STATE ESTIMATION OF A SCALAR DYNAMIC PRECIPITATION MODEL FROM TIME-AGGREGATE OBSERVATIONS

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ABSTRACT

Presented is a state estimator suitable for use with linear, scalar systems whose observables are time-integrals, over an interval Δt, of linear functions of the state. The ability of the ordinary Kalman filter to approximate the developed estimator in terms of prediction accuracy is examined by way of application to a scalar, dynamic, station precipitation model. For the application, the aggregation interval Δt takes on values commonly used in the official reporting of precipitation data.

INTRODUCTION

It is rather common in the study of hydrometeorological systems to use observations of time-aggregate quantities. Observational mechanisms measuring volume of water accumulated over a certain period of time will produce observations of the time-aggregate type. In particular, ground observations of the precipitation rate by rain gages often measure the volume of water accumulated in the gage receptacle over a period of time. Also, official reporting of hydrometeorological data is often done in terms of aggregate values (e.g., six-hourly accumulations of precipitation.)

When one uses a stochastic-dynamic model of the precipitation process, as is the one proposed by Georgakakos and Bras (1984a,b), one has observations $z(t_k)$ that are related to the model state $X(t)$ by

$$z(t_k) = \int_{t_{k-1}}^{t_k} \phi(t)\, X(t)\, dt + v(t_k) \ ; \ k = 1,2,... \tag{1}$$

with $\phi(t) \cdot X(t)$ being the instantaneous volumetric precipitation rate at time t predicted by the model. $X(t)$ is the system state at t and $\phi(t)$ is a time-varying coefficient. The initial and final times of aggregation are denoted by t_{k-1} and t_k for all k. $v(t_k)$ is a time-uncorrelated Gaussian random sequence that simulates the errors in the observation mechanism. $v(t_k)$ has zero mean and $R(t_k)$ variance, for all k. The observed volume of precipitation collected in the interval $[t_{k-1},t_k]$ is denoted by $z(t_k)$.

The observation equation in (1) does not directly fall into the framework of the Kalman filtering theory, which has proven useful in short-term, real-time hydrological and meteorological forecasting (e.g., Kitanidis and Bras, 1980a and b, Georgakakos and Bras, 1982b, 1984a and b).

It is the purpose of this work to derive the form that the Kalman filter equations take in the case of time-aggregate observations in scalar, linear systems with Gaussian statistics. For the case of the precipitation model, it will also quantify the decrease in forecast accuracy realized when the observation equation is approximated by the familiar form:

$$z(t_k) = \Delta t \; \phi(t_k) \; X(t_k) + v(t_k) \;\; ; \; k = 1,2,\ldots \tag{2}$$

with Δt given by

$$\Delta t = t_k - t_{k-1} \tag{3}$$

and an ordinary Kalman filter is used.

The case studies contain several storm events of both the convective and the stratiform types. The comparison of the two approaches is made for characteristic values of the time increment Δt used in the official reporting of rainfall in the United States.

MATHEMATICAL FORMULATION

Consider the system whose dynamics are given by the scalar stochastic differential equation:

$$\frac{dX(t)}{dt} = h(t) \; X(t) + f(t) + w(t) \tag{4}$$

with initial conditions for the mean $\hat{X}(t_o)$ and the variance $P(t_o)$ of $X(t)$ specified as: $\hat{X}(t_o) = \hat{X}_o$ and $P(t_o) = P_o$

where,

$X(t)$: system state at time t,

$h(t), f(t)$: time-varying functions dependent on the model input variables,

$w(t)$: a Gaussian white-noise process with spectral density equal to Q and with zero mean,

$\hat{X}_o$, P_o : the mean and the variance of the system Gaussian state at the initial time t_o.

The algebraic equation in (1) gives the relationship that the system state bears to the scalar observable variable $z(t_k)$ at time t_k.

For the case of the precipitation process, Georgakakos and Bras (1984a and b) give the expressions for $f(t)$, $h(t)$, and $\phi(t)$ as functions of the model input variables and parameters.

