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KALMAN FILTER ESTIMATION OF RADAR-RAINFALL FIELD BIAS

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1. INTRODUCTION

Radar rainfall estimates, when properly adjusted using a limited number of gages, can be a useful source of data for severe weather forecasting (flood and flash flood) and water management activities. The National Weather Service (NWS) plans to implement a real-time multi-stage/multisensor precipitation analysis system coincident with the implementation of the Next Generation Weather Radar (NEXRAD) beginning in 1989. The adjustment/merging of radar data and rain gage data is envisioned as taking place on hourly data in two distinct steps. This decision is based on the fact that a comprehensive merging of the radar and rain gage data should be performed only after the radar data have gone through quality control procedures which require satellite and other ancillary data not available on the NEXRAD computer system. Also, for flash flood forecasting, data are needed as soon as possible, while for river forecasting, timeliness can be sacrificed somewhat for the sake of improved accuracy. Thus, it was decided that a fast technique to correct for the radar mean field bias would be implemented within the NEXRAD computer system using limited gage data and an optimal interpolation procedure (Krajewski and Ahnert, 1986) would be performed later on a computer system external to NEXRAD using the more comprehensive gage data base available. A Kalman filter procedure to compute the mean field bias is proposed as the technique for a "fast" adjustment on the NEXRAD computer system. The adjusted rainfall estimates generated will be used for NEXRAD real-time graphical displays and for input to flash flood forecast models and procedures at the local forecast office and will be used as input to the optimal interpolation procedure performed during the next stage of processing (Ahnert et al., 1983, 1984; Hudlow et al., 1983).

The advantages of the Kalman filter over simply computing G/R ratios are: 1) the technique accounts for the "noise" in the measurements when updating the mean bias; 2) the technique provides an estimate of the error in the computed bias; 3) the technique should avoid the instability associated with G/R ratios when R is small; and 4) the technique combines an estimate of the bias and its error variance made an hour earlier with the current measurements and estimated measurement error variance to compute an updated bias estimate and new forecast for the next hour. It has these advantages while also being very efficient in terms of computer processing requirements.

To test the Kalman filtering approach, a fully controlled simulation experiment has been designed to perform a comprehensive test of the procedure. In the experiment, the bias in the radar field is simulated using various stochastic process models. Hourly radar-rainfall data are then generated by imposing an error field incorporating the simulated bias on a "ground truth" field. Similarly, unbiased rain-gage observations are generated. The Kalman filter is then used to update the estimate of the bias using the simulated observations. Using this method, performance may be tested for various rain-gage network densities, sensor measurement error characteristics, bias models, and Kalman filter parameters.

In the next sections we will describe the Kalman filter, the experimental design for testing it, and some preliminary test results.

2. THE KALMAN FILTER

Estimating the mean field bias from comparisons of radar areal estimates of rainfall and point estimates from gages which both contain errors in addition to the mean field bias is not straightforward. It is further complicated by the desire to incorporate information about the current bias contained in previous estimates of the bias. The technique used must incorporate: (a) a mathematical system model for the behavior of the bias; (b) an observation model for the radar and gage measurement behavior; and (c) an estimation method that combines (a) and (b). The Kalman filter is a technique that meets these requirements.

In order to design the Kalman filter for radar rainfall bias estimation, a mathematical model of the mean multiplicative radar rainfall bias had to be assumed. A reasonable system model seemed to be a random walk process; i.e., the bias is equally likely to increase or decrease over the next hour. For a random walk process:

$$B(t) = B(t-1) + S \qquad S \sim N(0,Q) \qquad (1)$$

A mathematical model of the relationship between the bias, the radar observations and gage observations also had to be assumed. It was assumed the radar and gage measurement noise could be combined into a single additive noise component. In this way, gage measurements ($\underline{G}=\langle G_i \rangle$) may be expressed as a function of the radar rainfall estimates ($\underline{R}=\langle R_i \rangle$), the bias (B), and a measurement noise vector ($\underline{M}$). Such representation accounts also for errors resulting from different sampling characteristics of radar and rain gages.

$$\underline{G}(t) = \underline{R}(t)\, B(t) + \underline{M} \qquad \underline{M} \sim N(\underline{0},\mathbf{F}) \qquad (2)$$

Eq. (2) is a vector equation since data from numerous gages are obtained. In fact, the length of the various vectors will vary with time depending on the gage locations where precipitation occurs.

