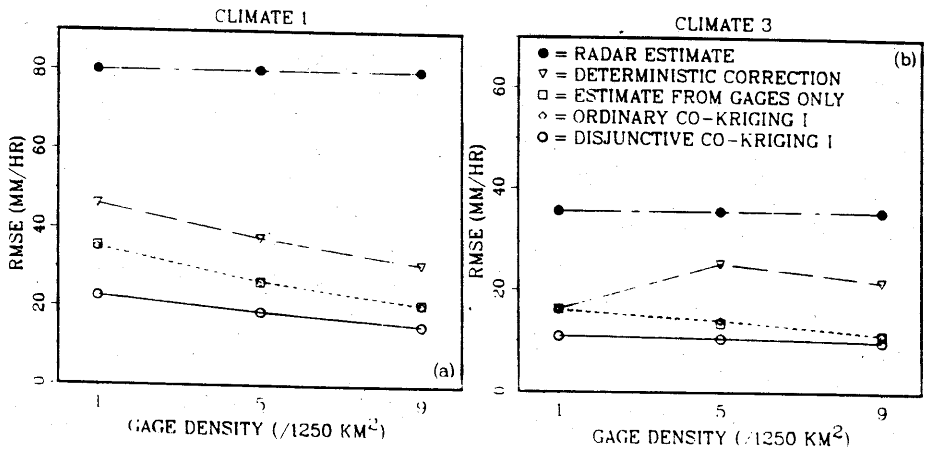

Figure 8. Effect of raingage network density on the RMSE of estimated rainfall fields

ous results, it appears that Ordinary Co-kriging I does not offer any significant improvements over just block kriging the gage field. Disjunctive Co-kriging I gives a 33% reduction in RMSE over Ordinary Co-kriging I for Climate 1, approximately 40% reduction for Climate 2, and a 21% reduction for Climate 3. Disjunctive Co-kriging II again shows the best performance for spatial rainfall estimation.

5 Conclusions

The main purpose of the research reported in this paper was to compare the performance of various (co)-kriging methods for spatial rainfall estimation under a radar umbrella. The results of numerical experiments indicate that spatial rainfall estimation by merging radar-rainfall and raingage data is more accurately performed using the Disjunctive Co-kriging techique than by using alternative linear Co-kriging procedures, the Brandes method or either raingage or radar-raifall data separately. One important conclusion that can be drawn from this study is that rainfall fields are generally described by nonlinear functions that cannot be adequately estimated using linear estimators. The theoretical advantage of disjunctive co-kriging, as presented in this paper, is, based on the ability, to estimate bivariate distributions of rainfall fields so that the conditional expectation of two observations at a time can be computed, using these distributions. The disjunctive co-kriging procedure specifically aims at deriving the probability distribution of rainfall fields, which in turn, preserves the spatial properties of rainfall fields more accurately. The main conclusions of the study are summarized as follows:

1) The disjunctive co-kriging estimator gives more accurate mean areal precipitation estimates over ordinary co-kriging, as is evident from root mean square error and mean error statistics obtained by averaging over entire estimated rainfall fields for each climate type. Disjunctive and ordinary co-kriging estimators provide a substantial increase in accuracy over the Brandes method. This supports the earlier findings of Austin (1987).

2) The power spectra of the estimated rainfall field residuals for all kriging estimators indicates that the best performance for the most uncorrelated noise over the low and high frequency intervals is provided by Disjunctive Co-kriging II, followed by Disjunctive Co-kriging I, and Ordinary Co-kriging II. Ordinary Co-kriging I, and Ordinary Block Kriging showed similar low-accuracy performance. The Brandes method was found to be poor for removal of noise and bias from the radar-rainfall field data.

3) Inclusion of radar-rainfall data in the rainfall field estimation by Disjunctive Co-kriging provides improvemed accuracy over only Block-kriging, but does not lead to significant improvements in the case of Ordinary Co-kriging.

4) Disjunctive co-kriging generally provides better estimates of spatial variability in the ground-truth rainfall field since it effectively removes significant correlation over low and high frequency intervals from the estimated rainfall fields.

5) The effect of correlation distance of the radar error field was not found to have a significant effect on rainfall estimation.

6) Evaluation of spatial rainfall estimation events under three different climatic conditions indicated that the relative performance of spatial rainfall estimators also depend on intensity levels and the spatial extent of precipitation clusters.

7) For approximation of the anamorphosis function, the analytical method is much more efficient in terms of CPU time and avoids certain instabilities of the numerical integration approach.

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References

Abramowitz, M.; Stegun, A. 1970: Handbook of mathematical functions, Dover Publications, New York, N.Y.

