STRUCTURE AND PARAMETER DETERMINATION OF A HYDROLOGICALLY USEFUL RAINFALL PREDICTION MODEL

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ABSTRACT

A physically based rainfall prediction model suitable for use with hydrologic catchment models has been developed in state-space form. The structure defined by uniform height-profiles for the updraft velocity and for the cloud layer-average diameter is examined in detail. The behavior of the model physical components during the storm duration is shown. Contour maps of the space of the two free model parameters for a number of performance criteria and for a convective and a stratiform storm group indicate model structure robustness to different criteria and storm-types.

INTRODUCTION

Related Previous Work

Georgakakos and Bras (1982) formulated a station precipitation model in state-space form. Based on the surface pressure, temperature and dew-point temperature, their model gives as an output the precipitation rate. The model state is the mass of the condenced liquid water equivalent in the area characterized by the input temperature and pressure indices. The model formulation is based on pseudo-adiabatic ascent of the air-masses and on simplified cloud microphysics with exponential particle-size distribution and linear dependence of the particle terminal fall-velocity on the particle diameter. Evaporation of the falling particles, for unsaturated sub-cloud layer is explicitly taken into account by their model. Predictions of snowfall vs. rainfall are based on the surface air-temperature.

Figure 1 presents a sketch of the physical mechanisms that are modeled. The upper part of the figure is a plan-view of the moving (velocity denoted by u) storm clouds, while the lower part is a cross-section through them. The shaded regions correspond to a cloud-column characterized by the input variables: air-temperature, T_0, air-pressure, p_0, and dew-point temperature, T_d, at the ground level. The model developed simulates the dynamics in this column. Air rises pseudo-adiabatically in the clouds with updraft velocity v (possibly height-varying), producing an input rate of condenced water equivalent I. The input mass of condenced water is distributed to different droplet diameters according to an exponential particle size distribution, n(D), whose parameters N_0 and c are possibly height-varying. Due to the action of the updraft at the cloud top, a portion

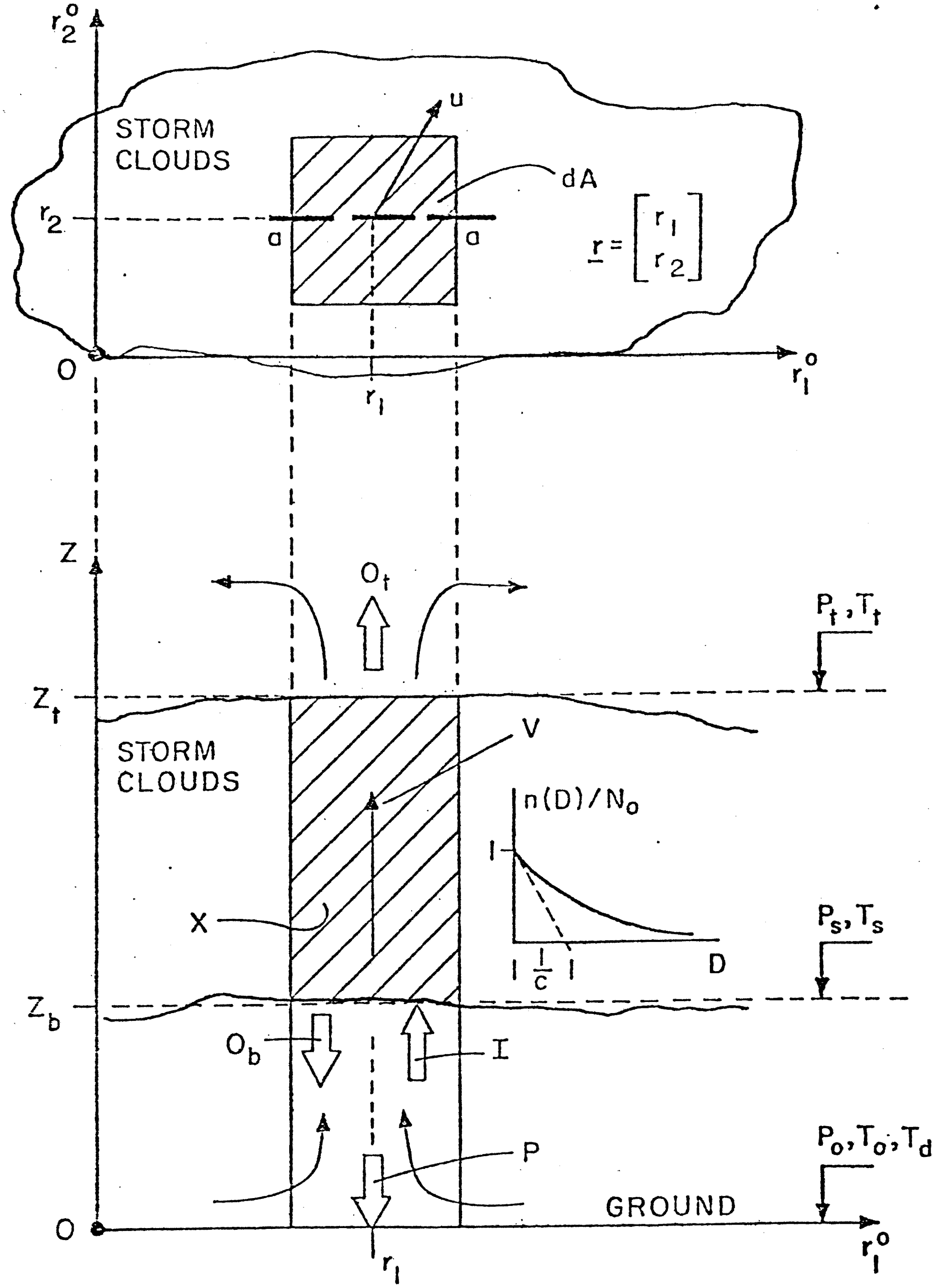

FIGURE 1 Schematic representation of the model physical components

of the water mass leaves the column with a rate O_t. The larger droplets fall through the cloud bottom with a rate O_b. The precipitation rate P at the ground level is computed from O_b by subtraction of the mass evaporated due to possible unsaturated conditions below the cloud base. The model dynamics equation consists of a statement of the conservation of the condenced water equivalent mass X within the cloud column. Heat-adiabatic ascent is used to determine the cloud-base (level Z_b) pressure, p_s, and temperature, T_s. Pseudo-adiabatic ascent and the terminal pressure p_t at the cloud-top (level Z_t) are used to determine the temperature T_t and, subsequently, the water vapor condenced per unit mass of moist air. The physical quantities v, c and P_t are parameterized using the input variables p_0, T_0 and T_d in an effort to obtain a storm and location invariant structure.

As a first step toward model verification, Georgakakos and Bras (1982) considered uniform profiles of updraft velocity and cloud particle layer-average diameter. In addition, the cloud particle average diameter was held constant independent of the input variables. The free model parameters in this case are:

1) the ratio EPS1 of the updraft velocity to the square root of the potential thermal energy per unit mass of the ascending air at the height of average updraft velocity, and

2) the time- and storm-constant cloud particle average diameter denoted by EPS4 (equal to 1/c).

Deterministic simulation runs were used to obtain values for the free parameters. Contour maps of different performance criteria indicated strong local gradients with saddle points. In addition, the optimum parameters differed considerably for different criteria. However, runs of the calibrated model running with a linear Kalman filter for a 1-hour forecast lead time showed good performance. The similar model performance for all cases tested suggested robust model structure for different conditions.

Objectives

This work demonstrates the properties of the stochastic precipitation model structure and presents parameter determination procedures. The model structure used is the one suggested by Georgakakos and Bras with uniform height-profiles of updraft velocity and cloud layer-average particle diameter. The following values (suggested in Georgakakos and Bras, 1982) were used for the filter parameters for all cases to be presented:

- No input error was assumed for p_0.
- A standard error of 1 degree Celcius was assumed for T_0 and T_d.
- A model error spectral density of 0.01 ($KG/M^2/SEC$) was used.
- An observation standard error of 1 (MM/HOUR) was assumed.
- The coefficient of variation of the initial state was 0.3.

The hourly storm data used for the parameter estimation runs were:

- Convective Group (CG): A line-storm and a tropical storm at the Logan airport, in Boston, Massachusetts, with a total of 100 wet-hours.
- A stratiform Group (SG): A frontal storm with persistent rains at the Logan airport, in Boston, Massachusetts, with a total of 60 wet-hours.