Eqs. (1) and (2) form the mathematical model of the "real world" upon which this filter is based. These can be used to derive the Kalman filter equations. The general vector equations for a discrete Kalman filter are summarized by Gelb (1974).

In order to use (1) and (2) for a Kalman filter bias estimation procedure, estimates of the system noise variance (Q) and measurement noise covariance matrix (**F**) are required. For simplicity, Q is assumed to be constant and known. The measurement noise covariance matrix must be estimated since it will be time varying and depend upon which gages are being used.

The Kalman filter runs once each hour using available hourly gage estimates and associated radar rainfall estimates at the gage locations. To ensure reasonableness of the input, gage-radar pairs are excluded if both the gage and radar hourly accumulation estimates are less than 0.6 mm. Next, to remove outliers, the mean and standard deviation of the ($\log(G_i) - \log(R_i)$) differences are calculated. Gage-radar pairs whose normalized ($\log(G_i) - \log(R_i)$) difference is greater than twice the standard deviation are excluded from the filter. The remaining gage-radar pairs are placed in corresponding elements of a gage observation vector ($\underline{G}$) and radar observation vector ($\underline{R}$).

Once current gage observations ($\underline{G}(t)$) and corresponding radar observations ($\underline{R}(t)$) are available we may use them to estimate **F**. From the observations a vector ($\underline{e}=\langle e_i \rangle$) of differences is obtained. Its elements are called the innovations and they give the difference in the current gage values from the corresponding radar estimates adjusted using the previous forecast of the bias for time t made at time t-1:

$$\underline{e}(t) = \underline{G}(t) - \underline{R}(t)\, \hat{B}(t|t-1) \qquad (3)$$

which can be manipulated to give:

$$\underline{e}(t) = \underline{G}(t) - \underline{R}(t)B(t) - \underline{R}(t)[\hat{B}(t|t-1) - B(t)] \qquad (4)$$

From Eq. (2), the definition of the measurement noise covariance **F** is:

$$\mathbf{F} = E\{[\underline{G}(t) - \underline{R}(t)B(t)][\underline{G}(t) - \underline{R}(t)B(t)]^T\} \qquad (5)$$

where the superscript T indicates the transpose and $E\{\cdot\}$ is the expectation operator.

Noting that the quantity $[\hat{B}(t|t-1) - B(t)]$ is uncorrelated with $\underline{G}(t)$ and $\underline{R}(t)$ and has a zero mean, Eq. (4) can be manipulated to yield:

$$E\{\underline{e}\,\underline{e}^T\} = \mathbf{F} + P(t|t-1)\, E\{\underline{R}(t)\underline{R}^T(t)\} \qquad (6)$$

where

$$P(t|t-1) = E\{[\hat{B}(t|t-1) - B(t)]^2\} \qquad (7)$$

and $P(t|t-1)$ is the previous forecast of the mean square error in the bias estimate and is estimated by the filter.

Eq. (6) provides the means to estimate the measurement noise covariance matrix **F**. The most straightforward method would be to simply use the sample estimates for each term in the required covariance matrices $E\{\underline{e}\,\underline{e}^T\}$ and $E\{\underline{R}(t)\underline{R}^T(t)\}$ and the estimated value of $P(t|t-1)$ provided by the filter.

Unfortunately, this straightforward procedure yields poor estimates of **F** (including frequent estimates which do not satisfy the requirement that **F** be a positive definite matrix). This instability is due fundamentally to the limited sample size. To avoid these difficulties, a somewhat more complex procedure has been adopted; Eq. (6) is replaced by the following:

$$\hat{\mathbf{F}} = \mathbf{X} - p\mathbf{Y} \qquad (8)$$

where **X** is a covariance matrix related to $E\{\underline{e}\,\underline{e}^T\}$
P is a scalar value related to $P(t|t-1)$
Y is a covariance matrix related to $E\{\underline{R}(t)\underline{R}^T(t)\}$.

The terms of the covariance matrices **X** and **Y** are defined by spatial covariance functions defined in the following way:

$$X_{ij} = a_x + b_x \exp[c_x \cdot d_{ij}] \qquad (9)$$

where X_{ij} is the i,j term of the matrix **X**
a_x, b_x, c_x are parameters of the covariance function
d_{ij} is the spatial distance from the i-th gage to the j-th gage.

