ADAPTATION OF MULTISOURCE REMOTELY SENSED DATA FOR HYDROLOGIC MODELING*

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ABSTRACT

A summary of a long-term study on the suitability of remote sensing capabilities for use in hydrologic models is reported. Seven hydrologic models used by government agencies were investigated. Also, six remote sensing capabilities were considered (precipitation estimates were not included). The effect of the models' structure and their impact on the applicability of remotely sensed information are discussed. A method of combining remotely sensed data with in situ collected observations is described and discussed. Particular attention is also given to the problem of updating the states of the models using remotely sensed information. The general results of the study indicate remote sensing information has only limited value for use with the hydrologic models in their present form. The usefulness of the remote sensing information would be greatly enhanced with minor modification of the models.

1. INTRODUCTION

In the past decade or so, many remote sensing technologies have been developed. They are capable of providing information on land characteristics such as topography, vegetative cover, soil moisture, water equivalent of the snow cover, and others. To some, these new technologies seemed to offer a panacea for many of the problems associated with the collection, processing, and analysis of data. Indeed, the remotely sensed data of terrain characteristics and vegetative cover have proven to be of considerable value, at least as useful as data obtained using standard technology and, in some cases, are obtained at a great saving in time and expense.

However, remotely sensed measurements of hydrological variables, such as soil moisture or water equivalent of the snow cover, have found very limited use in operational hydrology. There are a number of reasons for that situation. Limited accuracy, high volume of data requiring a lot of computer power, and variable frequency of observations all constrain the implementation of the new technology. Yet another factor, probably the most important one, is that the structure of existing operational hydrologic models is not suitable for straightforward applications of remotely sensed information. There is not a one-to-one correspondence between the hydrologic states as measured in the real world by the new technology and states as represented in the mathematical models of hydrologic processes presently in use. This is due to the fact that these models were developed prior to the availability of

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remotely sensed measurements. To add to the problem of non-correspondence, remote sensors may be sensitive and respond to more than the hydrologic variable of interest. For example, microwave sensors may infer both soil moisture and vegetative roughness.

Another problem is how to effectively combine the remotely sensed measurements with the existing in situ measurements in order to extract maximum information on the state of hydrological variables. Hydrological models generally represent the average conditions for a specific area such as a river drainage basin. Most models are lumped parameter and lumped input models and require areal average values for input data or for updating the states of the model. At the present, areal averages of hydrological variables are determined from standard point (in situ) measurements using various estimation techniques. Incorporation of remotely sensed measurements can substantially improve the accuracy of the estimated areal averages, especially under sparse ground network conditions, but it clearly requires changes to the standard in-situ-only estimation procedures.

This paper presents a summary of a long-term study, sponsored by NASA, that examined the suitability of operational hydrologic models to incorporate remotely sensed information and evaluated strategies for using remotely sensed information in the models. The detailed results of the research have been reported by Peck et al. (1981a, 1981b, 1983, 1984) and Johnson et al. (1982).

The interrelations among the problems described above can be presented as a diagram (Figure 1).

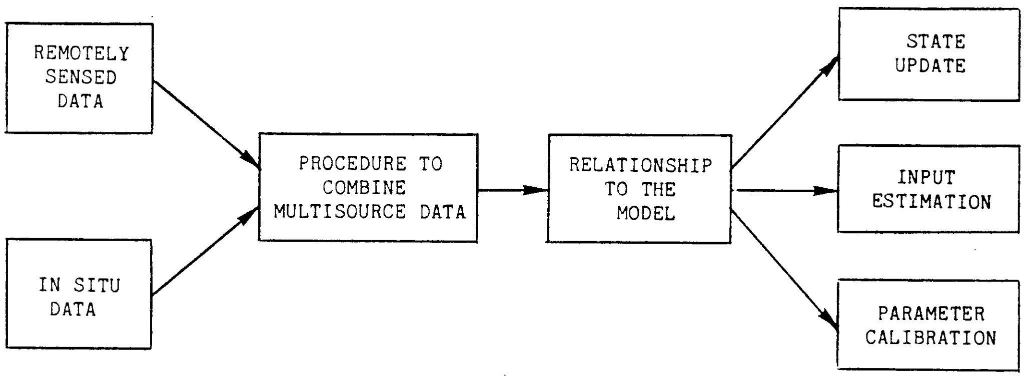

Figure 1. Remote sensing and hydrologic modeling--problems to be solved.