Hourly forecasts of the calibrated stochastic model are shown for a group of three storms from the International airport in Tulsa, Oklahoma, with a total of 100 wet-hours. In all cases to follow, only the periods within the storm duration are examined.

PARAMETER ESTIMATION PERFORMANCE CRITERIA

Several criteria were used in an effort to examine different aspects of the model performance.

Errors in the total volume of the storm-group precipitation were represented by the absolute proportional mean error (APME). This criterion is the absolute value of the ratio of the 1-step predicted residuals mean to the mean of the corresponding observations for the period under study. A value of zero represents optimal performance with respect to this criterion. The APME is most important when the model predictions will serve as input to hydrologic models.

The standard least-squares criterion is represented by the proportional standard error (PSE). It is the ratio of the 1-step predicted residuals standard deviation to the standard deviation of the corresponding observations. It gives the proportion of the observations standard deviation unexplained by the model. A value of zero corresponds to perfect performance with respect to PSE.

Maximum likelihood estimation is represented by the average value of the log-likelihood (ALL) over the period of interest. The greater the value of this criterion the better the model performance is. Optimization with respect to this criterion gives the parameter values with the highest probability of generating the observed sequence under the assumption that the model structure used is the true one.

The cross-correlation coefficient (CRCO) between predicted output and observed output is used as an indicator of timing of peaks and lows. Perfect performance with respect to CRCO is indicated by a value of one.

Due to the fact that the physically based model components are observable quantities, one can judge whether the parameter estimates give realistic values to these components.

PARAMETER DETERMINATION

The space of the two free model parameters was divided in grids and the value of each performance criterion was computed for each nodal grid-point for each of the two storm groups (convective and stratiform). Figures 2 through 9 present the contour maps for all cases, for the parameter space near the optimum values. In those figures, parameter EPS1 ranged from 10^{-4} to 3×10^{-3}, while the parameter EPS4 ranged from 10^{-5} M to 10^{-4} M. There were fifteen intervals for EPS1 and ten intervals for EPS4 where values of the performance indices were computed. EPS1 had a refined discretization due to the fact that preliminary contour maps showed that the index APME was particularly sensitive to changes in this parameter.

Table 1 gives the parameter and performance index values at the optimal points for each storm type and each performance measure. It also gives the discretization interval size for each of the two parameters. Table 2 gives the mean, standard deviation and coefficient of variation for the observed hourly data corresponding to the convective (CG) and the stratiform (SG) precipitation groups.

The group of figures 2 through 9 shows remarkable proximity of the locations of the optimal parameters in the parameter space for both storm groups and all performance measures. In addition, the fact that the gradients

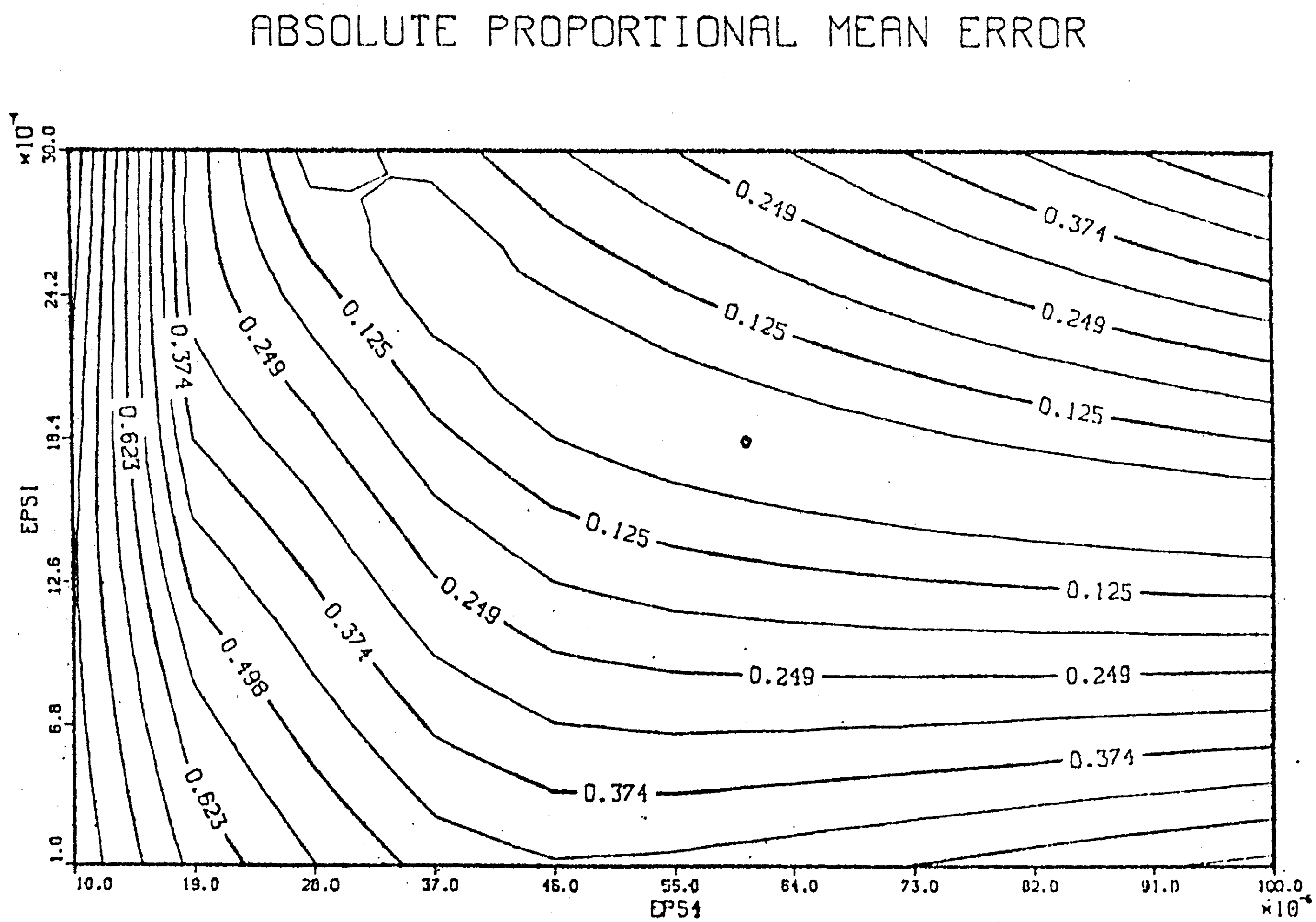

FIGURE 2 Criterion APME contours for the convective storm group in the space of parameters EPS1, EPS4

