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A SIMULATION STUDY OF RECURSIVE PARAMETER ESTIMATION TECHNIQUES FOR A STOCHASTIC-DYNAMIC FLOOD ROUTING MODEL

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Abstract. Recursive parameter estimation of a real-time, stochastic, flood-routing model is extensively studied. Following common hydrologic practice, flood routing is based on a series of nonlinear reservoirs. In order to utilize linear Kalman filtering and optimal estimation techniques for parameter estimation, statistical linearization of the nonlinear functions of the parameters is performed. The methodology is suitable for nonlinear multidimensional functions of random nonstationary processes. The linearization coefficients depend not only on the parameter means but on the whole probability density of the parameters. Implementation of the parameter estimation procedure requires the determination of the expected values of nonlinear multi-dimensional functions. A simple analytical method, called the Taylor-Gauss approximation, developed for the determination of the expected values, is compared to exact numerical integration procedures. The recursive estimation methodology based on statistical linearization is compared to an extended Kalman filter procedure based on ordinary linearization. Comparison of the different recursive parameter estimation techniques is done through a numerical simulation experiment. Sensitivity analysis with respect to measurement errors and sampling frequency is performed. Evaluation of the recursive parameter estimation techniques is based both on measures of accuracy and on measures of the CPU cost of each methodology.

Keywords. Parameter estimation; nonlinear filtering; modeling.

INTRODUCTION

Uncertainty in hydrologic systems is great because of the wide range of spatial and temporal scales of the hydrologic processes. Consequently, the development of conceptual models with parameters estimated from input-output data has flourished in recent years.

One of such models, the nonlinear reservoir, has been extensively used in hydrologic studies as a basic component for the generation of runoff (e.g., Mein et al., 1974; Peck, 1976; Kitanidis and Bras, 1980). In particular, flood routing hydrologic models rely heavily on it to simulate the storage effects of the various channel reaches (e.g., Mein et al., 1974; Georgakakos and Bras, 1982).

Denoting by I(t) the inflow to a channel reach and by O(t) the outflow at time t, the continuity equation yields:

$$\frac{dX(t)}{dt} = I(t) - O(t) \tag{1}$$

where X(t) is the volume of water in store in the reach. To complete the formulation a relationship between the storage X(t) and the rates I(t) and O(t) is required. A power function of the type:

$$O(t) = a \cdot X(t)^m, \tag{2}$$

where a and m are model parameters, has been found useful in runoff and streamflow routing studies (e.g., Peck, 1976; Unny and Karmeshu, 1984; Georgakakos and Bras, 1982). Therefore, the dynamic equation that governs the evolution of the state X(t) in time is:

$$\frac{dX(t)}{dt} = I(t) - a \cdot X(t)^m \tag{3}$$

In most of the real-world situations, the data available for model calibration (i.e., determination of the parameters a and m) are observations of the inflow I(t) and of the outflow O(t). Therefore, parameter estimation techniques based on input-output data are necessary.

In this paper we examine the utility of recursive parameter estimators in the parameter estimation problem associated with the nonlinear reservoir Eq. (2) and (3). The advantage of the recursive parameter estimators over the non-recursive ("batch") ones is that they can be used in real-time estimation problems, while the batch estimators, which rely on some gradient optimization procedure, process the whole time series a posteriori and thus have to be used "off-line". In addition, the recursive estimators process the time-series of data only once, while the batch estimators process the time series several times in order to find the optimum of the optimization criterion.

In general, non-recursive estimators at the expense of considerable computation time will produce more accurate estimates. However, the success of the recursive estimators highly depends on the type of nonlinearity involved and the sampling frequency of the data.

This work investigates the performance of the Extended Kalman Filter and of two linear filters, which are based on statistical linearization, in the estimation of the nonlinear reservoir parameters. The effects of the sampling frequency and of the expected measurement errors on the convergence properties of the estimators are examined in detail.