Analysis of Figure 1 makes it clear that a series of efforts were required. The first area of research focus was describing the relationship between hydrologic models and remotely sensed data in general terms. In order to do this, seven commonly used hydrologic models were analyzed in detail based on a common framework.

2. REVIEW OF HYDROLOGIC MODELS

The following five hydrologic models commonly used by Federal agencies were selected for review:

- Antecedent Precipitation Index (API)
- National Weather Service River Forecast System (NWSRFS)
- Storage, Treatment, Overflow, Runoff Model (STORM)
- Stanford Watershed Model IV (SWM)
- Streamflow Synthesis and Reservoir Regulation (SSARR)

Two other hydrologic models were reviewed. The Chemical, Runoff, and Erosion from Agricultural Management System (CREAMS) model was included because of its extensive use in the field of agriculture. The NWSRFS Snow Accumulation and Ablation model was also selected since it is commonly used with several of the basic hydrologic models.

Since the models were developed by different groups over a period of several years, it was necessary to analyze the models within a common framework that would also be suitable for considerations related to remote sensing technologies.

Modern systems analysis terms were adopted for that purpose and the inputs, processes, states, decision points, parameters, and outputs were identified for all the models. The complete findings of this phase were presented in an interim NASA report (Peck et al., 1981a).

3. REMOTE SENSING CAPABILITIES

The next question that arises is what are the capabilities of remote sensing as far as hydrologic modeling is concerned? Before an answer is given, a definition of remote sensing is required. For this study, remote sensing is taken to mean estimating the average value of a variable over some areal extent by examining the characteristics of the radiation from that area. The study focused on satellite-borne sensors, although the methodology is applicable to other platforms. Emphasis has been placed on those variables that have at least an intuitive connection to portions of existing hydrologic models, and that experience has shown can be remotely sensed with some degree of success.

Remote sensing capabilities of six variables were reviewed for use with the operational hydrologic models. These were:

- Areal extent of snow cover
- Impervious area
- Land cover
- Areal extent of frozen ground
- Soil moisture
- Water equivalent of snow cover

Of the six variables, only the first three can be considered to have operational measurement techniques at the present time. All three may be obtained through analysis of LANDSAT images.

None of the other three variables (areal extent of frozen ground, water equivalent of snow cover, and soil moisture), with the possible exception of frozen ground, can currently be measured effectively from satellites. All three are awaiting further extensive research.

In spite of the recognized potential value of remotely sensed data for water resource management, the Federal agencies responsible for river forecasting and water supply prediction are not using such data as a primary operational data base. To examine the reasons for this and to suggest improvements in remote sensing applications, it was necessary to couple the information from the review of the hydrologic models and the review of remote sensing capabilities.

4. STRATEGIES FOR USING REMOTELY SENSED DATA

There are several strategies for using remotely sensed data or, indeed, any type of data, in hydrologic models. The first is to estimate the inputs to the models. The second is to update the states of the models to be consistent with the observed data. A third strategy for using remotely sensed data is for calibration of the parameters of the model. In traditional applications, parameters are estimated only once based on current topographic and land cover data and hydrometeorological data for some calibration period. It is certainly possible to recalibrate a model based on new data. For example, remote sensed

observations of changes in land cover can be used for recalibrating of models, thus blurring the distinction between updating and calibrating the model.

Each of the seven hydrologic models was analyzed to determine the usefulness of the six selected remotely sensed variables for the present configuration of the model, with minor modification or adaptation. The most obvious conclusion from the analysis for the basic soil moisture accounting models is that, in their present configuration, there is not a significant potential for using the six target remotely sensed observations for input, updating, or calibration. With minor modification, some of the models can take advantage of the significant potential for applying remotely sensed data to hydrologic modeling.

A careful reader will have noticed that precipitation was excluded from consideration in spite of the fact that the ability to measure precipitation characteristics remotely has received more attention in the past than the ability to measure any of the other six variables. Remote sensing of precipitation would have direct use in hydrologic modeling since precipitation is normally the primary input to hydrologic models (although it may be a state as well). No model modifications would be required to benefit from such measurements. However, the problem that remote sensing of precipitation shares with the other six variables is that of how to optimally combine the data with the in situ collected measurements in order to increase the accuracy and reliability of the areal average estimation.

