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A GENERATOR OF RADAR RAINFALL DATA

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1. INTRODUCTION

In recent years, a number of methods have been proposed for merging radar and rain gage rainfall data in order to increase the accuracy of rainfall analysis [Brandes, 1975; Crawford, 1978; Eddy, 1979; Krajewski and Hudlow, 1983].

One of the generic problems faced while developing merging procedures is validation. Reliable synthesis of rainfall fields can be very helpful as an alternative to costly field experiments. Using rainfall synthesis, one can control the ensemble statistics of the generated fields so that the instrument-observation errors are simulated in the validation process. Thus, it becomes possible to evaluate various merging procedures for statistically different error fields. Following the idea proposed by Greene et al. [1980], we have developed a test system to validate merging procedures [Krajewski and Hudlow, 1984]. This paper describes part of the system - the radar data generator.

It is the purpose of this paper to present a methodology that avoids the explicit specification of the rainfall field statistics by acting on the point values of the observed fields. Thus, the original field which could be, for example, a high quality radar rainfall field, is taken as deterministic. A random noise, which is Gaussian, isotropic, and has predefined second-order statistics, is imposed at each point of the original field. The noise level varies from point to point, based on the local original field characteristics like magnitude and gradient.

Apart from the validation of merging procedures, the methodology can also be used in the design of rainfall observation systems and in testing mean areal precipitation estimators or rainfall-runoff models, to mention only a few applications.

In the next sections, the methodology is described, followed by a discussion of an example implementation.

2. THEORY

The idea is to generate fields from an existing high quality radar field by imposing a noise field of known statistics such that the ensemble of the resultant realizations meets certain conditions. The conditions pertain to the spatially-averaged second order statistics of the generated fields.

If $G(x,y)$ is the generated field and $O(x,y)$ is the original, high-quality radar field, then the error field $\Delta(x,y)$ is commonly [Hudlow et al., 1979b] expressed as:

$$\Delta(x,y) = \log_{10}\left(\frac{G(x,y)}{O(x,y)}\right) \quad \forall (x,y) \in A \tag{1}$$

where, x,y are the field-point coordinates, and A is the field domain.

For the purposes of this study we take $\Delta(x,y)$ to be the product of a random field and a deterministic component according to:

$$\Delta(x,y) = \varepsilon(x,y) \cdot S(x,y) \tag{2}$$

In Eq. (2), $\varepsilon(x,y)$ is a stationary and, in general, anisotropic, Gaussian random field of mean μ, variance σ^2, and correlation function $\rho(\tau_1,\tau_2)$ with τ_1,τ_2 denoting spatial lags in the two directions x and y. $S(x,y)$ is a deterministic function which makes $\Delta(x,y)$ a nonstationary random field. We adopt the form of $S(x,y)$ given by Greene et al. [1980]:

$$S(X,Y) = \frac{|\overline{\Delta O(X,Y)}| \cdot O_m(x,y) + O(x,y) \cdot |\Delta O(x,y)|_m}{2\,|\overline{\Delta O(x,y)}|_m \cdot O_m(x,y)} \tag{3}$$

where:

$\lvert\overline{\nabla O(x,y)}\rvert$	is the average absolute value of the gradient computed in four directions around the point (x,y) in the original field,
$\lvert\nabla O(x,y)\rvert_m$	is the maximum absolute gradient in the original field,
$O(x,y)$	is the original field value at the point (x,y),
$O_m(x,y)$	is the maximum value in the original field.

We note at this point that the development of the methodology is independent of the particular form of S(x,y) in Eq. (3), so that any deterministic, real function of (x,y) can be used.

Eliminating $\Delta(x,y)$ from Eqs. (1) and (2) yields

$$G(x,y) = O(x,y) \cdot 10^{\varepsilon(x,y) \cdot S(x,y)} \qquad (4)$$

If one has a mechanism of generating the random component field $\varepsilon(x,y)$, then, using Eq. (4) and the original high-quality radar field O(x,y), one can produce realization G(x,y).

There are several methods for the generation of $\varepsilon(x,y)$, given its statistical parameters μ, σ^2 and $\rho(\tau_1,\tau_2)$. The Turning Bands Method (TBM) presented by Mantoglou and Wilson [1982] seems to be an efficient one in terms of accuracy and cost.