MODEL STATE-SPACE FORM

The state-space form of the model in Eq. (2) and (3) is

$$\frac{dX(t)}{dt} = I(t) - a \cdot X(t)^m + w(t) \tag{4}$$

$$Z(t_k) = a \cdot X(t_k)^m + V(t_k) \; ; \; k=1,2,\ldots \tag{5}$$

where: $w(t)$ is a white noise process that models errors that are due to erroneous parameters, erroneous model structure and erroneous observations of the inflow $I(t)$. It is assumed Gaussian with mean zero and variance parameter Q.
$Z(t_k)$ is the observation of the outflow at time t_k, $k=1,2,\ldots$
$V(t_k)$ is a white noise sequence that models errors that are due to erroneous measurement of the outflow $O(t)$. It is assumed Gaussian with mean zero and variance R.

The system of the Eq. (4) and (5) is nonlinear in the state $X(t)$ and in the parameters a and m. In order to use a filtering algorithm to sequentially process the observations $Z(t_k)$ one has to linearize the system. In practice, two linearization procedures are employed. An ordinary linearization based on the first order terms of a Taylor series expansion about the current estimate of $X(t)$, which leads to the Extended Kalman Filter (EKF) formulation, and a statistical linearization procedure which leads to statistically linearized filtering algorithms. Both procedures are described in Bras and Rodriguez-Iturbe (1984) and are studied in the following.

THE LINEARIZATION TECHNIQUES

Given the nonlinear function $f_1(a,m,X) = a \cdot X^m$, linear approximations of it are sought of the type:

$$f_a(a,X,m) = N_o + N_a \cdot a \quad N_m \cdot m + N_x \cdot X \tag{6}$$

In the following, we denote the current estimates of a, X and m by $\hat{a}$, $\hat{X}$ and $\hat{m}$, respectively.

Ordinary Linearization

The ordinary linearization by Taylor series expansion about the point $(\hat{a},\hat{m},\hat{X})$ gives the following expressions for the coefficients N_a, N_m, N_x, N_o

$$N_a = \frac{\partial f_1(\hat{a},\hat{m},\hat{X})}{\partial a} \tag{7}$$

$$N_m = \frac{\partial f_1(\hat{a},\hat{m},\hat{X})}{\partial m} \tag{8}$$

$$N_x = \frac{\partial f_1(\hat{a},\hat{m},\hat{X})}{\partial X} \tag{9}$$

$$N_o = f_1(\hat{a},\hat{m},\hat{X}) - N_a \cdot \hat{a} - N_m \cdot \hat{m} - N_x \cdot \hat{X} \tag{10}$$

Statistical Linearization

When the linearization coefficients in Eq. (6) are obtained from a minimization of the expected value of the approximation error squared $e^2 = (f_1(a,m,X) - f_a(a,m,X))^2$ with respect to N_a, N_m, N_x, N_o, the linearization is called statistical linearization (Gelb, 1974, Georgakakos and Bras, 1982). In such a case, the coefficients of linearization are the solution vector $\underline{N}$ of the algebraic vector equation:

$$\underline{P} \cdot \underline{N} = \underline{\beta} \tag{11}$$

where

$$\underline{N} = [N_a \; N_m \; N_x \; N_o]^T \tag{12}$$

$$\underline{P} = \begin{bmatrix} E\{a^2\} & E\{am\} & E\{aX\} & E\{a\} \\ E\{ma\} & E\{m^2\} & E\{mX\} & E\{m\} \\ E\{Xa\} & E\{Xm\} & E\{X^2\} & E\{X\} \\ E\{a\} & E\{m\} & E\{X\} & 1 \end{bmatrix} \tag{13}$$

$$\underline{\beta} = [\; E\{a^2X^m\} \quad E\{amX^m\} \quad E\{aX^{m+1}\} \quad E\{aX^m\} \;]^T \tag{14}$$

with $E\{\cdot\}$ denoting expectation and upperscript "T" denoting transpose of a vector quantity.