DERIVATION OF THE STATE-ESTIMATOR EQUATIONS

The process $w(t)$ and the random variable $X(t_o)$ have been assumed Gaussian. Equation (4) is linear in the state $X(t)$. Therefore, in the absence of measurements, the joint density of the state $X(t)$ at various times remains Gaussian at least up until the first observation becomes available. Also, were the conditional density of $X(t_{k-1})$, conditioned on the observations $z(t_1)$, $z(t_2)$, ..., $z(t_{k-1})$, Gaussian, then equation (4) assures that the conditional joint density of any order of the process $X(t)$ with $t_{k-1} \leqq t \leqq t_k$, conditioned on the same observations, will remain Gaussian. Thus, if it is shown that, starting with a Gaussian state, updating across an observation preserves the Gaussian character of the conditional density, then that would

mean that the state remains Gaussian both at the propagation of the state in time between observations and at the updating of the state across an observation.

The integral $I = \int_{t_{k-1}}^{t_k} \phi(t)X(t)dt$ is a Gaussian random variable due to Gaussian process $X(t)$. The random variable $v(t_k)$ is independent of $X(t)$ with t in the closed interval $[t_{k-1}, t_k]$ and it is Gaussian. It follows that the random variable $z(t_k)$ given by equation (1) is also Gaussian. In addition, the random variables $X(t)$, for any t in $[t_{k-1},t_k]$, and $z(t_k)$ are also jointly Gaussian. This becomes evident if the integral I is written in summation form:

$$I = \sum_{i=1}^{N} \phi(t_i^*)\ X(t_i^*)\ \Delta t^*$$

with

$$t_{k-1} = t_o^* < t_1^* < \ldots < t_N^* = t_k$$

$$\Delta t^* = t_i^* - t_{i-1}^* \ ; \ i = 1,2,\ldots$$

and $\quad N \to \infty \ , \ \Delta t^* \to 0$

Then, form the linear matrix equation:

$$\begin{matrix} X(t_1^*) \\ X(t_2^*) \\ \cdot \\ \cdot \\ \cdot \\ X(t_N^*) \\ \\ z(t_k) \end{matrix} = \begin{matrix} 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & \cdots & 0 & 0 \\ \cdot & \cdot & \cdots & \cdot & \cdot \\ \cdot & \cdot & \cdots & \cdot & \cdot \\ \cdot & \cdot & \cdots & \cdot & \cdot \\ 0 & 0 & \cdots & 1 & 0 \\ \\ \phi(t_1^*)\Delta t^* & \phi(t_2^*)\Delta t^* & \cdots & \phi(t_N^*)\Delta t^* & 1 \end{matrix} \cdot \begin{matrix} X(t_1^*) \\ X(t_2^*) \\ \cdot \\ \cdot \\ \cdot \\ X(t_N^*) \\ \\ v(t_k) \end{matrix}$$

The random vector on the right-hand side is Gaussian. It follows that the random vector on the left-hand side is also Gaussian. As a consequence, $X(t_k)$ and $z(t_k)$ are jointly Gaussian and, furthermore, the density of $X(t_k)$ (already conditioned on the observations before time t_k) conditioned on $z(t_k)$ is Gaussian too.

The previous analysis proves that the density of $X(t)$ conditioned on the available observations is always Gaussian, given Gaussian initial conditions and Gaussian independent noise processes.

Update Equations

The first and second moments of $z(t_k)$ conditioned on $z(t_1),\dots,z(t_k)$, are known to be (e.g., Jazwinski, 1970):

$$\hat{X}(t_k|t_k) = \hat{X}(t_k|t_{k-1}) + P_{xz}(t_k|t_{k-1}) \cdot P_z^{-1}(t_k|t_{k-1}) \cdot \left(z(t_k) - \hat{z}(t_k|t_{k-1})\right) \quad (5)$$

$$P(t_k|t_k] = P(t_k|t_{k-1}) - P_{xz}^2(t_k|t_{k-1}) \cdot P_z^{-1}(t_k|t_{k-1}) \quad (6)$$

where,

$\hat{X}(t_i|t_j)$, $P(t_i|t_j)$: mean value and variance of the state at time t_i given observations up to and including time t_j,

$P_{xz}(\cdot)$: covariance of X and z,

$\hat{z}(\cdot)$, $P_z(\cdot)$: mean and variance of z.

Equations (5) and (6) update the mean and variance of the state across an observation. Next, the mean $\hat{z}(\cdot)$, variance $P_z(\cdot)$ and covariance $P_{xz}(\cdot)$ are determined based on the system stochastic equations.