5. MULTIPLE SENSOR DATA ANALYSIS

The main reason for combining measurements from remote sensing with other measurements (either remotely sensed or in situ) of the same or related variables, is to maximize the use of the information contained in each of the data types. Remotely sensed data have significant measurement errors; however, they provide areal extent information that is not provided by the in situ measurements. This is due to different sampling geometries of various sensors and physical principles of the measurement technology.

Mathematically, if $Z(\underline{u},t)$, $\underline{u}=(x,y)$ is process of interest at location $\underline{u}$ and time t, then point observations z_i of process Z can be expressed as:

$$z_i(\underline{u}_i,t) = Z(\underline{u}_i,t) + \varepsilon(\underline{u}_i,t) \qquad \text{for } i=1,2,\ldots,N \tag{1}$$

where $\varepsilon(\underline{u}_i,t)$ is measurement error that has a random nature. On the other hand, most of remotely sensed data, z_R, represent integrated (or averaged) values:

$$z_R(\underline{u}_j,t) = \frac{1}{A_j} \int_{A_j} Z(\underline{u}_j,t)\, dA + \varepsilon_R(\underline{u}_j,t) \qquad j=1,\ldots,K \tag{2}$$

The measurements $z_i(\underline{u}_i,t)$ and $z_R(\underline{u}_j,t)$ collected at the time t can represent instantaneous or accumulated data. Given data sets with observations of the type (1) and/or (2) the problem is to estimate:

$$Z_\Omega = \frac{1}{\Omega} \int_\Omega Z(\underline{u},t)\, d\Omega \tag{3}$$

where usually $\Omega \gg A_j$ and $A_j \subset \Omega$ and Ω is the basin of interest.

The problem is difficult, but several approaches are possible. In any case, for a technique to be of maximum value for estimating areal averages of hydrological variables from all available measurements, the following criteria should be satisfied:

(a) It should not be dependent on particular measurement technology.

(b) It should be an objective technique.

(c) It should produce an estimate, regardless of the mix of data available at any one time.

(d) It should explicitly recognize the sampling geometry of the data.

(e) It should explicitly recognize differences in measurement accuracy of different technologies.

(f) It must produce some estimate of the accuracy of the areal estimate.

Some techniques that meet these criteria have been developed in the area of radar-rainfall data analysis (e.g., Krajewski and Crawford (1982) and Krajewski and Hudlow (1983)). Other possible approaches are using universal co-kriging and disjunctive co-kriging (for a study of universal kriging and disjunctive kriging, see Puente and Bras, 1982). All these techniques, however, are extremely demanding as far as computational requirements are concerned and that makes them less attractive.

As an alternative, the correlation area method has been designed by Johnson et al. (1982). The method is a heuristic approach that takes liberties at certain points with a more theoretically correct technique in the interest of simplicity and operational capability. It also meets all the criteria listed above for estimating areal averages from data of various sampling characteristics. The algorithm assigns weights to each data type based on their accuracy and spatial influence. According to the method, an estimate $\hat{Z}_\Omega$ is computed as linear combination of various observations:

$$\hat{Z}_\Omega = \sum_{\ell=1}^{L} \sum_{i=1}^{N_\ell} \alpha_{\ell i} \, z_{\ell i} \tag{4}$$

where: L - is the number of observation types (point measurements, areal measurements, line measurements).

N_ℓ - number of observations from ℓth observation type

$\alpha_{\ell i}$ - weight assigned to the ith observation of the ℓth sensor

$z_{\ell i}$ - ith observation value of the ℓth sensor

The weights $\alpha_{\ell i}$ depend on the information value of each measurement which, in turn, is dependent upon how much area is assigned to that measurement and on how well that information correlates with the true variable in that area. The weight $\alpha_{\ell i}$ is equal to its correlation area $A_{\ell i}$ divided by the sum of all the correlation areas for the basin:

$$\alpha_{\ell i} = \frac{A_{\ell i}}{\sum_{\ell=1}^{L} \sum_{i=1}^{N_\ell} A_{\ell i}} \tag{5}$$