Assuming that the inverse $\underline{P}^{-1}$ of matrix $\underline{P}$ exists, the elements of vector $\underline{N}$ are obtained from

$$\underline{N} = \underline{P}^{-1} \cdot \underline{\beta} \tag{15}$$

The basic difficulty associated with the use of statistical linearization stems from the fact that one needs to compute the expected values of the nonlinear functions:

$$f_1 = a \cdot X^m$$

$$f_2 = a^2 \cdot X^m$$

$$f_3 = a \cdot m \cdot X^m$$

$$f_4 = a \cdot X^{m+1}$$

Under the usual assumption of normality for a,m and X, one needs to compute the integrals

$$I_1 = \iiint_{-\infty}^{+\infty} aX^m \cdot p(a,m,X) \, da \, dm \, dX$$

$$I_2 = \iiint_{-\infty}^{+\infty} a^2X^m \cdot p(a,m,X) \, da \, dm \, dX$$

$$I_3 = \iiint_{-\infty}^{+\infty} amX^m \cdot p(a,m,X) \, da \, dm \, dX$$

$$I_4 = \iiint_{-\infty}^{+\infty} aX^{m+1} \cdot p(a,m,X) \, da \, dm \, dX$$

where $p(a,m,X)$ is the joint probability density of a,m and X assumed Gaussian.

The most accurate determination of I_1, I_2, I_3 and I_4 is accomplished by direct numerical integration. However, this is a costly procedure, especially when the integration computations are performed repeatedly as is the case in recursive parameter estimation procedures.

Georgakakos and Bras (1982) proposed the Taylor-Gauss methodology to avoid numerical integration. According to that methodology, the nonlinear functions f_1, f_2, f_3 and f_4 are expanded in a

Taylor series expansion about the means $\hat{a}$, $\hat{m}$, and $\hat{X}$. Then, given the Gaussian joint density p(a,m,X), the expected value of each term in the series is determined. The higher order moments of a Gaussian density are given as functions of the first and second moments. Therefore, the net result contains only terms with the first two moments of a, m and X. The details of the formulation are given in Georgakakos and Bras (1982).

As an example, in the following we use the Taylor-Gauss methodology to approximate the expected value of the nonlinear function f_1.

Expansion of f_1 in a Taylor series about the point $(\hat{a},\hat{m},\hat{X})$ with second order terms retained gives:

$$
\begin{aligned}
a\cdot X^{m} \cong{} & \hat{a}\cdot\hat{X}^{\hat{m}} + \hat{X}^{\hat{m}}(a-\hat{a}) + \hat{a}\cdot\hat{m}\cdot\hat{X}^{(\hat{m}-1)}(X-\hat{X}) + \\
& + \hat{a}\cdot\hat{X}^{\hat{m}}\cdot\ln(\hat{X})\cdot(m-\hat{m}) + \frac{1}{2}\,\hat{a}\cdot\hat{m}\cdot(\hat{m}-1)\cdot\hat{X}^{(\hat{m}-2)}(X-\hat{X})^{2} + \\
& + \frac{1}{2}\,\hat{a}\cdot\hat{X}^{\hat{m}}\ln^{2}(\hat{X})(m-\hat{m})^{2} + \hat{m}\cdot\hat{X}^{(\hat{m}-1)}(a-\hat{a})(X-\hat{X}) + \hat{X}^{\hat{m}}\cdot \\
& \ln(\hat{X})(a-\hat{a})(m-\hat{m}) + \hat{a}\cdot\hat{X}^{(\hat{m}-1)}\left(1+\hat{m}\cdot\ln(\hat{X})\right)(X-\hat{X})(m-\hat{m})
\end{aligned}
\tag{16}
$$