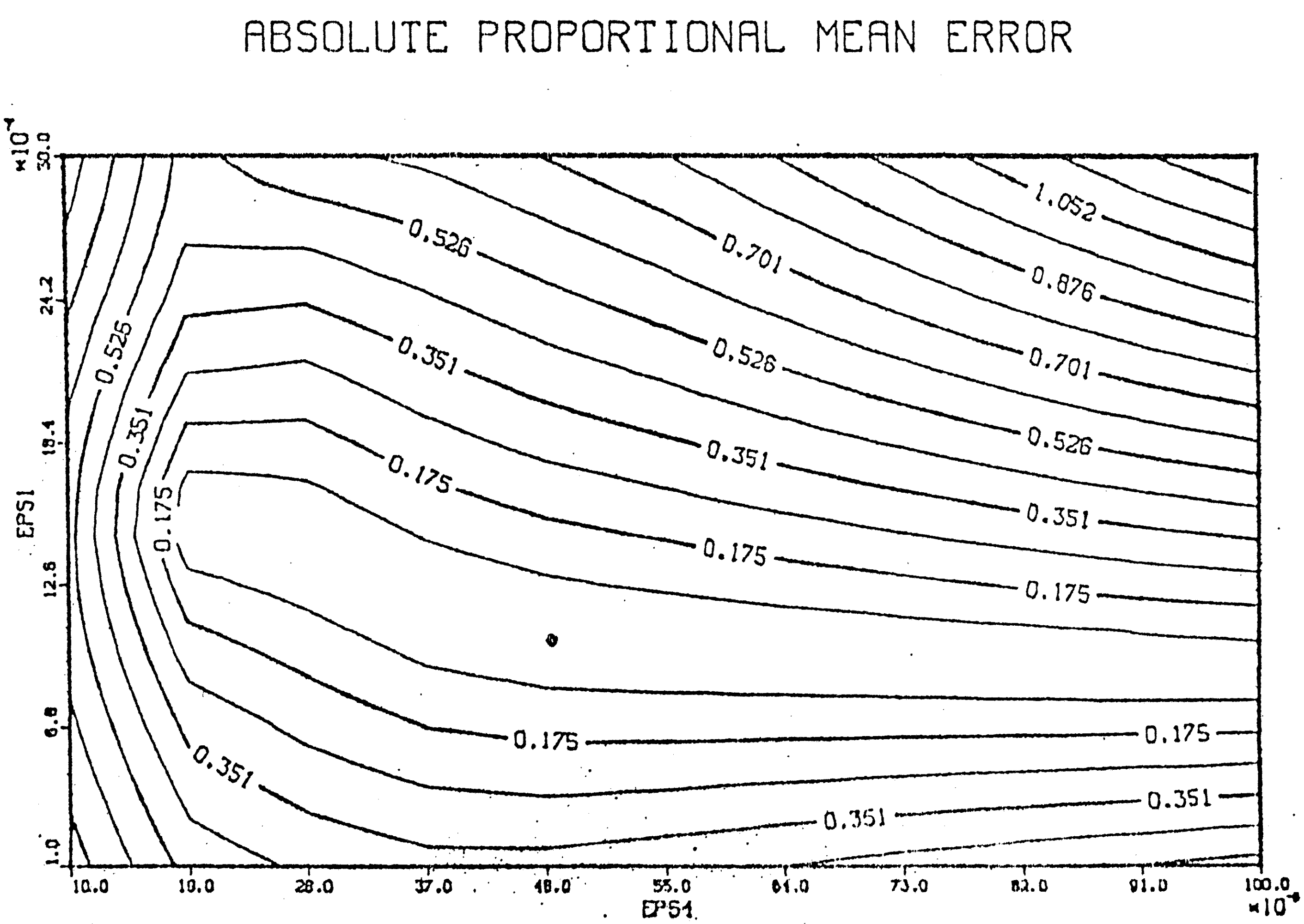

FIGURE 3 Criterion APME contours for the stratiform storm group in the space of parameters EPS1, EPS4

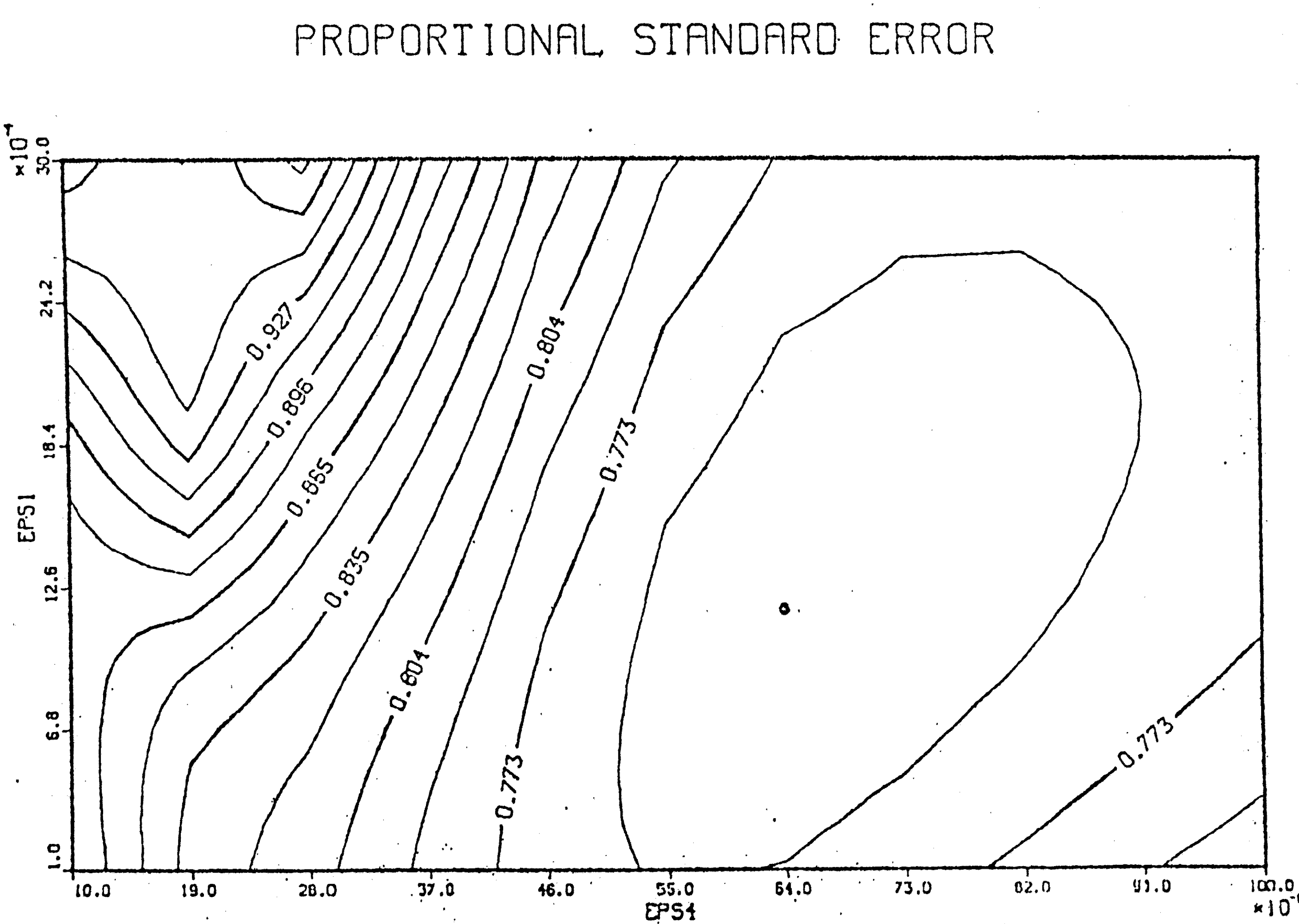

FIGURE 4 Criterion PSE contours for the convective storm group in the space of parameters EPS1, EPS4