Take the expected value of both sides in equation (1) conditioned on observations before time t_k to obtain:

$$\hat{z}(t_k|t_{k-1}) = \int_{t_{k-1}}^{t_k} \phi(t)\ \hat{X}(t|t_{k-1})\ dt \tag{7}$$

By definition,

$$P_{xz}(t_k|t_{k-1}) = E\left\{ [X(t_k) - \hat{X}(t_k|t_{k-1})] \cdot [z(t_k) - \hat{z}(t_{k-1})] \right\} \tag{8}$$

Using equations (1) and (7) in (8) one obtains,

$$P_{xz}(t_k|t_{k-1}) = \int_{t_{k-1}}^{t_k} \phi(t)\ P(t_k,t|t_{k-1})\ dt \tag{9}$$

because $v(t_k)$ is independent of $X(t_k)$.

$P(t_k,t|t_{k-1})$ is the cross-covariance of $X(t_k)$ and $X(t), t_{k-1} \leqq t \leqq t_k$, conditioned on observations at times before time t_k.

The variance $P_z(\cdot)$ is given by

$$P_z(t_k|t_{k-1}) = E\left\{[z(t_k) - \hat{z}(t_k|t_{k-1})]^2\right\} \tag{10}$$

In view of equations (1) and (7), (10) is written as

$$P_z(t_k|t_{k-1}) = E\left\{\left[\int_{t_{k-1}}^{t_k} \phi(t)\left(X(t) - \hat{X}(t|t_{k-1})\right)dt + v(t_k)\right]^2\right\} \tag{11}$$

The random variable $v(t_k)$ is uncorrelated to $X(t)$ for any t in the closed interval $[t_{k-1},t_k]$. Consequently, equation (11) is expanded to

$$P_z(t_k|t_{k-1}) = \int_{t_{k-1}}^{t_k} \int_{t_{k-1}}^{t_k} \phi(u)\,\phi(\tau)\,P(u,\tau|t_{k-1})\,du\,d\tau + R(t_k) \tag{12}$$

$P(u,\tau|t_{k-1})$ is the covariance of $X(u)$ and $X(\tau)$, $t_{k-1} \leq u,\tau \leq t_k$, conditioned on observations before time t_k.

Substitution of equations (7), (9) and (12) in (5) and (6) gives the following "updating" equations:

$$\hat{X}(t_k|t_k) = \hat{X}(t_k|t_{k-1}) + \left[\int_{t_{k-1}}^{t_k} \phi(t)\,P(t_k,t|t_{k-1})\,dt\right] \cdot$$
$$\cdot\left[\int_{t_{k-1}}^{t_k} \int_{t_{k-1}}^{t_k} \phi(u)\,\phi(\tau)\,P(u,\tau|t_{k-1})du\,d\tau + R(t_k)\right]^{-1}$$
$$\cdot\left[z(t_k) - \int_{t_{k-1}}^{t_k} \phi(t)\,\hat{X}(t|t_{k-1})dt\right] \tag{13}$$

and

$$P(t_k|t_k) = P(t_k|t_{k-1}) - \left[\int_{t_{k-1}}^{t_k} \phi(t)\,P(t_k,t|t_{k-1})\,dt\right]^2 \cdot$$
$$\cdot\left[\int_{t_{k-1}}^{t_k} \int_{t_{k-1}}^{t_k} \phi(u)\,\phi(\tau)\,P(u,\tau|t_{k-1})\,du\,d\tau + R(t_k)\right]^{-1} \tag{14}$$

The set of equations (13) and (14) reveals that the basic difference between the present formulation and the one corresponding to non-time-aggregate observations (i.e., Kalman filter) is that in the present formulation one needs to propagate the mean of $X(t)$ and the cross-covariance between $X(u)$ and $X(\tau)$, with t, u and τ in $[t_{k-1},t_k]$ for all k. In the ordinary formulation one only needs to propagate the mean and the variance of $X(t)$ for t in $[t_{k-1},t_k]$ for all k.