where correlation area $A_{\ell i}$ is defined as

$$A_{\ell i} = \int_{A^*_{\ell i}} \rho_{\ell i}(h) \, dA^*_\ell \quad , \; A_\ell \subset \Omega, \quad h = |\underline{u}_i - \underline{u}_j|, \quad \underline{u}_j \in A^*_{\ell i} \tag{6}$$

where $\rho_{\ell i}(h)$ is a correlation function of the ith observation of the ℓth sensor with the true value at a distance h, and $A^*_{\ell i}$ is the region defined by the points $\underline{u}_j$ such that:

$$\rho_{\ell i}(h) > \rho_{km}(h^*); \; h^* = |\underline{u}_m - \underline{u}_j|; \; k=1,2,\ldots,L; \; m=1,2,\ldots,N_k$$

Thus, the area $A^*_{\ell i}$ is the region within the basin where the ith observation of the ℓth sensor is more highly correlated with the true variable than is any other observation. The correlation area $A_{\ell i}$ is then the weighted average of the correlation itself over the region $A^*_{\ell i}$. The procedure is illustrated graphically in Figure 2 which shows the portion of the basin assigned to an areal sampling technology and to hypothetical line and point sampling technologies.

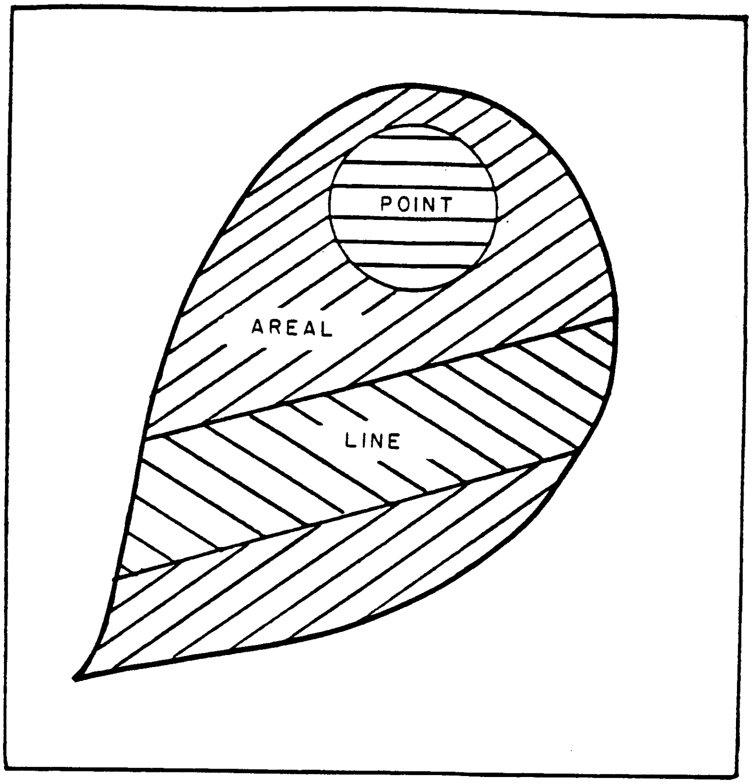

Figure 2. Illustration of correlation areas.

A measure, $\omega(\hat{Z}_\Omega)$, of the overall accuracy of the final estimate $\hat{Z}_\Omega$ is obtained by dividing the sum of the correlation areas $(A_{\ell i})$ by actual area of the basin B.

$$\omega(\hat{Z}_\Omega) = \frac{\sum_{\ell=1}^{L} \sum_{i=1}^{N_\ell} A_{\ell i}}{B} \tag{7}$$

By definition, $\omega(\hat{Z}_\Omega)$ must be between 0 and 1 and can be loosely referred to as the correlation of the estimated value $\hat{Z}_\Omega$ with the actual average Z_Ω of the variable Z over the catchment.

The algorithm has an intuitively reasonable behavior; more accurate measurements get a larger weight than less accurate observations, and samples in data sparse regions tend to get larger weight than those in data rich regions. Correlation areas which determine the weights

depend on the spatial correlation functions for each sensor type. Thus, some parameters that relate both to the accuracy of particular measurement technologies and to the correlation decay in space need to be estimated prior to implementation of the method. These parameters are:

(a) The correlation of each observation type with the corresponding true value of a variable (a_ℓ, $\ell=1,\ldots,L$).