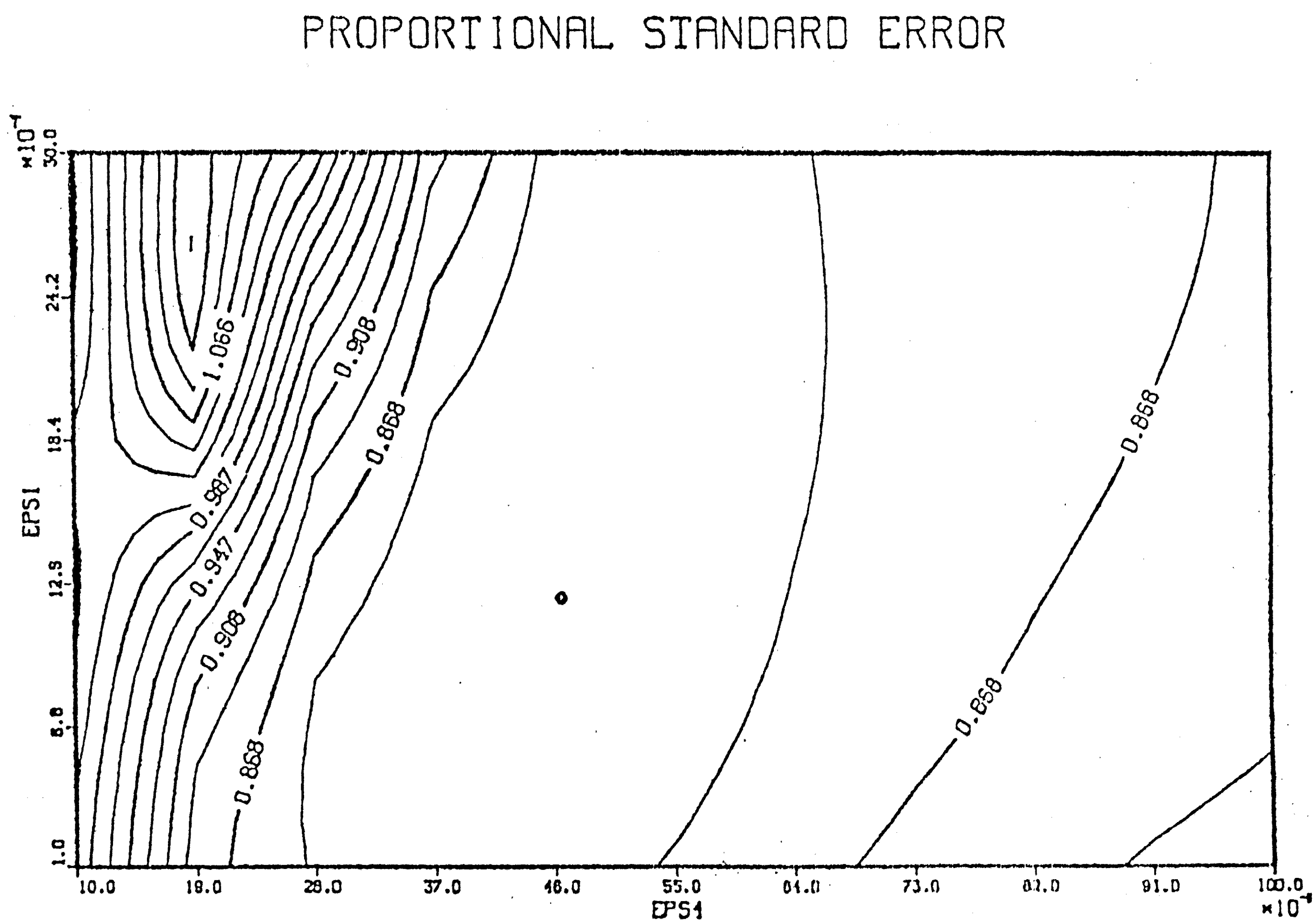

FIGURE 5 Criterion PSE contours for the stratiform storm group in the space of parameters EPS1, EPS4

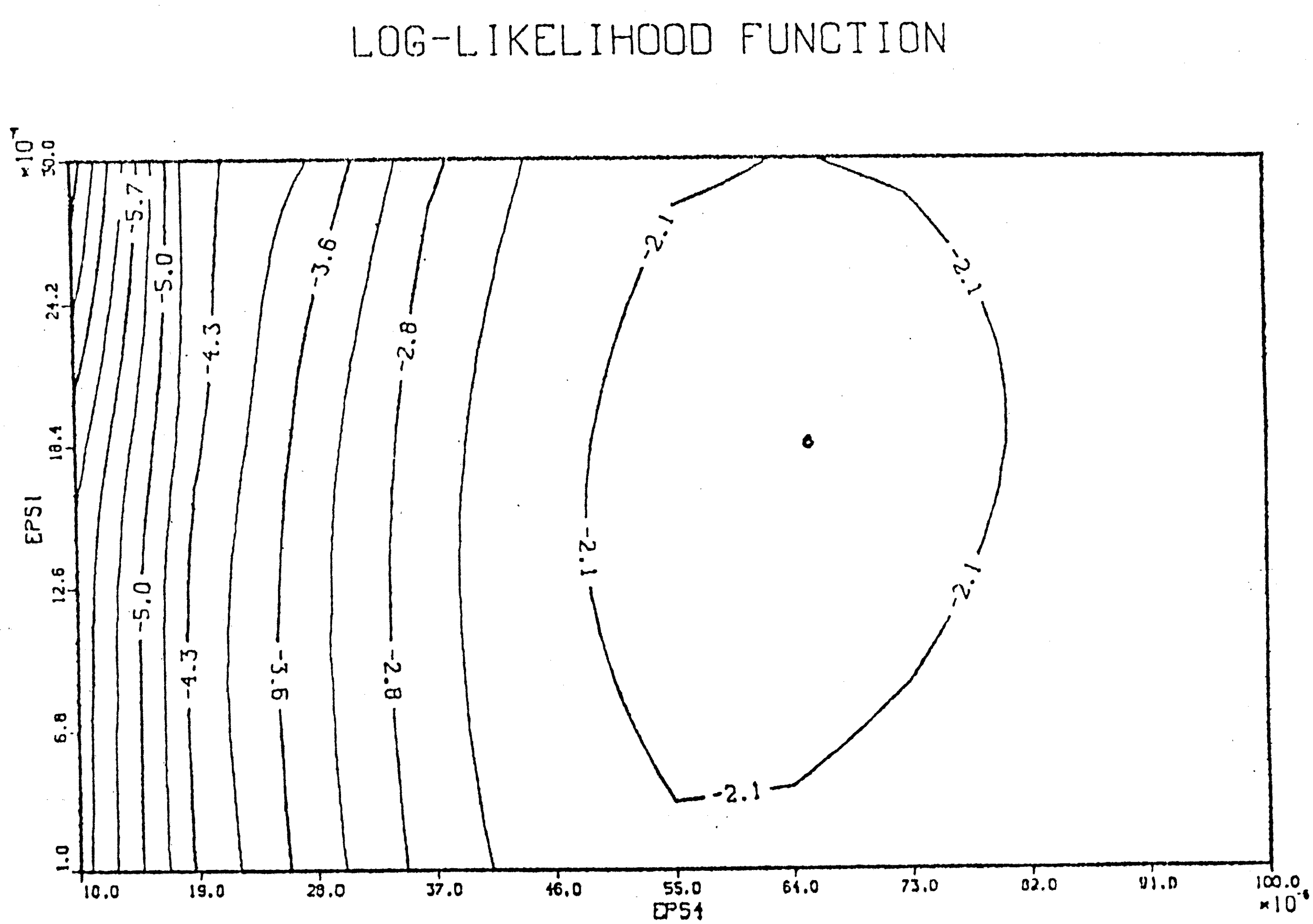

FIGURE 6 Criterion ALL contours for the convective storm group in the space of parameters EPS1, EPS4