Propagation Equations

The propagation equations for the mean $\hat{X}(t|t_{k-1})$ and variance $P(t|t_{k-1})$ of $X(t)$, $t_{k-1} \leq t \leq t_k$, conditioned on observations before time t_k, for a continuous-time dynamic system of the type in equation (4), are known to be (Gelb, 1974):

$$\frac{d\hat{X}(t|t_{k-1})}{dt} = h(t)\hat{X}(t|t_{k-1}) + f(t) \tag{15}$$

and

$$\frac{dP(t|t_{k-1})}{dt} = 2h(t)P(t|t_{k-1}) + Q \tag{16}$$

with initial conditions $\hat{X}(t_{k-1}|t_{k-1})$ and $P(t_{k-1}|t_{k-1})$. Next, the propagation equation for $P(u,\tau|t_{k-1})$, $t_{k-1} \leq u,\tau \leq t_k$, is derived.

Consider at first the case $u \geq \tau$.

The solution of equation (4) for time u, given initial conditions at time τ, can be written as (Gelb 1974):

$$X(u) = \Phi(u,\tau)X(\tau) + \int_{\tau}^{u} \Phi(u,t)\left[f(t) + w(t)\right]dt \tag{17}$$

with the transition function $\Phi(u,\tau)$ obtained from the solution of the homogeneous differential equation:

$$\frac{d\Phi(u,\tau)}{du} = h(u) \cdot \Phi(u,\tau) \tag{18}$$

with initial condition:

$$\Phi(\tau,\tau) = 1 \tag{19}$$

Take the expected value of equation (17) conditioned on observations before time t_k to obtain

$$\hat{X}(u|t_{k-1}) = \Phi(u,\tau)\hat{X}(\tau|t_{k-1}) + \int_{\tau}^{u} \Phi(u,t)\ f(t)dt \tag{20}$$

By definition.

$$P(u,\tau|t_{k-1}) = E\{[X(u) - \hat{X}(u|t_{k-1})] \cdot [X(\tau)-\hat{X}(\tau|t_{k-1})]\} \tag{21}$$

Substitution of equations (17) and (20) in (21) yields

$$P(u,\tau|t_{k-1}) = \Phi(u,\tau) \cdot P(\tau|t_{k-1});\ u \geqq \tau \tag{22}$$

because of the fact that $w(t)$ is independent of $[X(\tau) - \hat{X}(\tau|t_{k-1})]$ for t in $[\tau,u]$.

Similarly, one obtains:

$$P(u,\tau|t_{k-1}) = \Phi(\tau,u) \cdot P(u|t_{k-1});\ \tau \geqq u \tag{23}$$

The set of equations (15), (16), (18), (19), (22) and (23) together with the initial conditions $\hat{X}(t_{k-1}|t_{k-1})$ and $P(t_{k-1}|t_{k-1})$ represent the state-estimator propagation equations.

STATION PRECIPITATION MODEL APPLICATION

The stochastic-dynamic precipitation model of Georgakakos and Bras (1984a,b) uses as input, ground surface air temperature, pressure and dew-point temperature observations or forecasts. In the real-time forecasting of precipitation, the values of the input variables are constant over each time step. Given that $h(t)$, $f(t)$ and $\phi(t)$ are only functions of the input variables, it follows that these functions are constant over each time step. In such a case, the integrals in the state-estimator equations can be determined analytically.

For notation simplification, the constant values of $h(t)$, $f(t)$ and $\phi(t)$ for t in the interval $[t_{k-1}, t_k]$ will be denoted by h, f, and ϕ, respectively.

The precipitation model equations for t in the interval $[t_{k-1} \leq t \leq t_k]$ can be written as:

$$\frac{dX(t)}{dt} = hX(t) + f + w(t) \tag{24}$$

$$z(t_k) = \int_{t_{k-1}}^{t_k} \phi X(t)dt + v(t_k) \tag{25}$$

With the present formulation, the propagation equations (15), (16), (18), (19) and (22) give $[t_{k-1} \leq t \leq t_k]$:

$$\hat{X}(t|t_{k-1}) = e^{h(t-t_{k-1})} \hat{X}(t_{k-1}|t_{k-1}) + f \left(\frac{e^{h(t-t_{k-1})}-1}{h}\right) \tag{26}$$

$$P(t|t_{k-1}) = e^{2h(t-t_{k-1})} P(t_{k-1}|t_{k-1}) + Q \left(\frac{e^{2h(t-t_{k-1})}-1}{2h}\right) \tag{27}$$

$$\Phi(u,\tau) = e^{h(u-\tau)} \qquad ; \; t_{k-1} \leq \tau \leq u \leq t_k \tag{28}$$

$$P(u,\tau|t_{k-1}) = e^{h(u-\tau)} P(\tau|t_{k-1}); \; t_{k-1} \leq \tau \leq u \leq t_k \tag{29}$$