Taking expectations of both sides in Eq. (16) results in:

$$
\begin{aligned}
E\{aX^{m}\} \cong{} & \hat{a}\hat{X}^{\hat{m}} + \frac{1}{2}\,\hat{a}\hat{m}(\hat{m}-1)\hat{X}^{(\hat{m}-2)}\,\sigma_{X}^{2} + \frac{1}{2}\,\hat{a}\hat{X}^{\hat{m}}\ln^{2}(\hat{X})\sigma_{m}^{2} + \\
& \hat{m}\hat{X}^{(\hat{m}-1)}\sigma_{aX}^{2} + \hat{X}^{\hat{m}}\ln(\hat{X})\sigma_{am}^{2} + \hat{a}\hat{X}^{(\hat{m}-1)}\left(1+\hat{m}\ln(\hat{X})\right)\sigma_{Xm}^{2}
\end{aligned}
\tag{17}
$$

where: σ_X^2, σ_a^2, σ_m^2 are the current variances of X, a, and m,

and σ_{aX}^2, σ_{am}^2, σ_{Xm}^2 are the covariances of a and X, a and m, X and m, respectively.

In the case when higher order terms were retained in the expansion, Eq. (16), the higher order, even, joint moments of a, X, m, can be replaced by functions of the second order moments of the same variables based on the assumption of joint normality (Georgakakos and Bras, 1982). All the odd joint moments of the Gaussian a, X, m, are equal to zero.

Thus, the integral I_1 has been replaced with the approximate expression in Eq. (17).

The Taylor-Gauss methodology has given encouraging results in previous applications (Georgakakos and Bras, 1980; Georgakakos and Bras, 1982). Analysis of nonlinear power functions suggested that a few terms in the Taylor series expansion are necessary to obtain satisfactory results.

Note that the EKF uses only the first term on the right-hand side of Eq. (17) and, as a result, the expected value of the nonlinear function in the EKF formulation does not depend on the covariance properties of a, X and m.

RECURSIVE PARAMETER ESTIMATION

In the use of filtering theory for recursive parameter estimation, the system dynamics equation is augmented by first-order differential equations that describe the evolution of parameters in time. When the parameters of the system remain constant in time, the parameter differential equations become

$$\frac{d\underline{\alpha}}{dt} = \underline{0}$$

where $\underline{\alpha}$ is the vector of system parameters and $\underline{0}$ is a vector whose elements are zeros.

The system dynamics equation augmented by the parameter equations constitute a system of nonlinear (in general) differential equations. In order to apply the powerful techniques of linear filtering theory, some kind of linearization is required. The EKF uses ordinary linearization of the nonlinear functions based on a Taylor series expansion about nominal values of states and parameters with only the linear terms in the expansion retained. Within the EKF framework, the states and parameters are viewed as random processes with moments estimated recursively by the EKF. Furthermore, the nominal values of states and parameters in the Taylor series expansion are taken equal to the current estimates of those variables.

Several applications of the EKF as a parameter estimator have been reported in the literature (e.g., Nelson and Stear, 1976; Aidala, 1977; Chang, Whiting and Athans, 1977; Bowles and Grenney, 1978; Bras and Posada, 1980).

The main conclusions are that success depends on the type of nonlinearity and on the choice of the system model error statistics (i.e., the Q matrix). In many cases the estimates were biased even for a large number of observation points.

Georgakakos and Bras (1982) developed a recursive estimator that uses higher order terms in the Taylor series during the linearization process, thus reducing the linearization bias. It is based on the statistical linearization concepts presented in the previous section. The few applications of the estimator appeared encouraging.

In this work we studied the properties of the recursive estimators: EKF and statistical linearization filters (numerical integration and Taylor-Gauss method) for various conditions imposed on the system model error statistics, the sampling interval of the observations and the observations error. The analysis is based on simulation studies where: 1) time series of observations are generated by the model using the true parameters, prespecified sampling interval and noise statistics, and 2) the estimators are used to obtain the true parameters from the time series of observations.

DESIGN OF SIMULATION EXPERIMENT

The technique used to test the ideas of this paper is numerical simulation. As opposed to testing with real world data, simulation offers the opportunity to study the robustness of the various parameter estimators when various system statistics and the system input characteristics take a wide range of values. In fact, numerical simulation experiments can lead to conditions that the data should obey for maximum identifiability, thus setting the guidelines for data collection strategies and data quality control.