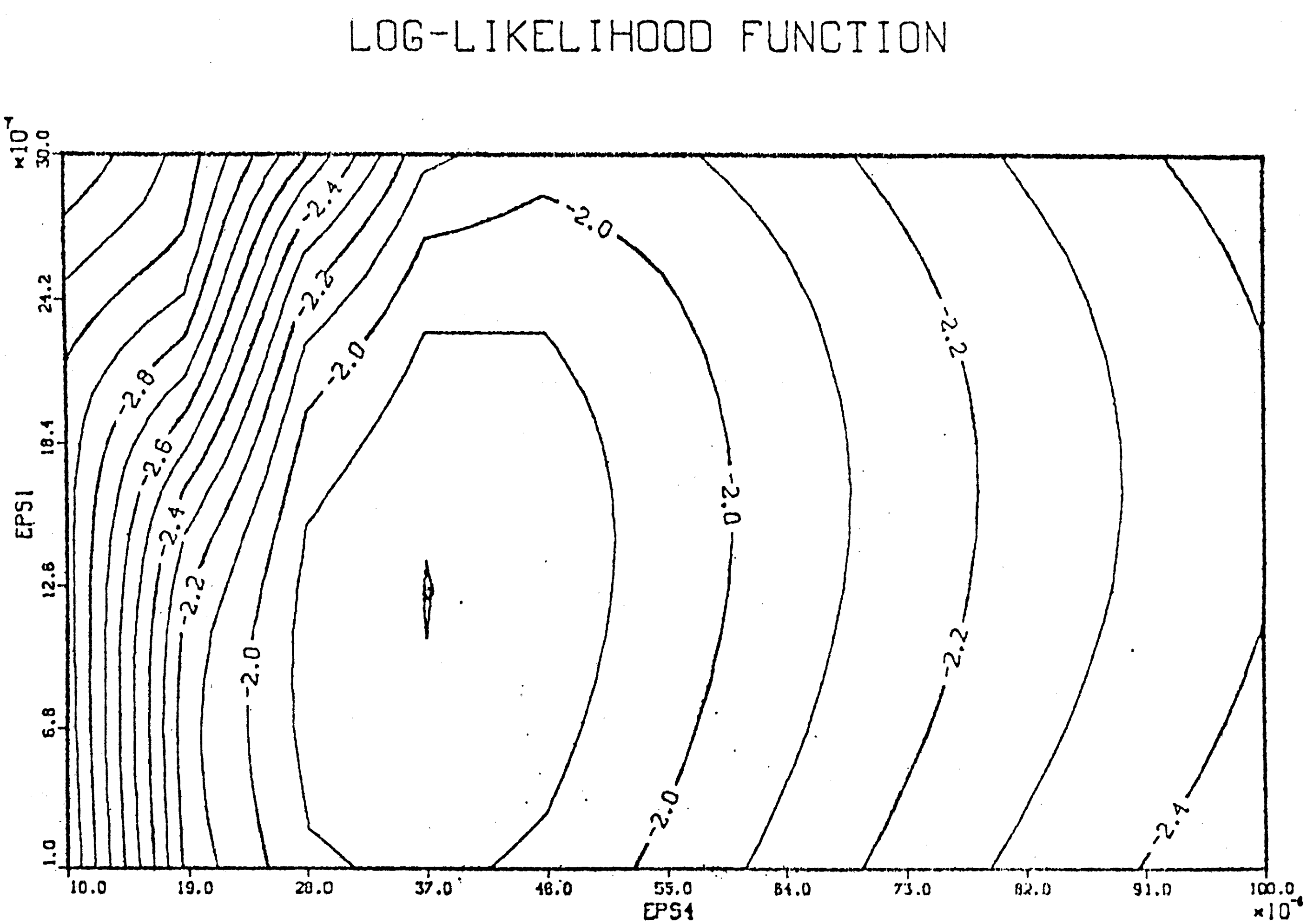

FIGURE 7 Criterion ALL contours for the stratiform storm group in the space of parameters EPS1, EPS4

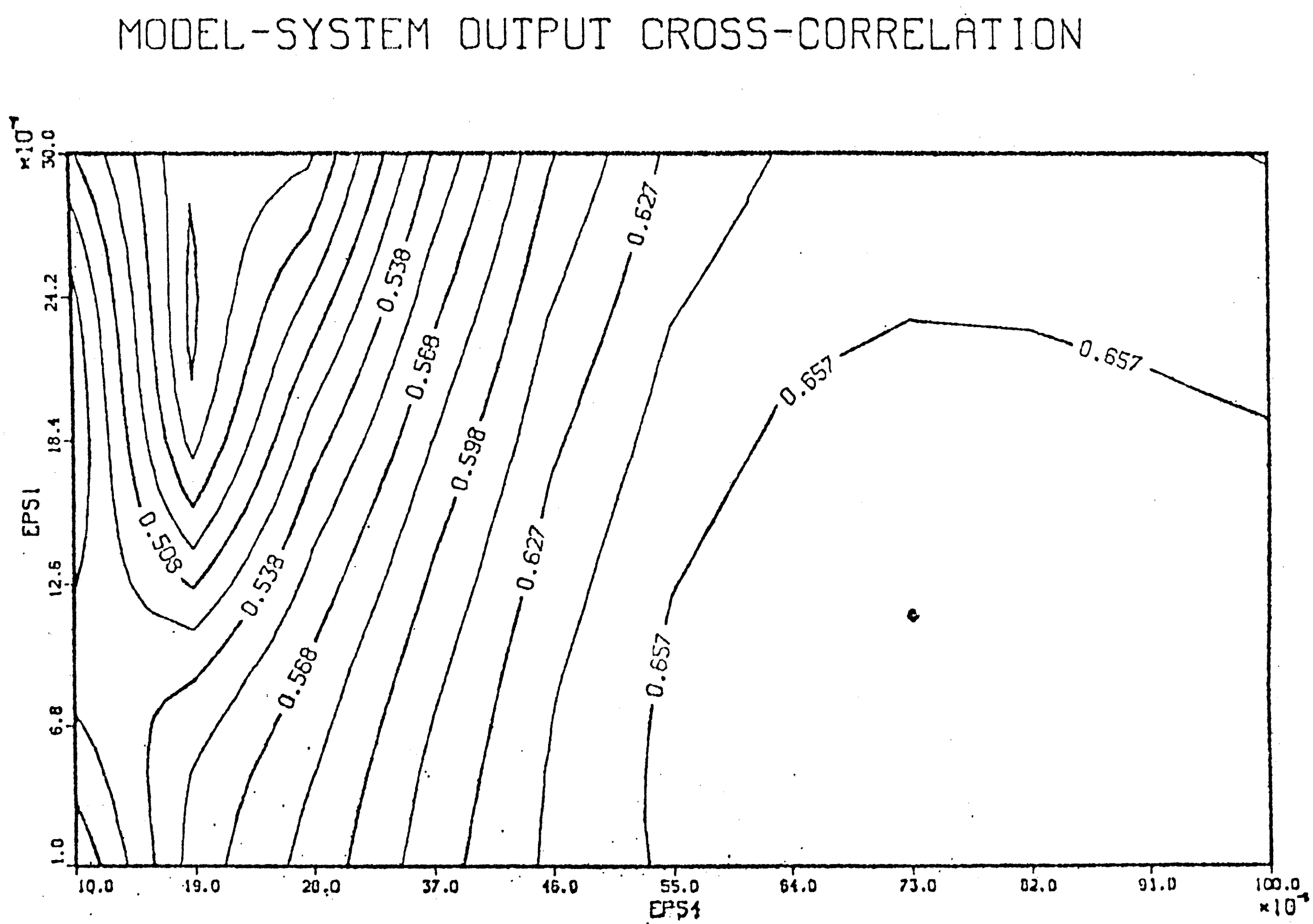

FIGURE 8 Criterion CRCO contours for the convective storm group in the space of parameters EPS1, EPS4