The TBM gives us a way to generate the field $\varepsilon(x,y)$ in Eq. (4) if its second order statistics are known. We obtain these statistics by imposing certain conditions on the generated fields.

Due to the fact that G(x,y) is a nonstationary random field, we need to specify operational measures of the statistical properties of it. Thus, we define the spatial mean R of the field as:

$$R = \frac{1}{|A|} \int_A E\{G(x,y)\} dA \qquad (5)$$

where A is the generation domain with area $|A|$, and $E\{\cdot\}$ denotes expectation of the value of G at the point (x,y).

The spatially-averaged variance P of the field G(x,y) is defined as:

$$P = \frac{1}{|A|} \int_A E\{[G(x,y) - E\{G(x,y)\}]^2\} dA \qquad (6)$$

Equations (5) and (6) describe the spatially averaged field-expected value and field-variance.

Another measure of variance used often in the radar literature [Hudlow et al., 1979b] in place of Eq. (6) is the variance V of the logarithmic ratio of Eq. (1) defined as:

$$V = \frac{1}{|A|} \int_A E\{\log_{10}^2(\frac{G(x,y)}{O(x,y)})\} dA - \frac{1}{|A|} \int_A E^2\{\log_{10}(\frac{G(x,y)}{O(x,y)})\} dA \qquad (7)$$

By setting the expressions in Eqs. (5) and (6) or (7) to prespecified values, one can, in principle, obtain expressions for μ, and σ^2. Then, one can generate the $\varepsilon(x,y)$ field using TBM with assumed $\rho(\tau_1,\tau_2)$ and, then, generate the G(x,y) field from (4).

The expressions are:

$$\frac{1}{|A|} \int_A O(x,y) \cdot \exp\{\frac{1}{2} (\ln 10)^2 \cdot S^2(x,y) \cdot \sigma^2 + (\ln 10) \cdot S(x,y) \cdot \mu\} dA = R_o \qquad (8)$$

$$\frac{1}{|A|} \int_A O^2(x,y) \cdot [\exp\{2(\ln 10)^2 \cdot S^2(x,y) \cdot \sigma^2 + 2(\ln 10) \cdot S(x,y) \cdot \mu\} - \exp\{(\ln 10)^2 \cdot S^2(x,y) \cdot \sigma^2 + 2(\ln 10) \cdot (S(x,y) \cdot \mu\}] dA = P_o \qquad (9)$$

$$\frac{\sigma^2}{|A|} \int_A S^2(x,y) dA = V_o \qquad (10)$$

where, R_o, P_o, V_o are prespecified values. Solving simultaneously the equations (8) and (9) or (10), one can obtain values for μ, and σ^2.

Figure 1 shows a schematic flowchart of the generation process. The next section presents an example of application of the proposed methodology.

STEP 1: SELECT ORIGINAL RADAR FIELD: O(x,y)

STEP 2: COMPUTE DETERMINISTIC COMPONENT: S(x,y) - Eq. (3)

STEP 3: SPECIFY DESIGN VALUES: R_o, P_o or V_o

STEP 4: COMPUTE μ, σ - Eqs. (8) and (9) or (10)

STEP 5: ASSUME $\rho(\tau_1,\tau_2)$

STEP 6: GENERATE ONE REALIZATION OF THE NOISE FIELD $\varepsilon(x,y)$ USING TBM - APPENDIX

STEP 7: GENERATE A RAINFALL FIELD: G(x,y) - Eq. (4)

STEP 8: MORE FIELDS?
IF YES -- GO TO STEP 6
IF NO -- STOP

Fig. 1. Schematic Flowchart for the Generation Process

3. NUMERICAL IMPLEMENTATION

This section presents an example generation of rainfall fields from an original, high-quality radar field. Accuracy of preservation of the design-statistics in the generated ensemble of rainfall fields is studied in connection to the number of generated fields and the magnitude of the statistics themselves.

The original field consisted of daily radar data from the international GARP Atlantic Tropical Experiment (GATE) conducted in 1974. Detailed description of the GATE data is given by

Hudlow and Patterson [1979a]. The original radar field corresponds to spatially averaged daily accumulations for July 28, 1974. Spatial averages were computed in 4 km x 4 km domains.