In this work we will concentrate on the generic element of many hydrologic models, that is the nonlinear reservoir. Work is underway to extend the present results to a series of nonlinear reservoirs where identification of the number of them (order of model) from input-output data is of central importance.

Estimation of both a and m is performed using the EKF, a statistically linearized filter based on numerical integration (NSLF), and a statistically linearized filter based on the Taylor-Gauss methodology (TGSLF).

The parameters are estimated based on a train of input hydrographs and on generated noisy output hydrographs. Observations of input and output are sampled at various time intervals ranging from 15 minutes to 6 hours duration.

Sensitivity of the parameter estimation techniques with respect to the level of the model error variance parameter, Q, and to the observation error variance, R, is studied.

The primary performance criterion is convergence to the true parameter values. In addition, the mean squared error, the normalized residual standard error, and the CPU time on a PR1ME 750 computer with a PRIMOS operating system were studied.

The steps followed during the simulation experiments are given next.

STEP 1. Select the sampling interval Δt, and the input hydrograph characteristics (such as time to peak and ratio of peak flow to base flow). Note that the inverse Gaussian function (Johnson and Kotz, 1970) was used to generate the input hydrographs.

STEP 2. Integrate the dynamics, Eq. (3), from 0 to Δt using baseflow initial conditions and a train of identical input hydrographs.

STEP 3. Specify the variance of the model error in the interval $(0,\Delta t)$ as (Gelb, 1974):

$$Q_{\Delta t} = \int_0^{\Delta t} \Phi^2(\Delta t,\tau)\cdot Q\cdot d\tau$$

where $\Phi(\Delta t,t)$ is the transition matrix of the linearized system. For the purposes of this work the interval $(0,\Delta t)$ was subdivided to 5-minute intervals and $\Phi(t,\tau)$ was computed in each one of them from

$$\Phi(t,\tau) = e^{-F(t-\tau)}$$

with $F = \hat{a}\cdot\hat{m}\cdot\hat{X}^{(\hat{m}-1)}$ and "^" denoting current estimate at each 5-minute interval.

STEP 4. Generate a Gaussian random number with mean zero and variance $Q_{\Delta t}$, and add this number to the state variable X resulted from the integration in STEP 2.

STEP 5. Based on the modified X computed at STEP 4, compute the system output O(t) based on Eq. (2).

STEP 6. Generate a Gaussian random number with mean zero and variance R, and add this number to the system output computed at STEP 5. The result is the noisy observation at Δt.

STEP 7. Repeat the sequence STEP 1 to STEP 6 for as many observation points as desired.

STEP 8. Using: 1) the input-hydrographs values at the end points of the intervals Δt, and 2) the generated observations at the same points, perform parameter estimation with each one of EKF, NSLF and TGSLF. The initial parameter values used in parameter estimation STEP 8 were selected from the boundaries of the set of physically realistic true values.

STEP 9. Repeat sequence STEP 1 to STEP 8 for different Δt, Q, R and input characteristics.

RESULTS OF SIMULATION EXPERIMENT

This section presents initial results of the simulation experiment. For all of the cases to be presented in the following, we used a single nonlinear reservoir with a generic input hydrograph (generated by the inverse Gaussian function) that was characterized by a base flow of 1 mm/hour, a peak flow of 10 mm/hour, and a time to peak equal to 11 hours. Both the parameters a and m were estimated by the recursive algorithms. The "true" parameter values were chosen as:

$$a_{true} = 0.5\ [hr^{-1}\cdot mm^{-0.6}]$$

$$m_{true} = 1.6$$

Note that the value of m chosen corresponds to a physical upper bound on m in cases of flood routing (see Mein et al., 1974).