Similarly, the terms of the covariance matrix **Y** are defined by:

$$Y_{ij} = a_y + b_y \exp[c_y \cdot d_{ij}] \qquad (10)$$

The covariance parameters, a_x, b_x, and c_x, are basically estimated by a least-squares fit to the covariance of the innovations ($\underline{e}$). Likewise the covariance parameters, a_y, b_y, and c_y, are estimated by a least-squares fit to the covariances of the radar observations ($\underline{R}(t)$). Constraints are imposed on the values of the covariance parameters, i.e.,

$$a_x,\ b_x,\ a_y,\ b_y \geq 0 \quad \text{and} \quad c_x,\ c_y < 0 \qquad (11)$$

Actually, the constraints, Eq. (11), are applied after the least squares estimation procedure. They are applied in three steps described below:

Step 1: Estimate all parameters a_x, b_x, and c_x, and test against constraints, Eq. (11).

Step 2: If step 1 violates constraints, then re-estimate covariance parameters with the additional constraint that $a_x \equiv 0$. (This is a two-parameter covariance model.)

Step 3: If step 2 violates constraints, then re-estimate covariance parameters with $b_x \equiv c_x \equiv 0$ and a_x allowed to vary. (This is a one-parameter covariance model.)

The two sets of covariance parameters are linked in such a way that if a one-, two-, or three-parameter model is used for the innovations (**X**), a similar model is used for the radar observations (**Y**).

Finally, the scalar $\underline{p}$ in Eq. (8) must be defined in such a way that $\hat{\mathbf{F}}$ is positive definite. This constraint is satisfied by

$$p = \text{Min}\ \{\hat{P}(t|t-1),\ 0.5(a_x+b_x)/(a_y+b_y)\} \qquad (12)$$

with the factor of 0.5 times the ratio of the variance of the innovations to the variance of the radar observations as an essentially arbitrary choice.

In fact, several of the estimation steps to define $\hat{\mathbf{F}}$ are arbitrary in essence, but they do assure that the the estimate ($\hat{\mathbf{F}}$) of the measurement noise covariance matrix (**F**) is a well-behaved positive definite matrix. With the value of the system noise variance (Q) assumed constant and known, we now have all the information required to operate the Kalman filter. The update begins with the calculation of the Kalman gain vector ($\underline{K}$):

$$\underline{K}(t) = \hat{P}(t|t-1)\ \underline{R}^T(t)\ \{\underline{R}(t)\hat{P}(t|t-1)\ \underline{R}^T(t) + \mathbf{F}(t)\}^{-1} \qquad (13)$$

where the superscript -1 indicates a matrix inversion and $\hat{P}(t|t-1)$ is the previous forecast of the mean square error in the bias estimate for time t made at time (t-1).

The new bias estimate for time t is then given by:

$$\hat{B}(t|t) = \hat{B}(t|t-1) + \underline{K}^T(t)\ \underline{e}(t) \qquad (14)$$

New bias error variance estimate:

$$\hat{P}(t|t) = [1 - \underline{K}^T(t)\ \underline{R}(t)]\ \hat{P}(t|t-1) \qquad (15)$$

Based on our random walk model the new forecast bias and forecast mean square error estimates are:

$$\hat{B}(t+1|t) = \hat{B}(t|t) \qquad (16)$$

$$\hat{P}(t+1|t) = \hat{P}(t|t) + Q \qquad (17)$$

The adaptable parameter considered to be most crucial to the filter performance is Q, the estimate of the mean square error in the drift of the bias from one hour to the next. Determining a reasonable value for Q is a major goal of these tests.

For a detailed functional specification of the adjustment algorithm for NEXRAD, incorporating this Kalman filter, see the NEXRAD Algorithm Report (NEXRAD, 1985).

3. TEST DESIGN

One way of testing the Kalman filter would be to make use of a set of radar data and a corresponding set of rain gage data having a density such that it may be used as "ground truth" (O'Bannon and Ahnert, 1986). Such an evaluation is hampered by errors inherent in the use of point gage observations as "ground truth" for areal radar rainfall estimates, by the limited availability of data sets containing both radar and dense rain gage observations, and by the large data management effort required to handle enough data to give the experiment statistically meaningful results.

