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# Optimization of Complex Hydrologic Models Using Random Search Methods

Larry E. Brazil[1] and Witold F. Krajewski[2]

The primary objective of automated calibration of complex conceptual hydrologic simulation models is to find the global optimum of a specified response surface. While direct search techniques such as gradient or Newton methods may be valuable tools for determining local optimum points, they present many practical and theoretical difficulties in real applications and often are of limited utility for global problems. As an alternative, four random search techniques have been proposed and analyzed in this study. A comparison experiment was performed on synthetic data using a state-space version of the Sacramento Soil Moisture Accounting Model. Experiment results are presented and implementation details are discussed.

## Introduction

One important use of conceptual hydrologic simulation models is hydrologic forecasting. Information obtained from forecasting models is used as input to decisions concerning items such as water supply, irrigation, power production, reservoir operation, and navigation. Most importantly, hydrologic forecast models are used in the preparation of river forecasts which include the issuance of flood forecasts and warnings.

Forecast models must be calibrated for the specific area for which they are to be used. A particular model's accuracy usually is dependent on the accuracy of the calibration. The calibration process generally consists of estimating the values for parameters which will minimize the differences between observed historical streamflows and streamflow values computed by the model. The actual procedures used in calibrating hydrologic models vary considerably depending on the form of the model; however, most calibration strategies include a combination of manual and automatic fitting techniques.

A variety of automatic parameter identification procedures have been developed and adapted to an assortment of models. Many of the models have more than five parameters, making an exhaustive search of the parameter space infeasible. As a result, most of the parameter estimation algorithms are based on some type of directed search procedure which attempts to find the global optimum on the objective function response surface. These procedures typically have two major drawbacks: 1) The final result is strongly influenced by the

[1,2] Research Hydrologists, Hydrologic Research Laboratory, National Weather Service, Silver Spring, MD. 20910.

parameter starting values. If the values are inaccurate, the search algorithm often converges to an unrealistic local optimum. 2) The search typically is driven by one objective function which may or may not be the best fitting criterion for the model and its application. The main advantages of the automatic techniques, though, are that they are computer rather than labor intensive and often can provide insight into modeling problems that may have been overlooked in manual calibration efforts.

Random search procedures offer a means of overcoming some of the major automatic technique disadvantages. Although they often are more computationally expensive, random procedures are less prone to local optima, since they search within an area rather than along a path determined by a starting point, and do not necessarily need to be driven by a single objective function. Random techniques are becoming particularly more attractive as computer hardware prices decrease and processing time becomes more available.

## Application

The National Weather Service (NWS) is responsible for providing hydrologic forecasts for rivers and watersheds throughout the United States. Most of the forecasting is performed with the aid of computer simulation models such as the Sacramento Soil Moisture Accounting Model (Burnash et al., 1973). A major problem faced by the NWS is how to calibrate basins that are added to the forecast network and recalibrate existing modeled basins to reflect changing watershed conditions.

Ongoing research is being performed to improve the techniques used to calibrate NWS models. The purpose of this study was to determine the feasibility of using a random search optimization procedure with an NWS conceptual forecast model. The model used in the study was a modified version of the Sacramento model. The modifications resulted from the transformation of the model from its original FORTRAN algorithm into a set of state-space equations. The version being used in this study is a first order approximation of the integral of the nonlinear model developed by Georgakakos and Bras (1982). A detailed discussion of the model will be published soon.

## Random Search Methods

Four random search optimization algorithms were used in this study. A brief description of the algorithms follows.

### <u>Algorithm 1</u>

This is a uniform random (UR) search method. Parameter space $\Omega$ is searched through a random independent drawing and the best point $\underline{\alpha}_{OPT}$ is selected as

$$\underline{\alpha}_{OPT} = \{\underline{\alpha}_{OPT}: \; f(\underline{\alpha}_{OPT}) = \min f(\underline{\alpha}^{j}), \; j = 1,2,\ldots,K\}$$