For all the runs the initial parameter estimates were set equal to 1 for m and 0.01 for a. The choice of the m-value was based on normal practice to initially assume a linear system, and the choice for the a-value was made based on our intent to have the initial estimate be as far as physically constraints would allow from the "true" a-value.

The initial value of the state was set equal to the known initial true value of the state that corresponds to baseflow conditions.

The variances for the initial a and m estimates were set equal to 1 for the EKF. The integration scheme used for the statistical linearization filters was sensitive to high values of the initial variances for a and m due to the proximity of the initial a and m values to the infeasible negative region of the real axis. Thus, the initial variance was reduced to 0.05 for those filters.

Four values of the sampling interval were examined. These were 15 minutes, and 1, 3, and 6 hours.

Sensitivity to the intensity of the model error and observation error noises was studied by the examination of four cases of noise intensity as follows:

Case 1: $Q = 0.00285\ (mm^2/hr)$
$R(t_k) = 0.01\ (mm/hr)^2$

Case 2: $Q = 0.00285\ (mm^2/hr)$
$R(t_k) = (0.1 + 0.1 \times Z(t_k))^2\ (mm/hr)^2$

Case 3: $Q = 0.285\ (mm^2/hr)$
$R(t_k) = 0.01\ (mm/hr)^2$

Case 4: $Q = 0.285\ (mm^2/hr)$
$R(t_k) = (0.1 + 0.1 \times Z(t_k))^2\ (mm/hr)^2$

The value of the model error variance parameter Q in Cases 1 and 2, is such that at each hour the state standard deviation is augmented by an amount equal to approximately 2 percent of the average state magnitude. In Cases 3 and 4, the state standard deviation is augmented at each hour by an amount equal to approximately 20 percent of the average state magnitude. Thus, Cases 1 and 2 represent the low model error noise, and Cases 3 and 4 represent the high model error noise.

In terms of the observation error variance $R(t_k)$ at time t_k, Cases 1 and 3 are the low noise cases with the standard deviation of the observation error set equal to 10 percent of the baseflow level. In the high observation noise Cases 2 and 4, the standard deviation of the observation error is proportional to the concurrent magnitude of the flows with coefficient of proportionality equal to 0.1. That corresponds to streamflow data of good quality (see Georgakakos and Bras, 1982) which is expected in hydrologic calibration studies.

Accuracy of the final parameter estimates was measured by 1) the percent proportional error denoted by E_a for parameter a and by E_m for parameter m, and 2) the difference S_a and S_m between final estimates and true values for a and m expressed in number of standard deviations. Thus, if $\hat{a}$, σ_a denote the final estimate for a and its standard deviation, and $\hat{m}$ and σ_m the corresponding quantities for m, then the accuracy measures E_a, E_m, S_a, and S_m are given by

$$E_a = \left(\frac{\hat{a} - a_t}{a_t}\right)\cdot 100$$

$$E_m = \left(\frac{\hat{m} - m_t}{m_t}\right)\cdot 100$$

$$S_a = \frac{\hat{a} - a_t}{\sigma_a}$$

$$S_m = \frac{\hat{m} - m_t}{\sigma_m}$$

where a_t and m_t are the true parameter values. S_a and S_m provide a measure of the reliability of the recursive estimators in predicting the variance (σ_a^2 and σ_m^2) of the final estimates $\hat{a}$ and $\hat{m}$.

We start by comparing the EKF with the TGSLF for all cases. Table 1 presents the values of the accuracy measures defined previously together with the CPU time used by the recursive estimators on a PR1ME 750 computer with the PRIMOS operating system, to process 1800 data points. The sampling interval and the noise intensity cases presented previously enter as parameters in Table 1. The results are displayed both for the EKF and the TGSLF. The statistical linearization procedure in the TGSLF had up to and including second order terms in the Taylor series expansion of the nonlinear functions f_1, f_2, f_3, and f_4 (see previous section).