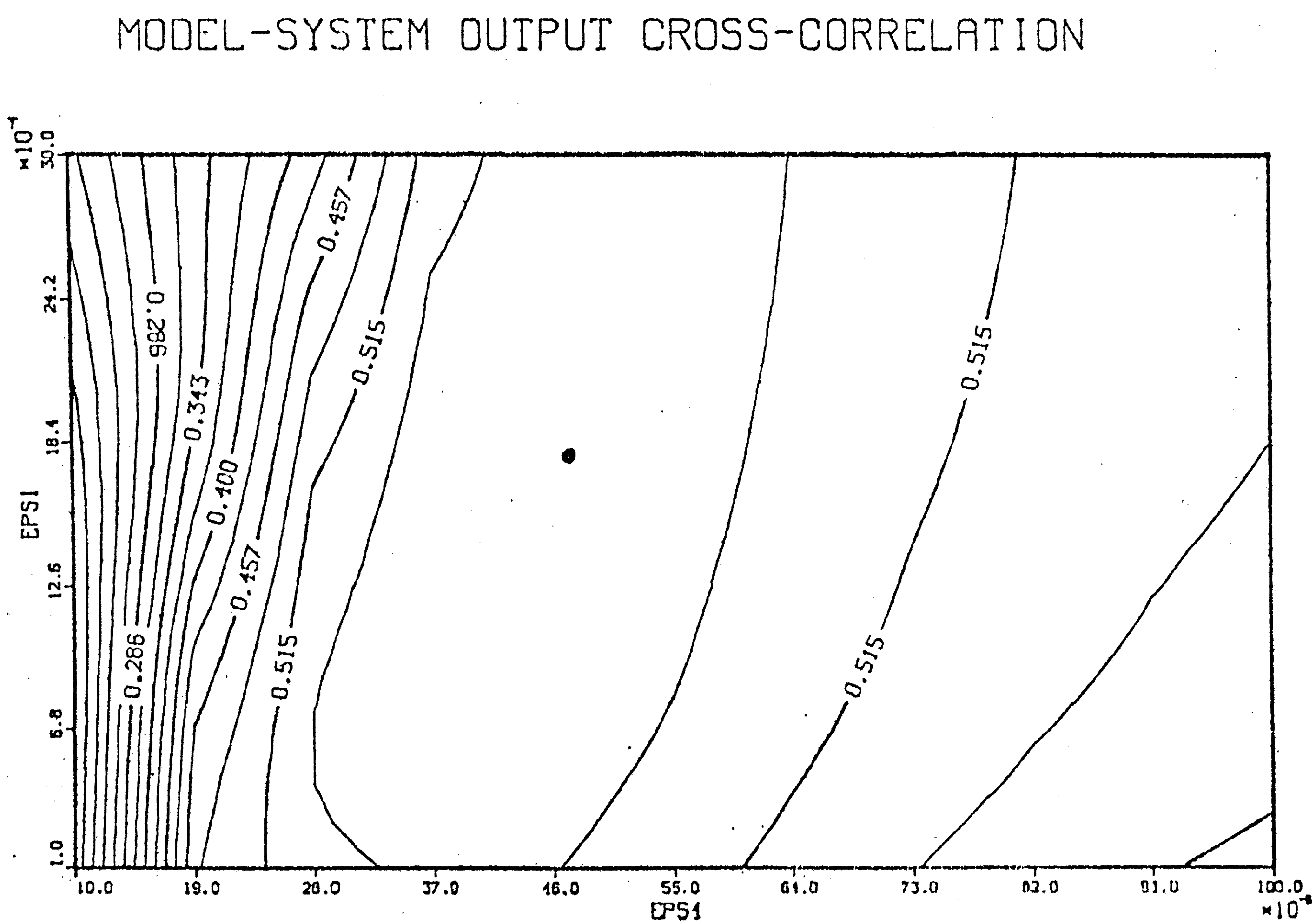

FIGURE 9 Criterion CRCO contours for the stratiform storm group in the space of parameters EPS1, EPS4

Table 1

PERFORMANCE INDEX AND PARAMETER VALUES AT OPTIMUM

	APME		PSE		ALL		CRCO	
	CG	SG	CG	SG	CG	SG	CG	SG
PERFORMANCE INDEX :	0.01	0.00	0.75	0.83	-2.03	-1.74	0.67	0.57
PARAMETER EPS1 ($x10^3$) :	1.84	1.07	1.07	1.26	1.84	1.26	0.87	1.84
PARAMETER EPS4 ($x10^5$M) :	6.40	4.60	6.40	4.60	6.40	3.70	7.30	4.60

COMPUTATIONAL INTERVAL SIZE

EPS1 = 0.19×10^{-3}

EPS4 = 0.90×10^{-5} METERS

Table 2

STORM GROUP STATISTICS

		CG	SG
MEAN (MM)	:	2.59	1.58
STANDARD DEVIATION (MM)	:	2.73	1.79
COEFF. OF VARIATION	:	1.05	1.13

of all performance indices near the optimum are relatively low for both storm groups, suggests model structure robustness to deviations of the parameters from the optimal values. It is also worth noticing that the hydrologically significant performance index APME is relatively sensitive to the value of the updraft velocity regulating parameter EPS1, while it is relatively insensitive to the cloud average particle diameter EPS4 for EPS4 greater than 50 micrometers and EPS1 near the optimum.

The time trace of the estimated updraft velocity in M/SEC, of the estimated cloud height in KILOMETERS is shown in Figures 10, 11 and 12, respectively, for the convective storm group, with parameter values:

$$EPS1 = 1.84 \times 10^{-3}$$

$$EPS4 = 6.4 \times 10^{-5}$$

Those figures show that the three observable quantities took reasonable values. Characteristic is the fact that the updraft velocity took values in the vicinity of 10 CM/SEC with small variation with time. These values are usually attributed to the stratiform precipitation systems and to the mesoscale areas associated with individual cells in intense convection (see for example Mason, 1971, and Houze and Betts,1981). Naturally, the values in Figures 10 through 12 are hourly averaged estimates. The corresponding fit of the hourly predictions of the precipitation rate (MM/HOUR) in dashed line to the corresponding observations in thick solid line is shown in Figure 13. Shaded regions present the forecast errors. Figure 14 gives the time trace of the filter gains, suggesting non-divergence conditions of filter operation.

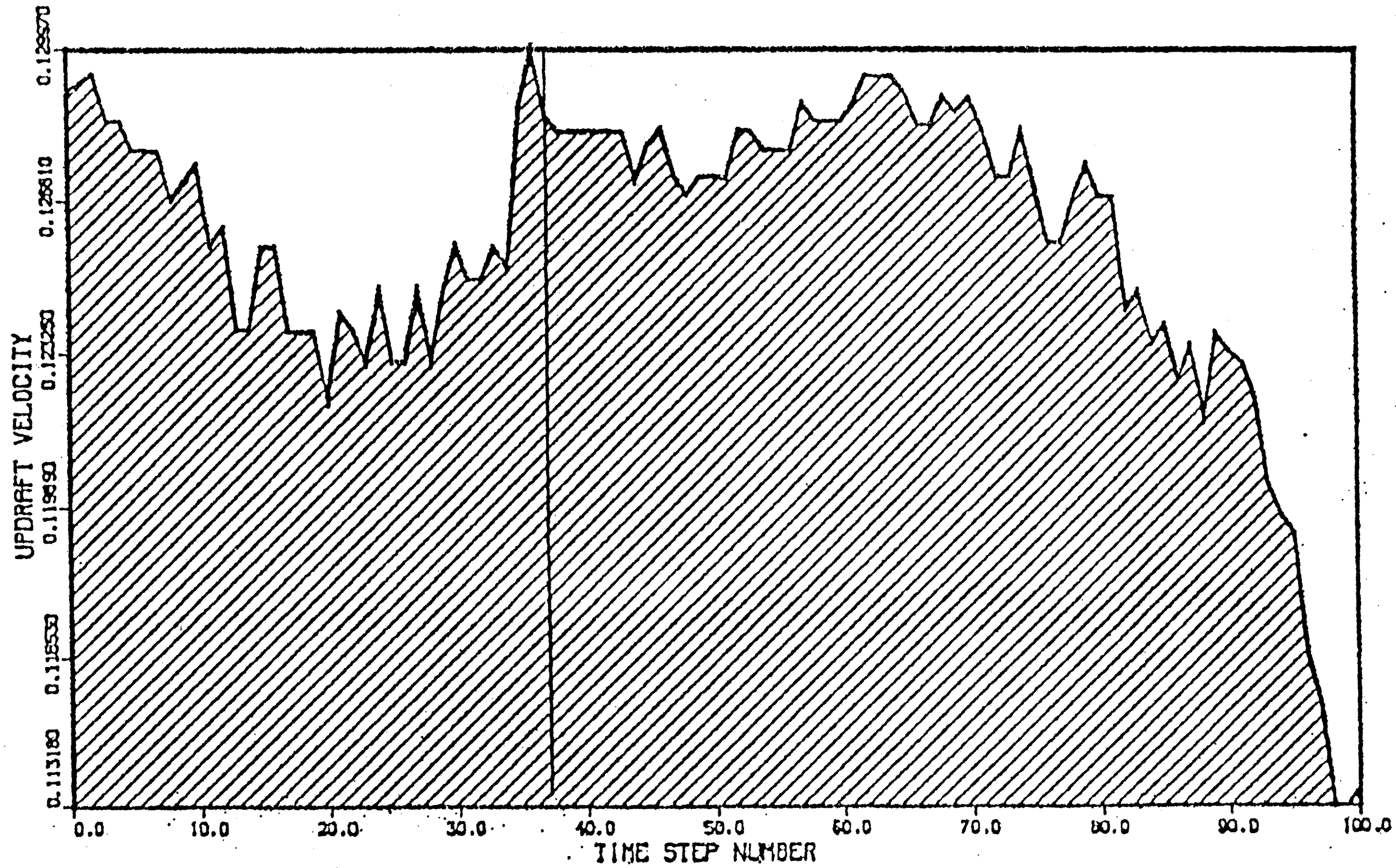

FIGURE 10 Time trace of the estimated hourly averaged updraft velocity in M/SEC for the convective storm group used in calibration