Table 1. Best Run for each Optimization Algorithm

| Parameter | Algorithm 1 | 2 | 3 | 4 | True Value | Lower Bound | Upper Bound |
|---|---|---|---|---|---|---|---|
| UZTWM | 131. | 121. | 111. | 117. | 120. | 100. | 150. |
| UZFWM | 15.1 | 15.1 | 14.8 | 15.6 | 15.0 | 10. | 30. |
| LZTWM | 200. | 165. | 193. | 159. | 160. | 100. | 200. |
| LZFPM | 167. | 168. | 158. | 150. | 140. | 100. | 200. |
| LZFSM | 14. | 14. | 15. | 12. | 14. | 10. | 60. |
| UZK | .280 | .303 | .382 | .276 | .300 | .2 | .4 |
| LZPK | .0144 | .0150 | .0098 | .0140 | .0130 | .001 | .02 |
| LZSK | .138 | .102 | .167 | .130 | .126 | .02 | .2 |
| ZPERC | 62. | 53. | 81. | 49. | 48. | 10. | 100. |
| REXP | 2.90 | 2.70 | 2.57 | 2.29 | 2.10 | 1.5 | 4. |
| PFREE | .017 | .053 | .073 | .037 | .020 | 0. | .1 |
| ADIMP | .135 | .171 | .175 | .188 | .170 | .1 | .2 |
| PCTIM | .015 | .000 | .000 | .000 | .001 | 0. | .05 |
| MSE (mm) | .0141 | .0026 | .0148 | .0031 | | | |
| Average MSE (mm) | .0158 | .0060 | .0166 | .0058 | | | |

References

Burnash, R.J.C., Ferral, R. L., and McGuire, R. A., A Generalized Streamflow Simulation System: Conceptual Modeling for Digital Computers, Joint Federal-State River Forecast Center, Sacramento, CA (1973), 204 pages.

Georgakakos, K. P., and Bras, R. L., "A Precipitation Model and It's Use in Real-Time River Flow Forecasting," MIT TR286, Ralph M. Parsons Laboratory, Cambridge, MA. (1982), 302 pages.

Pronzato, L., Walter, E., Venot, A., and Lebruchec, J. F., "A General-Purpose Global Optimizer: Implementation and Applications," Mathematics and Computers in Simulation, 26 (1984), 412-422.



where $\underline{\alpha}^j = \{\alpha_i^j: \alpha_i^j = U(\alpha_i^L, \alpha_i^U), i = 1,\dots,N\}$. Number of iterations K is predetermined and usually based on economic considerations. U(a,b) denotes uniform distribution on the range with lower bound a and upper bound b. This algorithm computes several objective functions at each iteration. The user selects the objective function(s) of interest after the program ends and determines the optimum point in a multi-objective analysis of the results.

Algorithm 2

This algorithm is based on an adaptive random search (ARS) algorithm described by Pronzato et al. (1984). The method is based on the following considerations: instead of uniformly searching the whole feasible region $\Omega$, we can concentrate on those locations which show a potential for having a global optimum. This potential is based on the preceding search of the whole space $\Omega$. The best point found is suspected of being within the vicinity of the optimum and its proximity is subject to an additional search. Since this can lead to a local optimum, a mechanism is provided to escape from such regions and continue the search in other parts of $\Omega$. If, however, the search returns to the same vicinity a predetermined number of times, the best point in this vicinity is declared the global optimum. The algorithm consists of the following steps.

1. Select the criterion to be minimized $f(\underline{\alpha})$ and the admissible range of the parameters

$$\alpha_i^L \le \alpha_i \le \alpha_i^U \qquad \text{for } i = 1,\dots,N$$

$$R_i = \alpha_i^U - \alpha_i^L$$

2. Select the starting point as

$$\alpha_i^0 = \frac{1}{2}(\alpha_i^L - \alpha_i^U) \qquad \text{for } i = 1,\dots,N$$

3. Set MAX, LOC, K, $L_{stop}$, and $k = 1$, $L_{opt} = 0$

4. Compute $R_i^{(k)} = 10^{1-k} R_i \qquad$ for $i = 1,\dots,N$

5. Perform MAX iterations of uniform random search so that

$$\alpha_i^{j+1} = \alpha_j^j + U(\beta_i^L, \beta_i^U) \qquad \text{for } i = 1,\dots,N$$

where

$$\beta_i^L = \max\{\alpha_i^L, \alpha_i^j - \tfrac{1}{2}R_i^{(k)}\}$$

$$\beta_i^U = \min\{\alpha_i^U, \alpha_i^j + \tfrac{1}{2}R_i^{(k)}\}$$

Store the best found point and the corresponding k as $\underline{\alpha}^*(k)$.