The expressions for $\hat{z}(t_k|t_{k-1})$, $P_{xz}(t_k|t_{k-1})$ and $P_z(t_k|t_{k-1})$ become ($\Delta t = t_k - t_{k-1}$):

$$\hat{z}(t_k|t_{k-1}) = \phi\left(\frac{1-e^{-h\Delta t}}{h}\right) e^{h\Delta t} \hat{X}(t_{k-1}|t_{k-1}) + \frac{\phi f}{h}\left(\frac{e^{h\Delta t}-1}{h} - \Delta t\right) \tag{30}$$

$$P_{xz}(t_k|t_{k-1}) = \phi\left(\frac{1-e^{-h\Delta t}}{h}\right) e^{2h\Delta t} P(t_{k-1}|t_{k-1}) + \frac{\phi Q}{2}\left(\frac{e^{h\Delta t}-1}{h}\right)^2 \tag{31}$$

$$P_z(t_k|t_{k-1}) = \phi^2\left(\frac{1-e^{-h\Delta t}}{h}\right)^2 e^{2h\Delta t} P(t_{k-1}|t_{k-1}) +$$

$$+\frac{\phi^2 Q}{2h^2}\left(\frac{e^{2h\Delta t}+3-4e^{h\Delta t}+2h\Delta t}{h}\right) + R(t_k) \tag{32}$$

The coefficient h in the dynamics equation (1) of the precipitation model is negative or zero. It follows that the right-hand side of equations (30), (31) and (32) is non-negative. Consequently, $\hat{z}$, P_{xz}, P_z are non-negative at all times.

Substitution of equations (30), (31) and (32) in the updating equations (5) and (6), and use of equations (26) and (27) yields:

$$\hat{X}(t_k|t_k) = \hat{X}(t_k|t_{k-1}) + K(t_k) \cdot \left[z(t_k) - \phi\left(\frac{1-e^{-h\Delta t}}{h}\right)\hat{X}(t_k|t_{k-1})\right] -$$

$$\frac{\phi f}{h} K(t_k)\left[\left(\frac{1-e^{-h\Delta t}}{h}\right) - \Delta t\right] \tag{33}$$

$$P(t_k|t_k) = \left[1-K(t_k)\phi\left(\frac{1-e^{-h\Delta t}}{h}\right)\right]P(t_k|t_{k-1}) +$$

$$+ K(t_k)\phi\,\frac{Q}{2}\,e^{h\Delta t}\left(\frac{1-e^{-h\Delta t}}{h}\right)^2 \tag{34}$$

$$K(t_k) = \frac{\phi\left(\frac{1-e^{-h\Delta t}}{h}\right)\left[P(t_k|t_{k-1})+\frac{Q}{2}\left(\frac{1-e^{h\Delta t}}{h}\right)\right]}{\phi^2\left(\frac{1-e^{-h\Delta t}}{h}\right)^2\left[P(t_k|t_{k-1})+\frac{Q}{2h}\right] + \frac{\phi^2 Q}{h^2}\left(\Delta t-\frac{e^{h\Delta t}-1}{h}\right) + R(t_k)} \tag{35}$$

By contrast, in the ordinary case, when equation (2) is the observation equation, the updating equations become:

$$\hat{X}(t_k|t_k) = \hat{X}(t_k|t_{k-1}) + K(t_k)\cdot\left[z(t_k)-\phi\Delta t\hat{X}(t_k|t_{k-1})\right] \tag{36}$$

$$P(t_k|t_k) = \left[1-K(t_k)\phi\Delta t\right]\,P(t_k|t_{k-1}) \tag{37}$$

$$K(t_k) = \frac{\phi\Delta t P(t_k|t_{k-1})}{\phi^2\Delta t^2 P(t_k|t_{k-1})+R(t_k)} \tag{38}$$

Even for the simple case of constant f, h, and ϕ during each time step, it can be seen that the exact filter equations ((26), (27), and (33) through (35)) are much more complex than the approximate Kalman filter equations ((26), (27), and (36) through (38)). They require significantly more CPU time than the Kalman filter equations do. Therefore, an examination of the usefulness of the Kalman filter in cases of time-aggregate observations is warranted.