An alternative method for testing the Kalman filter is to use simulated radar and rain gage data. The simulation system used here was developed by Krajewski and Georgakakos (1984) based on earlier work of Greene et al. (1980). The idea is to generate fields from an existing high quality radar field by imposing a noise field of known statistics. The generated radar and rain gage field may then be used as input to the Kalman filter to update the estimated bias. For comparison, the actual bias can be independently calculated from the original and generated radar fields. Each step in the testing process is described below. Preliminary test results are described in the next section.

<u>Step 1</u>: Select Original Field O(x,y): The original field consists of hourly radar data from the GARP Atlantic Tropical Experiment (GATE) conducted in 1974. A detailed description of the GATE data is given by Hudlow and Patterson (1979). The 24 hours used in preliminary testing were from September 2, 1974. Data are on a 100 x 100 grid of 16 km^2 spatial averages.

<u>Step 2</u>: Calculate New Theoretical Bias $\beta(t)$: The theoretical bias specifies the multiplicative bias to be introduced into the original field when generating the radar field. Two models for the theoretical bias are being used. The first is simply that the bias is constant. The second is that the bias follows a random walk process.

Step 3: Generate Radar Field $R(x,y)$:
Noise is added to the original field to generate a radar field with specified second order statistics (Krajewski and Georgakakos, 1984). The statistics specified for $R(x,y)$ are the bias $\beta(t)$, the variance in $\log[R(x,y)/O(x,y)]$, (V_r), and the correlation distance of the noise $1/h$. The generated fields statistics are approximately those specified (an ensemble of fields would have these statistics exactly). Therefore, to compute the error in the biases estimated later we compute the actual bias $B(t)$ in the radar field.

$$B(t) = \Sigma O(x,y)/\Sigma R(x,y) \quad \text{for } R(x,y) \neq 0$$

Step 4: Generate a Set of Rain Gage Values {G}:
A specified number of rain gage locations are randomly scattered across the (x,y) grid. The same grid of 30 gages was used in all preliminary runs. A rain gage value is then generated in a way that accounts for the differences in the point sampling characteristics of the gage versus the areal estimate given in the original field. The procedure is described by Krajewski (1986). The measurement error expressed in terms of standard deviation given as a percentage of the mean was then imposed on the adjusted point value. This error was 10 percent in all the runs.

Step 5: Form Gage Observation Vector $\underline{G}(t)$ and Radar Observation Vector $\underline{R}(t)$:
The radar field value directly over each gage is paired with the gage value to form gage-radar sets. The gage-radar pairs are then subjected to the tests described above to assure reasonableness.

Step 6: Update Bias Estimates From $\underline{G}(t)$ and $\underline{R}(t)$:

$$\hat{B}_K(t),\ \hat{B}_{GR}(t),\ \hat{B}_{GRS}(t)$$

The Kalman filter bias $\hat{B}_K(t)$ is computed using the equations derived in section 2 and a specified value of the system noise variance (Q). In addition, for comparison, two additional bias estimates are generated by computing G/R ratios. The equations used are:

$$\hat{B}_{GR}(t) = 10^{\frac{1}{N}\Sigma\log(G_i/R_i)}$$

$$\hat{B}_{GRS}(t) = 0.5\ \hat{B}_{GRS}(t-1) + \hat{B}_{GR}(t)$$

The same gage observation vector ($\underline{G}(t)$) and radar observation vector ($\underline{R}(t)$) are used in the Kalman filter and in computing the G/R ratios.

Step 7: Repeat Steps 1-6 Beginning with Next Hour Original Field.

Step 8: Repeat Steps 1-6 with New Set of Specifications for the Theoretical Bias Model, Radar Noise Parameters, Numbers and Locations of Gages, and/or Kalman Filter System Noise Parameter.

Step 9: Compute Error Statistics:
For each run of 24 hours, the mean square error of the three bias estimates is computed.

$$\hat{MSQ} = \frac{1}{24}\Sigma\left[\hat{B}(t) - B(t)\right]^2$$

4. PRELIMINARY RESULTS AND CONCLUSIONS

Table 1 presents the results from the first set of test runs. The theoretical bias (β) was set to a constant value of 0.5, 1.0, or 2.0. The radar noise parameter (V_r) was set to near 0 and for this reason the correlation distance (1/h) had little effect on the results.

Table 1. Parameter Values and Mean Square Errors of the Kalman Filter Bias Estimates (MSQ_K), G/R Ratio Bias Estimates (MSQ_{GR}), and Smoothed G/R Ratio Bias Estimates (MSQ_{GRS}) for tests with a Constant Theoretical Bias (β) and V_r = 0.005.