(b) The rate at which the correlation decays in space (typically it is assumed that the correlation is only a function of distance), (b_ℓ, $\ell=1,\ldots,L$).

These two parameters define a simple isotropic correlation model, for example:

$$\rho_{\ell i}(h) = a_\ell \cdot \exp\,(b_\ell \cdot h) \tag{8}$$

with h defined as above.

For most technologies, with perhaps the exception of in situ measurements, it is not easy to obtain the values of these parameters. For ground sampling technologies, it is expected that the correlation between point measurement and the true values is generally "large," i.e., a_1 (if $\ell=1$ corresponds to ground measurement) is approximately 0.9 or better. For remote sensing technologies (line-type and areal), a number of expensive field experiments would be required to estimate those parameters. In general, it is expected that it would be somewhat less than 0.9.

As far as estimation of the decay parameter b_ℓ is concerned, three approaches are possible: the use of historical data, real time estimation, and a conceptual model. Of these three, the historical approach and the conceptual model approach offer the most promise for operational estimates of decay parameters:

Historical Data. Historical data on soil moisture or snow water equivalent can be analyzed to estimate decay parameters. Then its value can be assumed to apply to current conditions. The difficulties with this approach are procuring a historical data base and developing a procedure to "stratify" the data for different values related to some easy-to-identify properties of current conditions.

Real Time Data. If enough pointwise data are available in real time, a value of the decay parameter can be estimated for the current condition. Using remotely sensed data for this approach is rather difficult because of the sample-averaging properties of this type of data.

Conceptual Model. The conceptual model approach can be illustrated by considering soil moisture as the product of two random fields, the field capacity, and the fraction of field capacity that is filled. The variability of the field capacity can be related to a soil map. If a conceptual model is developed to relate the statistics of the "fraction of field capacity" to some easy-to-estimate parameters (e.g., the antecedent precipitation index), it may be possible to estimate a correlation decay for the soil moisture without actual measurements of soil moisture.

The correlation area method has been implemented in a computer code, but has not been subjected to extensive testing. The lack of adequate "ground truth" information precludes a direct approach to this testing. As the final phase of the long-term research study, an indirect evaluation was planned, but has not yet been conducted due to funding limitations. This evaluation would determine the degree of improvement in streamflow predictions obtained by incorporating remotely sensed measurements (using the correlation area method) in hydrological models as compared with predictions using only standard measurements.

Another way the method could be tested is via a numerical simulation experiment, similar to the one described by Krajewski and Hudlow (1983) for testing radar and rain gage rainfall data merging procedures.

According to the diagram in Figure 1, once the remotely sensed data are combined with other types of data to produce the best estimate of hydrologic variables of interest, the next step is to find the relationship between that variable and the elements of the hydrologic model.

6. STRUCTURE OF THE MODELS

The real-world state of a basin is extraordinarily complex. It is a function of the actual distribution in time and space of interaction of weather with geology and biota of the basin. Compared with the complex processes of the real world, the hydrologic processes in the models are simple and provide inexact estimation of the states in the real world. Thus, there is uncertainty in the models. Remotely sensed observations, as well as standard measurements, are also imperfect measures of the states of the real world due to measurement errors. To make things worse, they have a less than perfect relationship with modeled states.

For example, a remotely sensed measurement of soil moisture may be considered to represent only the total water in the top 10 cm of the soil. Thus, such a measurement for 10 cm does not directly relate to the total soil moisture represented by the combined soil moisture states of the upper zone of the hydrologic model which often covers more than the top 10 cm. The relative depth of the model state can be an order of magnitude greater than the 10 cm soil moisture measurement in the real world.

For the models where there is not a single state that relates to a particular field measurement, either the model must be restructured or a complex measurement model must be developed to relate the measurement to the modeled state. A detailed analysis of the structure of operational hydrologic models and proposed modifications is described by Peck et al. (1983).

When an observation is made, there are two sources of information on the states of the nature: one from the real-world observations and one from the model. The basic idea behind updating a model is to obtain an updated state that better reflects the real world. This is done by using knowledge from both sources of information to compute the so-called gain. The gain procedure weights the relative accuracy of the observation and the model estimates--more vigorously modifying the model's estimates when the observations are relatively more accurate, and vice versa. The updated state (i.e., soil moisture storage or water equivalent of the snow cover) must be chosen somewhere between the values indicated by the model alone and the observations alone. The conceptual framework for updating is presented in Figure 3.