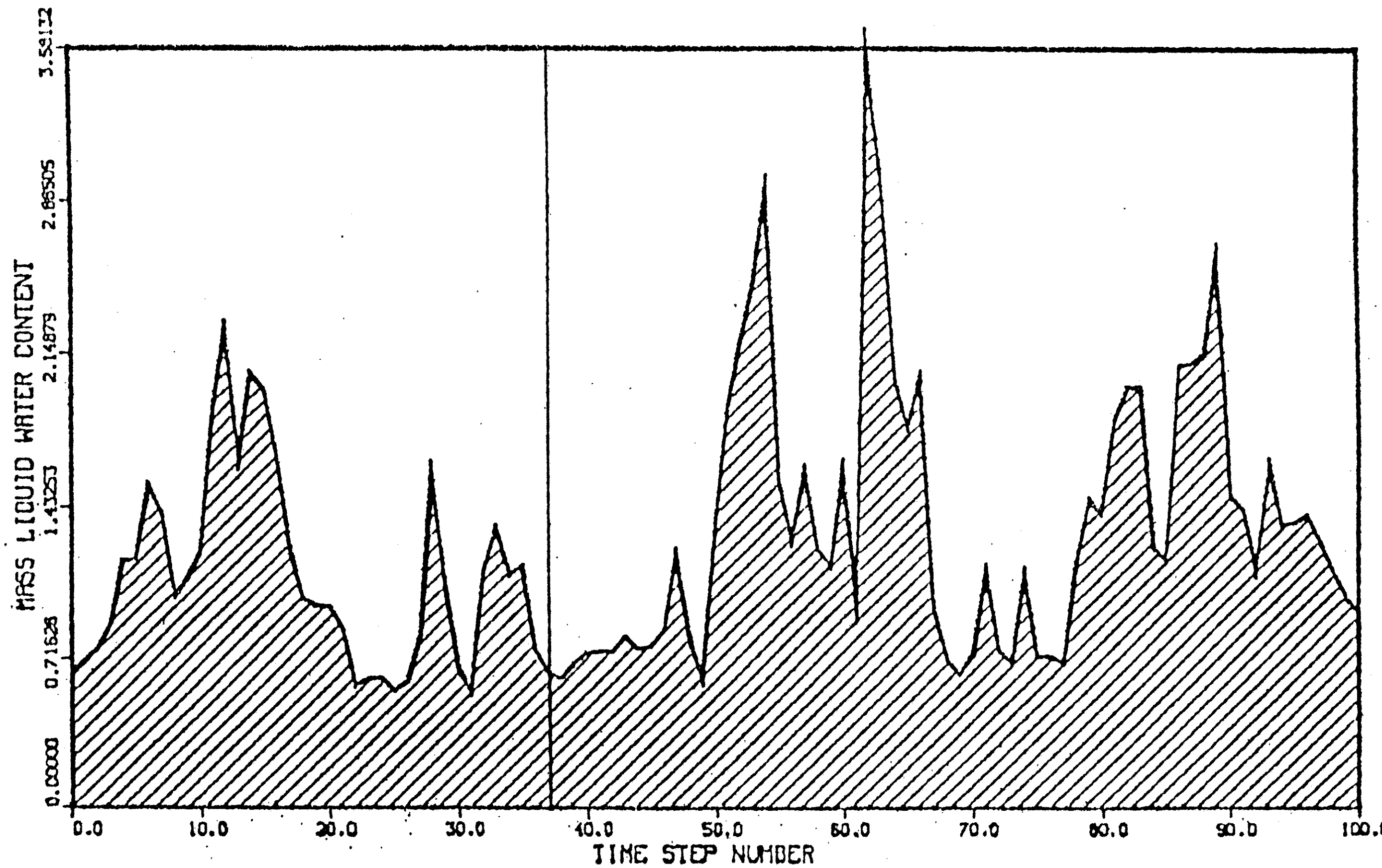

FIGURE 11 Time trace of the estimated hourly averaged mass liquid water content in GRAMS/M^3 for the convective storm group used in calibration

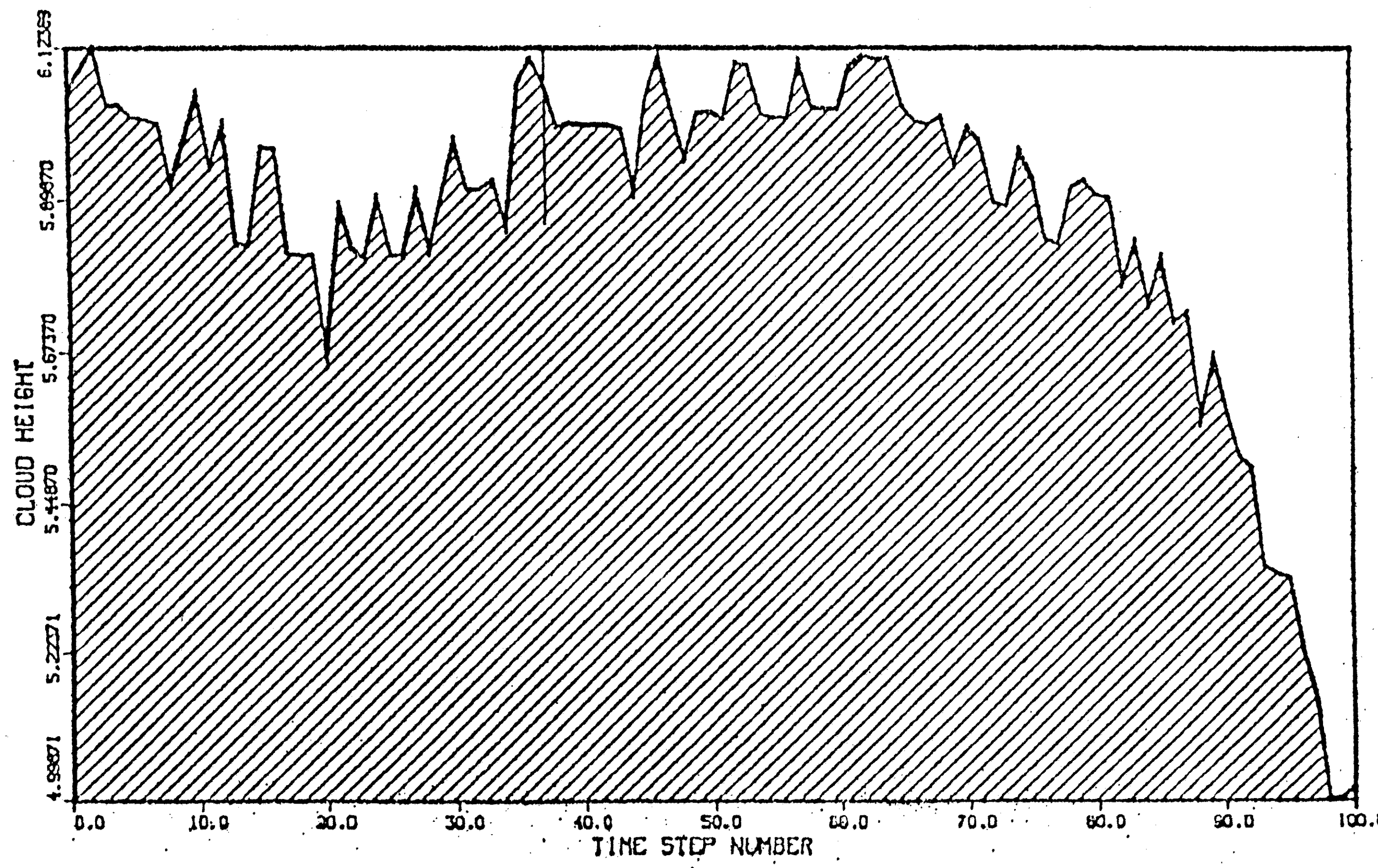

FIGURE 12 Time trace of the estimated hourly averaged cloud height in KILOMETERS for the convective group used in calibration