For the purposes of this example, the design equations (8) and (10) were studied.

During the example runs, we generated rainfall fields from the original radar field with mean R_o equal to the spatial average of the original field (1.16 mm/hour), and with pre-specified values for the logarithmic-ratio variance V_o (0.005, 0.01, 0.03). The correlation condition consisted of specifying values for the correlation distance $(\frac{1}{h})$ of the $\varepsilon(x,y)$ field (4 km, 12 km, 20 km) assuming an isotropic, exponential correlation function:

$$\operatorname{cor}(\tau) = e^{-h \cdot \tau} \tag{11}$$

For all the combinations of R_o, V_o and $(\frac{1}{h})$, we generated three ensembles of rainfall fields, with number of fields per ensemble, NS, equal to 10, 25, and 50. We then computed the statistics R and V for each ensemble and we compared them with their theoretical values obtained from Eqs. (8) and (10), respectively. We also computed μ and σ^2 from each ensemble and compared them to the ones obtained by solving Eqs. (8) and (10). Thus, we were able to evaluate the accuracy of the TBM generator.

The accuracy of the preservation of R was in all the cases better than 1 percent while the maximum error of μ and σ^2 was 12.5 percent. The accuracy of V is shown in Figure 2.

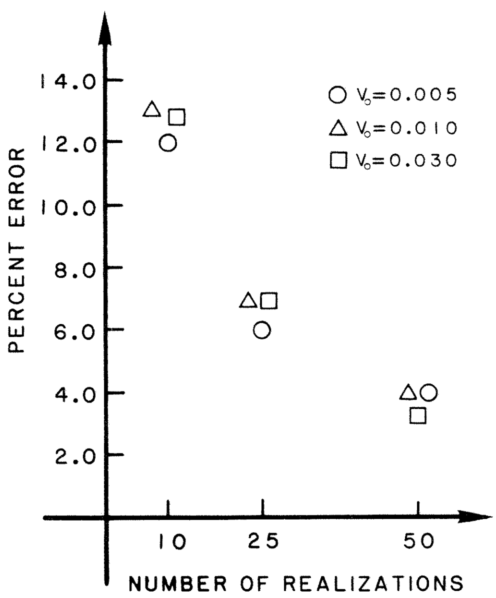

Fig. 2. Percent error of V computed for different number of realizations.

Due to the fact that the specification of high V and P statistics will sometimes yield physically unacceptable values of the precipitation rates, we monitored the number of values exceeding an arbitrarily chosen rate, set to 50 mm/hour, which is close to the observed World Record value of 53 mm/hour for daily data [Chow, 1964]. Table 2 contains the monitored values. Consequently, we give guidelines on the specification of V, which is a normalized measure, so that generation of realistic rainfall fields results.

Table 2. Number of Generated Rainfall Values that Exceed 50 mm/hr.

	NS=10			NS=25			NS=50		
1/h (km)	4	12	20	4	12	20	4	12	20
V_o									
0.005	1	1	0	3	2	0	5	4	1
0.010	1	1	0	11	7	3	27	21	15
0.030	32	41	36	92	113	99	193	232	219

4. SUMMARY AND CONCLUSIONS

A method for generation of radar precipitation fields was described. The method works by imposing a noise field on high quality radar rainfall fields. The noise parameters are determined based on a set of conditions pertaining to the resultant field. In that way, non-stationary, non-ergodic fields can be simulated.

Since the original and the 'observation' (original + noise) fields are known, the method can be used in the validation procedures of different hydrologic models (radar and rain gage data merging, mean areal precipitation estimation, rainfall-runoff). A separate simulation model has been developed to generate rain gage observations starting from the same original field [Krajewski and Hudlow, 1984]. The two simulation models are being used for testing and evaluation of an objective analysis of the precipitation data system. The example given shows that the accuracy of the preservation of the required statistics is very good, especially for realistic values of the variance measure V (< 0.01 for daily data), even for a relatively small number of realizations (NS ⩽ 25). The method proposed is flexible in that one can generate fields with a wide range of second-order statistics from one high-quality radar field.

Work is in progress to include the correlation function $\rho(\tau_1, \tau_2)$ in the design process.

5. ACKNOWLEDGEMENTS

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