Ahnert, P.R.; Krajewski, W.F.; Johnson, E.R. 1986: Kalman filter estimation of radar-rainfall field bias. Preprints of the 23rd conference of radar meteorology and the conference on cloud physics, AMS, Boston, Mass., JP33-JP37

Austin, P.M. 1987: Relation between measured radar reflectivity and surface rainfall, Monthly weather review, 115, 1053-1070

Battan, L.J. 1973: Radar observation of the atmosphere, Univ. Chicago Press, Chicago

Brandes, E.A. 1975: Optimizing rainfall estimates with the aid of radar, J. Appl. Meteorol. 14, 1339-1345

Byers, H.R. 1948: The use of radar in determining the amount of rain falling over a small area. Trans. American Geoph. Union, 29, 187-196

Crawford, K.C. 1979: Considerations for the design of a hydrologic data network using multivariate sensors, Water Resour. Res. 15(6), 1752-1762

Creutin, J.D.; Delrieu G.; Lebe
cal approach, J. Atm. anc

Doviak, R.J.; Zrnic, D.S. 1984:

Eddy, A. 1979: Objective anal

Greene, D.R.; Flanders A.F. 1
hydrometeorology, AMS

Greene, D.R.; Hudlow, M.D.;
ing radar and raingage c
Mass., 470-479

Hiatt, W.E. 1956: Radar in t
U.G.G.F., 286-290

Hudlow, M.D.; Arkell, R.; P
parison of the gate C-bar
D.C.

Journel, A.G.; Huijbregts Ch. J

Kessler, E.; Wilk, K.E. 1968:
Report No. 5, Secretariat

Krajewski, W.F. 1987: Co-krig

Krajewski, W.F.; Azimi-Zonc
Note No. 10, Hydrologic

Krajewski, W.F.; Georgakako
764-768

Krajewski, W.F.; Hudlow, M.
areal precipitation from
of natural hazards, Sacra

Larson, L.W.; Peck, E.L 197
Resour. Res. 10(4), 857-

Mantoglou, A.; Wilson, J.L.
generation by a spectral

Matheron, G. 1976: A simple
NATO A.S.I. "Geostat 7

Puente, C.E.; Bras, R.L. 1982
Parsons Lab., Technical

Seo, D.J.; Krajewski, W.F.; I
and radar using co-krigir

Seo, D.J.; Krajewski, W.F.;
data from raingages and

Valdes, J.B.; Rodriguez-Iturb
mensional model. Water

Waymire, E.; Gupta, V.K.; Rc
scale, Water Resour. Re

Wilson, J.W. 1970: Integratic
Meteo. 9, 489-497

Wilson, J.W.; Brandes, E.A.
60(9), 1048-1958

Yates, S.R. 1986: Disjunctive

Zawadzki, I. 1973: Statistical

Zawadzki, I. 1984: Factors a
Conference on radar me

Accepted January 27, 1989

Creutin, J.D.; Delrieu G.; Lebel T. 1988: Rain measurement by raingage-radar combination: A geostatistical approach, J. Atm. and Oceanic Tech. 5(1), 102-115

Doviak, R.J.; Zrnic, D.S. 1984: Doppler radar and weather observations. Academic Press, Orlando, FL.

Eddy, A. 1979: Objective analysis of convective scale rainfall using gages and radar, J. Hydrol. 44, 125-134

Greene, D.R.; Flanders A.F. 1976: Radar hydrology - the state of the art. Preprints of the conference on hydrometeorology, AMS, Boston, Mass., 66-71

Greene, D.R.; Hudlow, M.D.; Johnson, E.R. 1980: A test of some objective analysis procedures for merging radar and raingage data. Preprints of the 19th conference on radar meteorology. AMS, Boston, Mass., 470-479

Hiatt, W.E. 1956: Radar in flood hydrology. Publication of 42nd International hydrologic Association, U.G.G.F., 286-290

Hudlow, M.D.; Arkell, R.; Patterson, V.; Pytlowany, P.; Richards, F. 1979: Calibration and intercomparison of the gate C-band radars. NOAA Tech. Rep. EDIS 31, U.S. Dept. of Commer., Washington D.C.

Journel, A.G.; Huijbregts Ch. J. 1978: Mining geostatistics. Academic Press, New York, N.Y.

Kessler, E.; Wilk, K.E. 1968: Radar measurements of precipitation for hydrological purposes. WMO/IHD Report No. 5, Secretariat of the World Meteorological Organization. Geneva, Switzerland

Krajewski, W.F. 1987: Co-kriging radar-rainfall and rain gage data, J. of Geoph. Res. 92(D8), 9571-9580

Krajewski, W.F.; Azimi-Zonooz, A. 1987: Hermite approximation of anamorphosis function, Technical Note No. 10, Hydrologic Research Laboratory, National Weather Service, Silver Spring, Maryland

Krajewski, W.F.; Georgakakos, K.P. 1985: Synthesis of radar rainfall data, Water Resour. Res., 21(5), 764-768

Krajewski, W.F.; Hudlow, M.D. 1983: Evaluation and application of a real time method to estimate mean areal precipitation from rain gage and radar data. Proceedings of the WMO conference on mitigation of natural hazards, Sacramento, California

Larson, L.W.; Peck, E.L 1974: Accuracy of precipitation measurements for hydrologic modeling. Water Resour. Res. 10(4), 857-863

Mantoglou, A.; Wilson, J.L. 1982: The turning bands method for simulation of random fields using line generation by a spectral method, Water Resour. Res., 18(5), 1379-1394

Matheron, G. 1976: A simple substitute for conditional expectation: the disjunctive kriging, Proceedings, NATO A.S.I. "Geostat 75", D. Reidel Publishers, Dordrecht, Netherlands

Puente, C.E.; Bras, R.L. 1982: A comparison of linear and nonlinear random field estimators, Ralph. M. Parsons Lab., Technical Report 287, M.I.T., Cambridge, Mass.