6. Set $k = k + 1$.

   If $k > K$, go to 7, otherwise $MAX = MAX/k$, go to 4.

7. Select $\min\{\alpha^*(k): k = 1,\ldots,K\}$. Record the "optimal" k as $k^*$.

   If $k^* = K$, then $L_{opt} = L_{opt} + 1$. If $L_{opt} = L_{stop}$, go to 10. If $k^* \neq K$, set $L_{opt} = 0$.

8. Perform LOC iterations of uniform random search around $\underline{\alpha}^*(k)$ within the neighborhood $\underline{R}^{(k^*)}$ corresponding to the optimal $k^*$.

9. Reset the parameters MAX and $k = 1$. Go to 4.

10. Stop. The best point is $\underline{\alpha}_{OPT} = \underline{\alpha}^*(k)$.

The values of K, MAX, LOC and $L_{stop}$ suggested by Pronzato et al. (1984) were 5, 100, 100, 5, respectively. We decided to use K = 3 and MAX = 200 for this problem. This decision was based on preliminary runs and economic considerations.

### Algorithm 3

Algorithm 3 is the same as algorithm 1 except the Ω is now modified. This modification takes into account some functional relationships that are believed to exist between certain parameters of our model. Parametric relationships were used for three pairs of parameters in this study. For example, a quadratic relationship is believed to exist between the two percolation parameters REXP and ZPERC. This parametric relationship is used to restrict the search space to a band along the curve relating the parameters. The other two restrictions were based on relationships that are assumed between the interflow and baseflow components in two-dimensional space.

### Algorithm 4

Algorithm 4 is the same as algorithm 2 with Ω modified as described above. Figure 1 presents the concept of the ARS method for both the algorithms 2 and 4.

## Synthetic Data Experiment

In order to evaluate the performance of the above algorithms, a synthetic data simulation experiment was designed and conducted. Seven years of 6-hourly streamflow data were generated using the model described previously, and corresponding 7 years of actual rainfall record from the Bird Creek basin in Oklahoma. The original parameter values were obtained by manual calibration of the basin. The observations of streamflow are taken as error-free data; therefore, the difference between the parameter values used to generate the data



(the optimal parameters) and the estimated parameters are due to sampling error and the estimation algorithm only. The resulting hydrograph is characterized by an average annual flow of 0.60 mm of runoff per day with flows ranging from 0.0 to 16.5 mm per 6 hours.

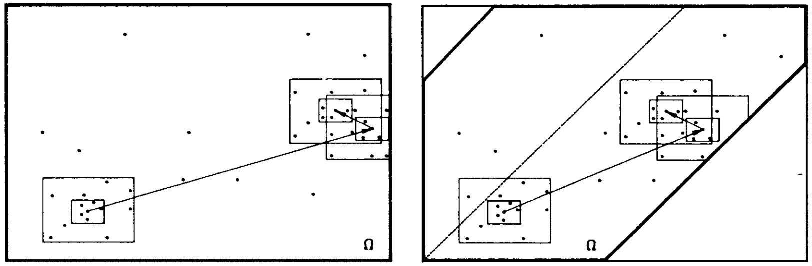


Figure 1. ARS concept representation. Left diagram (algorithm 2) shows search of a two-parameter space within upper and lower parameter bounds. Right diagram (algorithm 4) shows search being further restricted by the parametric relationship.

## Results

The mean square error (MSE) was used as the objective function for all four algorithms. Five realizations were performed for each of the algorithms to evaluate bias introduced by the random seed choice. The results of the realization for each method are presented in Table 1. It shows that the best overall run was produced by algorithm 2 and that the mean of the MSE's for the ARS runs was only slightly improved by restricting the search space with parametric relationships. Relatively small improvement, due to parametric relationships among the model parameters, can be explained by the fact that, for this experiment, the implemented relationships only slightly modified the size of Ω space. The ARS algorithm was consistently better than the UR search. It should be pointed out, however, that each of the 20 runs found different solutions -- we did not find the global optimum -- and in the case of the UR search, we cannot even claim that the solutions are local optima.

## Conclusions

The main conclusion resulting from the study is that the random search algorithms provide an attractive alternative to other nonrandom search techniques. ARS is more accurate than the UR search and also less expensive (about 3 times). Preliminary runs with real data confirm these findings. The random search methods soon will become a component of the system used to calibrate NWS hydrologic simulation models.