	Parameters			Mean Square Errors		
Run #	β	1/h	Q	MSQ_K	MSQ_{GR}	MSQ_{GRS}
1	0.5	20.0	.01	.037	.024	.005
2	0.5	20.0	.05	.036	.024	.005
3	0.5	20.0	.10	.038	.024	.005
4	0.5	4.0	.01	.029	.021	.006
5	0.5	4.0	.05	.032	.021	.006
6	0.5	4.0	.10	.036	.021	.006
7	1.0	20.0	.01	.048	.078	.058
8	1.0	20.0	.05	.054	.078	.058
9	1.0	20.0	.10	.056	.078	.058
10	1.0	4.0	.01	.043	.067	.046
11	1.0	4.0	.05	.043	.067	.046
12	1.0	4.0	.10	.046	.067	.046
13	2.0	20.0	.01	.253	.736	.430
14	2.0	20.0	.05	.206	.736	.430
15	2.0	20.0	.10	.191	.736	.430
16	2.0	4.0	.01	.255	.790	.419
17	2.0	4.0	.05	.208	.790	.419
18	2.0	4.0	.10	.195	.790	.419

The Kalman filter system noise parameter's effect on $\hat{B}_K$ seemed to depend on the value of the bias. For a bias of 0.5 and 1.0 a smaller Q resulted in a smaller error in $\hat{B}_K$ while for a bias of 2.0 a larger Q reduced the error. This is probably due to the assumption in Eq. (1) that the bias (B) follows a random walk process. This initial model assumes a change in the bias from 0.1 to 0.2 is equally likely to a change from 2.1 to 2.2. However, since it is a multiplicative bias, the change from 0.1 to 0.2 results in a far larger change in the radar estimates and probably is far less likely than a change in the bias from 2.1 to 2.2. An easily implementable correction is to let Q become a function of $B_K(t)$. The proper form of this function and its effects are being investigated.

The Kalman filter procedure did better than G/R ratios when the bias equaled 2.0 (radar underestimation), and worse when the bias equaled 0.5 (radar overestimation). For the parameters chosen, this is understandable since gage values included noise, while radar noise was set to near 0. For a bias of 2.0, values of

R are reduced and noise in G produces larger errors in the G/R ratios. The reverse is true when the bias is 0.5. Once radar noise is added, this effect may be less apparent. The smoothed G/R ratio (B_{GRS}) also is aided by the fact that the theoretical bias (β) was held constant.

For a more realistic test the radar noise parameter was increased and the theoretical bias was generated from a random walk process model as in Eq. (1). To distinguish it from the Kalman filter system noise (Q) parameter the symbol (q) is used for the variance parameter used when generating the theoretical bias. The value of q was set to 1.0. This large value of q results in a rapidly varying theoretical bias. Bias error statistics for some preliminary runs using this theoretical bias time series are presented in Table 2. An interesting result is the apparent reverse in the dependence of MSQ_K on Q for the two values of 1/h. Values chosen for 1/h represent estimated minimum and maximum values. The actual range of this statistic is not known.

From Table 2, it is apparent that the Kalman filter remains a good candidate method. More testing is in progress to see how the performance of the bias estimates degrades as the number of gages is decreased. The availability of 30 real-time hourly gages assumed in tests presented here exceeds the number currently available for most sites. The Kalman filter method should be able to handle reduced amounts of gage data better than G/R ratio methods.

Table 2. Parameter Values and Mean Square Errors of the Kalman Filter Bias Estimates (MSQ_K), G/R Ratio Estimates (MSQ_{GR}), and Smoothed G/R Ratio Estimates (MSQ_{GRS}) for tests with a Random Walk Model for the Theoretical Bias (β) and V_r = 0.15.

	Parameters			Mean Square Errors		
Run #	q	1/h	Q	MSQ_K	MSQ_{GR}	MSQ_{GRS}
1	1.0	20.0	.05	.319	.267	.326
2	1.0	20.0	.10	.266	.267	.326
3	1.0	20.0	.25	.260	.267	.326
4	1.0	4.0	.05	.231	.362	2.754
5	1.0	4.0	.10	.236	.362	2.754
6	1.0	4.0	.25	.320	.362	2.754

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