Seo, D.J.; Krajewski, W.F.; Bowles, D.S. 1989a: Stochastic interpolation of rainfall data from raingages and radar using co-kriging: 1. Design of experiments. Sumitted to Water Resour. Res.

Seo, D.J.; Krajewski, W.F.; Azimi-Zonooz, A.; Bowles, D.S. 1989b: Stochastic interpolation of rainfall data from raingages and radar using co-kriging: 2. Results. Submitted to Water Resour. Res.

Valdes, J.B.; Rodriguez-Iturbe, I.; Gupta, V.K. 1985: Approximation of temporal rainfall from a multidimensional model. Water Resour. Res. 21(8), 1259-1270

Waymire, E.; Gupta, V.K.; Rodriguez-Iturbe, I. 1984: A spectral theory of rainfall intensity at the Meso-β scale, Water Resour. Res., 20(10), 1453-1465

Wilson, J.W. 1970: Integration of radar and raingage data for improved rainfall measurements, J. Appl. Meteo. 9, 489-497

Wilson, J.W.; Brandes, E.A. 1979: Radar measurement of rainfall - a summary, Bull. Am. Meteorol. Soc., 60(9), 1048-1958

Yates, S.R. 1986: Disjunctive kriging 3. Co-kriging, Water Resour. Res. 22(10), 1371-1376

Zawadzki, I. 1973: Statistical properties of precipitation patterns, J. Appl. Meteorol., 12, 459-472

Zawadzki, I. 1984: Factors affecting the precision of radar measurements of rain, Preprints of the 22nd Conference on radar meteorology. AMS, Boston, Mass., 251-256

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Technical comments

Comment

F. Ashkar, B. Bobée, D. Leroux and D. Morisette: The generalized method of moments as applied to the generalized gamma distribution. Stochastic Hydrol. Hydraul. 2 (1988) 161–174

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Ashkar et al. (1988) have brought to the attention of hydrologists and hydraulicists the generalized gamma (*GG*) distribution and pointed out its practical advantages - flexibility of shape, a zero lower bound and no upper bound.

It may be of interest that the late Russian hydrologist Kalinin (1962) suggested theoretical reasons for using this distribution for annual runoff totals. His rationale is based on a simplified description of the mechanism of runoff formation, taking into account alternation of rainy and rainless periods on the one hand and, on the other, the empirically often documented nonlinear relationship between annual precipitation and runoff volumes. I have reproduced Kalinin's rationale in connection with the occasionally observed phenomenon of a negative skew in historic samples of annual runoff (Klemes, 1970) and pointed out that the *GG* distribution may be regarded as a first step to a physically based model for the distribution of annual runoff (Klemes, 1978). More detailed studies involving transformation of stochastic inputs by nonlinear storage systems (Klemes, 1982) suggest that a theoretically accurate mathematical description of distributions of hydrological phenomena may be very complex and may not even be obtainable in a closed form (e.g., see Moran, 1967); however, this is the very reason why a *GG* distribution or, more specifically, a power transformation of the simple 2-parameter gamma distribution may provide a good approximation to distributions of many hydrological variables.

I would also like to supplement and correct some historic information given by Ashkar et al. (1988) since the authors' sources were obviously incomplete. They correctly state that the *GG* distribution is widely used in the Soviet Union; however, it has been used there not only for flood frequencies but perhaps even more often for fitting distributions of annual and monthly runoff totals (or mean flows). Actually, in the USSR and countries where the Russian hydrology school has become more familiar (e.g., China and Eastern Europe), the power-transformed gamma distribution is known as the Kritskiy-Menkel distribution although Kritskiy and Menkel themselves have consistently called it "three parameter gamma" distribution. They originally proposed the use of this distribution in hydrology neither in 1969 as Ashkar et al. (1988) imply, nor in 1950 as I have long believed (Klemes, 1970), but in 1946 (Kritskiy and Menkel, 1946) as Professor Kritskiy wrote to me shortly before his recent death. Extensive analyses of sample parameter estimates by an approximate maximum likelihood method were done for this distribution by Blokhinov (1974) whose results, together with a wealth of other relevant information (e.g., ordinates of the normalized *GG* distribution function for various ratios of its coefficients of variation and skew), are summarized in the last two books of the two eminent late Russian hydrologists and water resource engineers, Professors S.N. Kritskiy and M.F. Menkel (Kritskiy and Menkel, 1981, 1982).