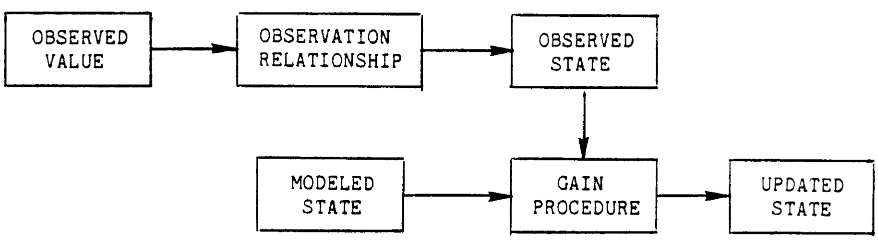

Figure 3. Conceptual framework for updating the states.

There are two possible approaches to computing the gain for updating the states of a hydrologic model based on remotely sensed and other observations: either a full probabilistic approach implemented in the form of a Kalman filter, or a heuristic approach which contains some probabilistic elements.

The full probabilistic approach accomplishes the gain and model adjustment process simultaneously. In the heuristic procedure, the gain and adjustment are done sequentially.

The adjustment process modifies all of the model states to be consistent with the updated value produced by the gain procedure.

Heuristic Approach to Updating. It is possible to develop a variety of heuristic techniques for updating hydrologic models. For example, the model could be updated with a constant gain parameter, e.g., the updated state is always halfway between the modeled state and the observed state. The gain computation could be outside of the model entirely, e.g., the updated state is computed by a hydrologist based on his or her belief in the modeled and measured states. Peck et al. (1983) describe the heuristic algorithm that is especially designed to mesh with the correlation area method in which the measure of estimate accuracy is given in terms of $\omega(\hat{Z}_\Omega)$ of Eq. (7). In this heuristic approach, the relative accuracy of the modeled state is assumed constant while the relative accuracy of the observed state varies with the mix of observations available at any time.

Full Probabilistic Approach. There are various methods for applying a full probabilistic approach to updating mathematical models. One that has been used in hydrology is the Kalman filter approach (see Gelb, 1974). A number of investigations have applied modern stochastic process theory to the NWSRFS. These studies originally focused on updating the soil moisture model using observed discharge (see Kitanidis and Bras, 1980a and 1980b; TASC, 1980; TASC, 1981). Later studies focused on investigating maximum likelihood parameter identification (Restrepo-Posada and Bras, 1982), more sophisticated channel inflow routing procedures (Georgakakos and Bras, 1980), and inclusion of a precipitation model (Georgakakos and Bras, 1982).

In order to apply a Kalman filter approach, it is first necessary to represent the hydrologic model in the form of a stochastic state-space model, i.e., a set of simultaneous differential (or integral) equations with error terms. Two sets of equations are required: one representing the relationship of any observations to the modeled states, and the second representing the dynamics of the model states themselves. If the model is nonlinear, it is necessary to linearize it.

Since the Kalman filter procedure continuously accounts for variability in the accuracy of the modeled states, an updating procedure based on this approach is sensitive to both variation in model accuracy and variations in the observation mix. It is necessary to relate the correlation value $\omega(\hat{Z}_\Omega)$ to the observation error variance. A derivation of this approach is described by Peck et al. (1983) for the Sacramento Soil Moisture Accounting procedure augmented by addition of two state variables and changes in the model dynamics so that a single state variable in the augmented model has a one-to-one relationship to fairly shallow (5-10 cm) observations of soil moisture.

7. CONCLUSIONS

The basic conclusions from the study reported by Peck et al. (1981a, 1981b) and Johnson et al. (1982) are:

- Hydrologic models in their present configuration do not have a significant potential for using remotely sensed information (excepting precipitation estimates).
- The use of remotely sensed data could be significant with minor modification of existing models, although it takes substantial work to do that.
- Hydrologic modeling can be improved through the development of new generation of models or subroutines for existing models which recognize the characteristics of new remote sensing capabilities.
- Remotely sensed data should be combined with in situ data and with the hydrologic models in order to be of maximum benefit.

8. ACKNOWLEDGMENTS

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