Examination of the values in Table 1 leads to the conclusion that both the EKF and the TGSLF give better estimates for short sampling intervals and small noises. The bias of the EKF estimates appears to be higher than the TGSLF bias, especially for the high noise and long sampling interval cases. Both estimations are reliable at the 95 percent probability level for the 15-minute interval and low observation noise levels (Cases 1 and 3). For most cases, a much higher probability level is required for reliability.

Perhaps the most important conclusion from a practical point of view is that the observation noise intensity affects the final estimates much

TABLE 1 Values of the Accuracy Measures E_a, S_a, E_m, S_m, and CPU Time

METHOD:		EKF				TGSLF			
MEASURE		Case 1	Case 2	Case 3	Case 4	Case 1	Case 2	Case 3	Case 4
E_a (%)	Δt = 0.25 hr	1	-72	18	3	-1	-58	19	1
	Δt = 1.00 hr	-22	-57	25	66	1	-37	41	66
	Δt = 3.00 hr	-60	-66	-34	-43	-17	-43	-4	-2
	Δt = 6.00 hr	-74	-66	-80	-68	-54	-46	-63	-42
S_a	Δt = 0.25 hr	0.1	-15.0	1.9	0.2	-0.1	-9.1	2.1	0.1
	Δt = 1.00 hr	-3.6	-8.6	3.4	3.4	0.2	-4.1	5.5	3.8
	Δt = 3.00 hr	-15.0	-8.7	-7.7	-4.3	-4.9	-6.5	-1.2	-0.2
	Δt = 6.00 hr	-36.9	-13.8	-40.0	-13.7	-90.0	-13.4	-100.0	-9.9
E_m (%)	Δt = 0.25 hr	-2	26	-1	-4	0	17	-8	-7
	Δt = 1.00 hr	4	2	-11	-25	-3	-7	-14	-27
	Δt = 3.00 hr	31	49	-4	25	-13	18	13	17
	Δt = 6.00 hr	3	10	24	15	-13	-10	16	10
S_m	Δt = 0.25 hr	-0.9	5.5	-0.1	-0.6	0.2	3.9	-2.8	-1.6
	Δt = 1.00 hr	2.1	0.6	-4.9	-6.5	-1.9	-2.0	-6.9	-8.2
	Δt = 3.00 hr	8.4	4.9	-1.6	2.5	-10.7	3.8	3.3	3.0
	Δt = 6.00 hr	1.5	1.8	5.5	2.4	-60.0	-4.3	4.6	1.9
CPU(min)	Δt = 0.25 hr	4	3	5	5	11	11	15	12
	Δt = 1.00 hr	6	5	8	7	16	15	23	18
	Δt = 3.00 hr	11	12	11	12	24	24	31	30
	Δt = 6.00 hr	10	13	15	14	32	31	33	32

more than the model error noise intensity does. Therefore, good quality data are essential for accurate parameter estimates. Furthermore, the table shows that, for the steep input hydrograph used in the runs, short sampling interval will yield more accurate parameter estimates. In terms of the relative accuracy of parameters a and m, Table 1 shows that m was easier to identify, especially for the long sampling intervals.

Finally, the values of CPU time in Table 1 show that the TGSLF, with up to and including second order terms in the Taylor series expansion, is approximately three times slower than the EKF. However, even for the longest sampling (and forecast) intervals, the requirements of both methods in CPU time are quite minor (up to 33 minutes per 1800 data points) so that they can be easily used on mini- or micro-computers.

Next, we examined whether model predictions were sensitive to inaccurate parameter estimates. The variance R_n of the normalized residuals and the variance R^2 of the step-one prediction residuals was computed for all the EKF and TGSLF final parameter sets that correspond to each of the sampling intervals and each of the cases (Table 1). For all runs, the EKF was used to produce the model predictions, with the parameters a and m remaining constant throughout the run. A total of 600 data points were used for each run. The base values of R_n and R^2 were established by runs with parameters constant and equal to the "true" parameter values. Table 2 presents the results for all the cases and sampling intervals of interest. The values in Table 2 indicate that:

1. Both parameter estimators gave parameter estimates that produced the minimum possible R^2 value.

TABLE 2 Variance R_n of Normalized Residuals and Variance R^2 of Prediction Residuals

	R_n			R^2 (mm/hr)^2		
TRUE	EKF	TGSLF	TRUE		EKF	TGSLF
Δt = 15 min						
Case 1	1.03	1.03	1.03	0.01	0.01	0.01
Case 2	1.02	1.06	1.04	0.24	0.25	0.25
Case 3	1.15	1.07	1.19	0.18	0.18	0.18
Case 4	1.05	1.06	1.09	0.45	0.45	0.45
Δt = 1 hr						
Case 1	1.17	1.17	1.17	0.02	0.02	0.02
Case 2	1.04	1.08	1.08	0.30	0.31	0.31
Case 3	1.72	1.81	1.79	0.68	0.68	0.68
Case 4	1.41	1.47	1.48	0.98	0.98	0.98
Δt = 3 hr						
Case 1	1.56	1.54	1.65	0.03	0.03	0.03
Case 2	1.08	1.08	1.08	0.27	0.27	0.27
Case 3	2.94	3.51	2.63	2.21	2.22	2.22
Case 4	2.04	1.97	1.88	2.49	2.49	2.49
Δt = 6 hr						
Case 1	1.98	2.08	2.10	0.05	0.05	0.05
Case 2	1.16	1.16	1.16	0.31	0.31	0.31
Case 3	4.09	4.90	4.57	4.07	4.07	4.07
Case 4	2.67	2.94	2.75	4.28	4.28	4.28

2. For long sampling interval and high intensity of the noises, the variance of the normalized residuals was considerably different from its optimal value of 1, indicating non-optimal filter parameter Q.

Thus, even though the parameter estimates were not accurate for all cases (see Table 1), they can be used for prediction with the same predictive capability as the true parameters. Furthermore, the nonlinear reservoir model is robust with respect to parameter changes of the magnitude in Table 1.

The Q approximation in STEP 3 of the generation algorithm (see previous sections) resulted in the deterioration of the R_n and R^2 values for the true parameters for Cases 3 and 4 and Δt > 1 hour shown in Table 2. This points to the importance of accurate Q values in parameter estimation studies with recursive parameter estimators based on filtering algorithms. Work in progress will study the utility of real-time recursive estimators of Q of the type in Georgakakos, 1984, in the recursive estimation of the parameter a and m of the nonlinear reservoir.

Since the EKF and the TGSLF gave final estimates that lead to as good predictions (Table 2) as those obtained from the true parameter values, it was not considered necessary to run the costly NSLF for all the cases of sampling intervals and noise intensities. For illustration purposes and to show the relative cost of the procedure, we run the NSLF parameter estimator for a sampling interval of 6 hours for all noise cases for a total of 100 data points. The results follow:

	E_a(%)	E_m(%)	CPU (min)
Case 1	-35	-31	187
Case 2	-35	-31	193
Case 3	19	-11	200
Case 4	23	-30	238

The estimate accuracy is comparable to the accuracy of the EKF and TGSLF estimates with CPU tiome increased dramatically.

CONCLUSIONS

An examination of the performance of recursive parameter estimators based on filtering theory in the estimation of the parameter of a nonlinear reservoir was undertaken. The extended Kalman filter, and two filters based on statistical linearization procedures were utilized. The performance of the estimators was measured by the proximity of the final parameter estimates to the true values and by least squares criteria concerning the step-one prediction residuals.

Initial results based on a simulation experiment indicate that the value of the model error variance parameter Q is very important for effective parameter estimation by all the estimators. The estimation procedure is more sensitive to Q than it is to changes in the parameter estimators. Even though, for the cases examined, the estimators gave different parameter estimates, the predictive capability of the resultant nonlinear reservoir model was equally good suggesting robust model behavior.

For the steep nonlinear hydrograph input used in the present study, sampling intervals less than an hour and very good quality data are necessary for accurate parameter estimation.

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