When the Kalman filter is used with time-aggregate observations, a model for the approximation error is needed. The simplest model that would

not modify the Kalman filter equations represents the error as an additive zero-mean, Gaussian random sequence of given variance (dependent on Δt), that is added to the observation noise $v(t_k)$. The net effect of this kind of a model is to inflate the observation noise variance $R(t_k)$.

In the following the model of approximation error is employed for the examination of the usefulness of the Kalman filter formulation in rainfall forecasting when the observations are time-aggregate quantities.

The observation error variance is given by

$$R(t_k) = \left(R_o + C_R \cdot z(t_k)\right)^2 \qquad (39)$$

where R_o is a constant component and C_R is a coefficient that allows R to be analogous to the magnitude of the observed rainfall $z(t_k)$ at time t_k.

The parameters R_o and C_R for both the exact and approximate filters for all the values of Δt considered in the study were obtained by sensitivity analysis. Only results corresponding to the best R_o and C_R estimates were compared. In this way, the estimates of R_o and C_R for the approximate filter incorporated the variance of the approximation error.

The reporting interval Δt (see equation (3)) took the values 1, 3, and 6 hours commonly used by the Environmental data and Information Service, National Oceanic and Atmospheric Administration, in the documentation of rainfall data. A storm group with a total of 302 wet hours was formed with convective and stratiform storms from Boston, Massachusetts, and it became the data base for this study. Description and statistical characteristcs of the individual storms can be found in Georgakakos and Bras (1982a).

The precipitation model parameters and the model error variance parameter Q were set equal to the best estimates obtained from the identification study of Georgakakos (1984) with hourly data.

The standard deviation, s, and the mean, m, of the one-step-ahead predicted residuals were the performance indices.

Table 1 presents the values of s and m obtained for all three of the values of Δt using 1) the filter for aggregate observations (FAO), and 2) the Kalman Filter (KF). On the same table, the best estimates of R_o and C_R for the Kalman filter are presented. For the exact filter and for all the values of Δt: $R_o = 0.05$ mm/Δt and $C_R = 0.2$. Finally, Table 1 also shows the mean μ and standard deviation σ of the observed rainfall data for all the values of Δt.

The results indicate that the calibration of R_o and C_R for the Kalman filter leads to results that are excellent in terms of the standard deviation of the prediction residuals even for $\Delta t = 6$ hours. However, a significant increase in bias is observed once Δt becomes greater than 3 hours. Those conclusions are expected given the model chosen for the approximation error. It is the increased variance of the approximation error that causes the inflation of R_o and C_R for increasing Δt.

Table 1. Filter Intercomparison Results

(Units are mm/Δt; C_R is dimensionless)

	Δt = 1 hr	Δt = 3 hrs	Δt = 6 hrs
s_{FAO}	1.8	4.8	8.4
m_{FAO}	0.1	0.1	0.2
s_{KF}	1.8	4.8	8.4
m_{KF}	0.1	0.2	1.0
R_o	0.1	2.6	7.0
C_R	0.2	0.1	2.0
μ	1.3	5.5	11.0
σ	2.4	6.0	9.9

CONCLUSIONS

A recursive estimator was presented, suitable for use with linear or linearized systems that have time-aggregate observations. The estimator differs from the commonly used Kalman filter estimator in that it requires the propagation in time of the cross-covariance between the system state at time t_1 and the system state at time t_2, with t_1 and t_2 being any two values of time within the aggregation interval of the observations.

The formulated filter and the ordinary Kalman filter were compared in terms of the prediction errors for the case of the precipitation model of Georgakakos and Bras (1984a) running with historical data for Boston, Massachusetts. For short time intervals of aggregation of the reported rainfall data ($\Delta t \leq 3$ hours), the Kalman filter with inflated observation noise variance approximated the formulated filter well in terms of prediction error mean and variance. For longer Δt the Kalman filter developed considerable prediction bias.

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