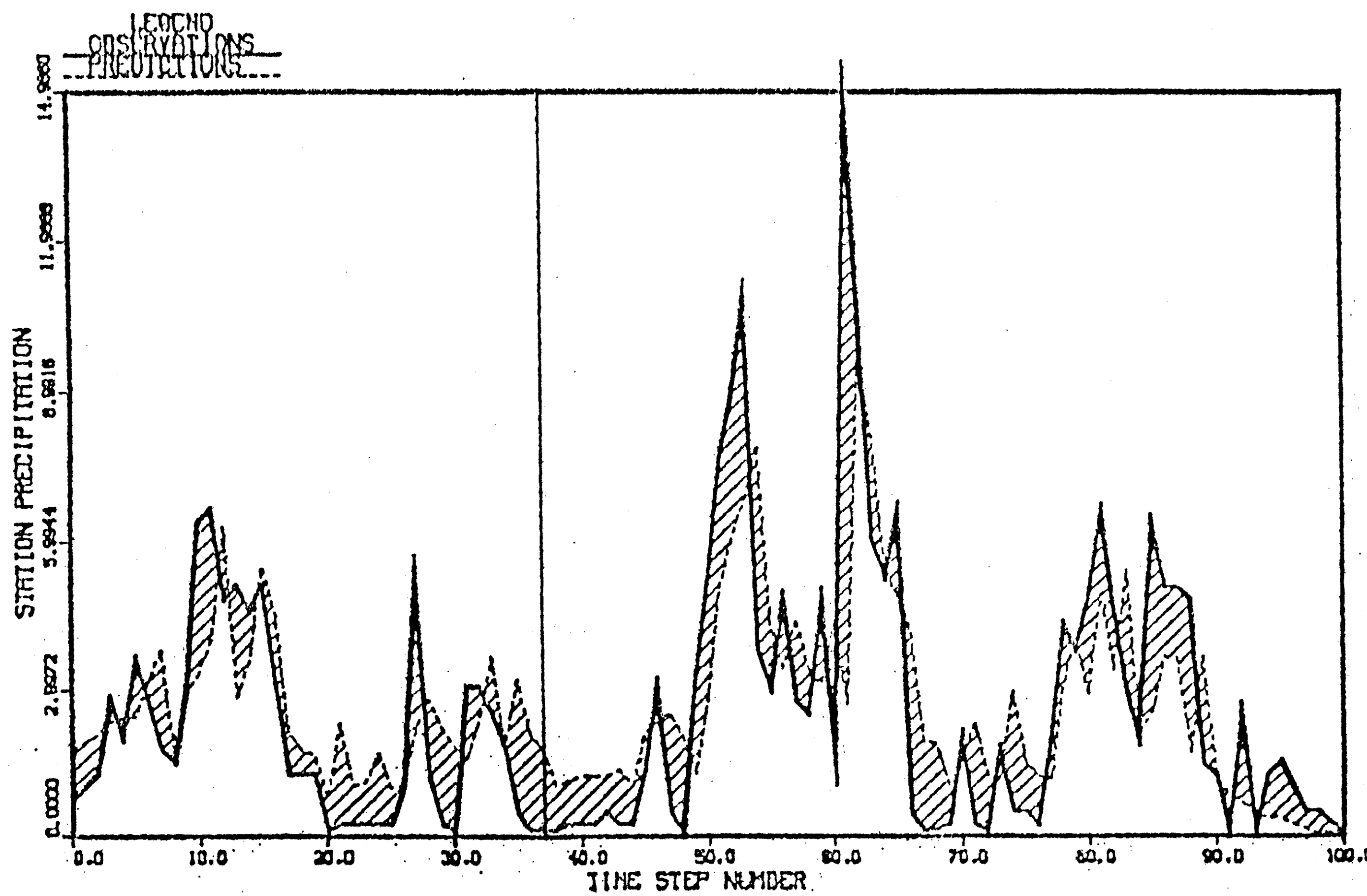

FIGURE 13 Forecasts (dashed line) vs. observations (thick solid line) for the convective storm group used in calibration(BOSTON, MA). One hour forecast lead time.

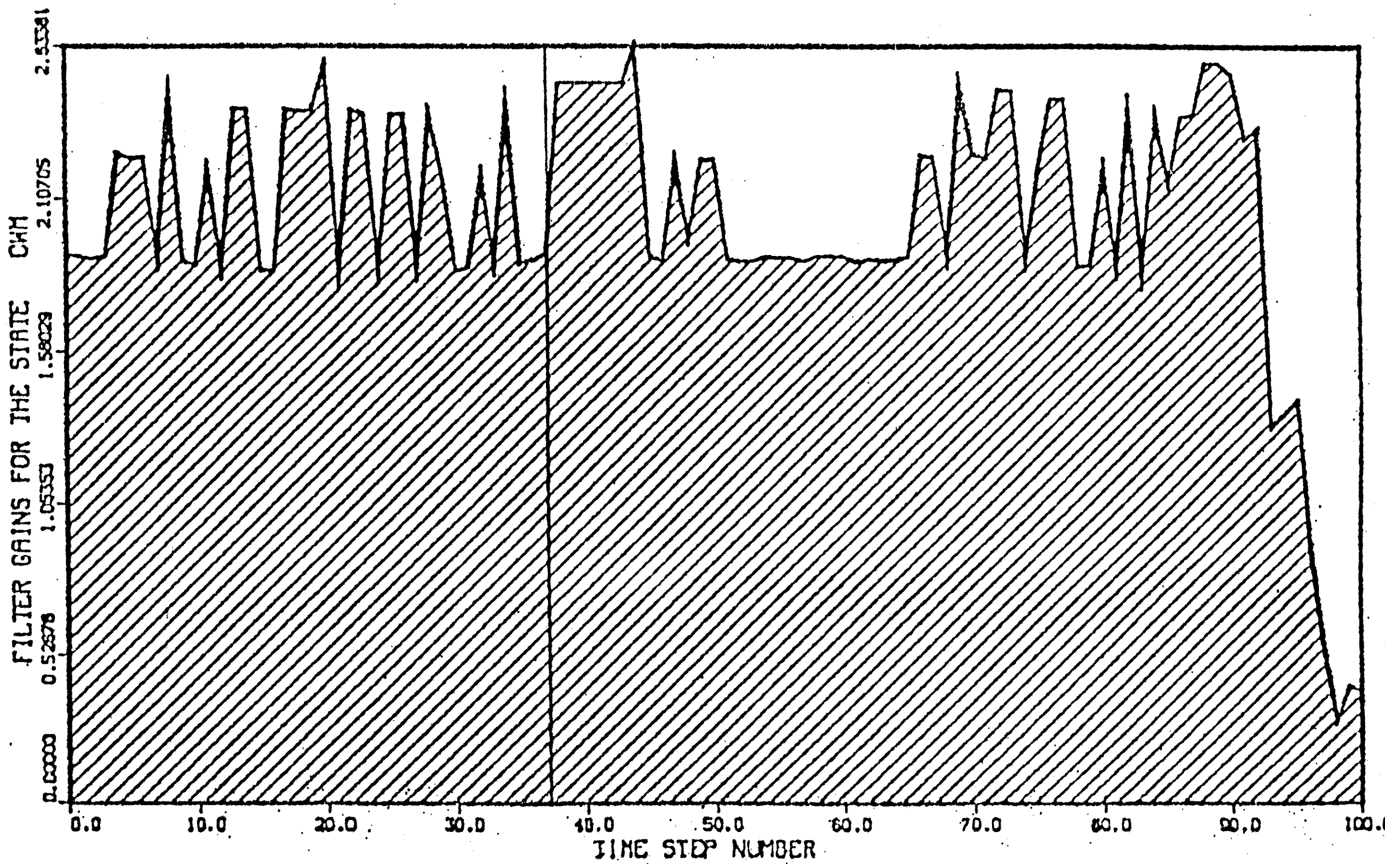

FIGURE 14 Time trace of the Kalman filter gains in KGxHOURS/MM for hourly feedback and for the convective storm group used in calibration

VERIFICATION

The parameter values:

$$EPS1 = 1.65 \times 10^{-3}$$

$$EPS4 = 5.5 \times 10^{-5}$$

were adopted for the verification run. Those values represent a compromise among the sets of optimal values indicated by the contours in Figures 2 to 9 and by the values in Table 1.

Three storms from Tulsa, Oklahoma, were combined to form a verification group. Figure 15 presents the hourly forecasts of the precipitation rate in MM/HOUR and the corresponding observations. Again, the thick solid line corresponds to the observations.

The absolute mean error was 20% of the observations mean. The proportional standard error was 0.83, indicating a 30% variance reduction. The persistence coefficient was 0.10 showing improvement over the predictions of a simple persistence scheme whose predictions are the current observations. The proximity of the APME and PSE values corresponding to the verification run to the ones in Table 1, corresponding to the parameter estimation runs for Boston, supports model structure and parameter values robustness to location changes.

Reasonable values of the physical model components were observed in the verification run. For example, the updraft velocity had an average value of 12 CM/SEC, the average liquid water content of the cloud column was 1.39 GRAMS/M^3 and the cloud height average was 5.86 KILOMETERS.

The low, -0.12, lag-1 correlation coefficient of the step-1 predicted residuals indicated satisfactory filter performance.

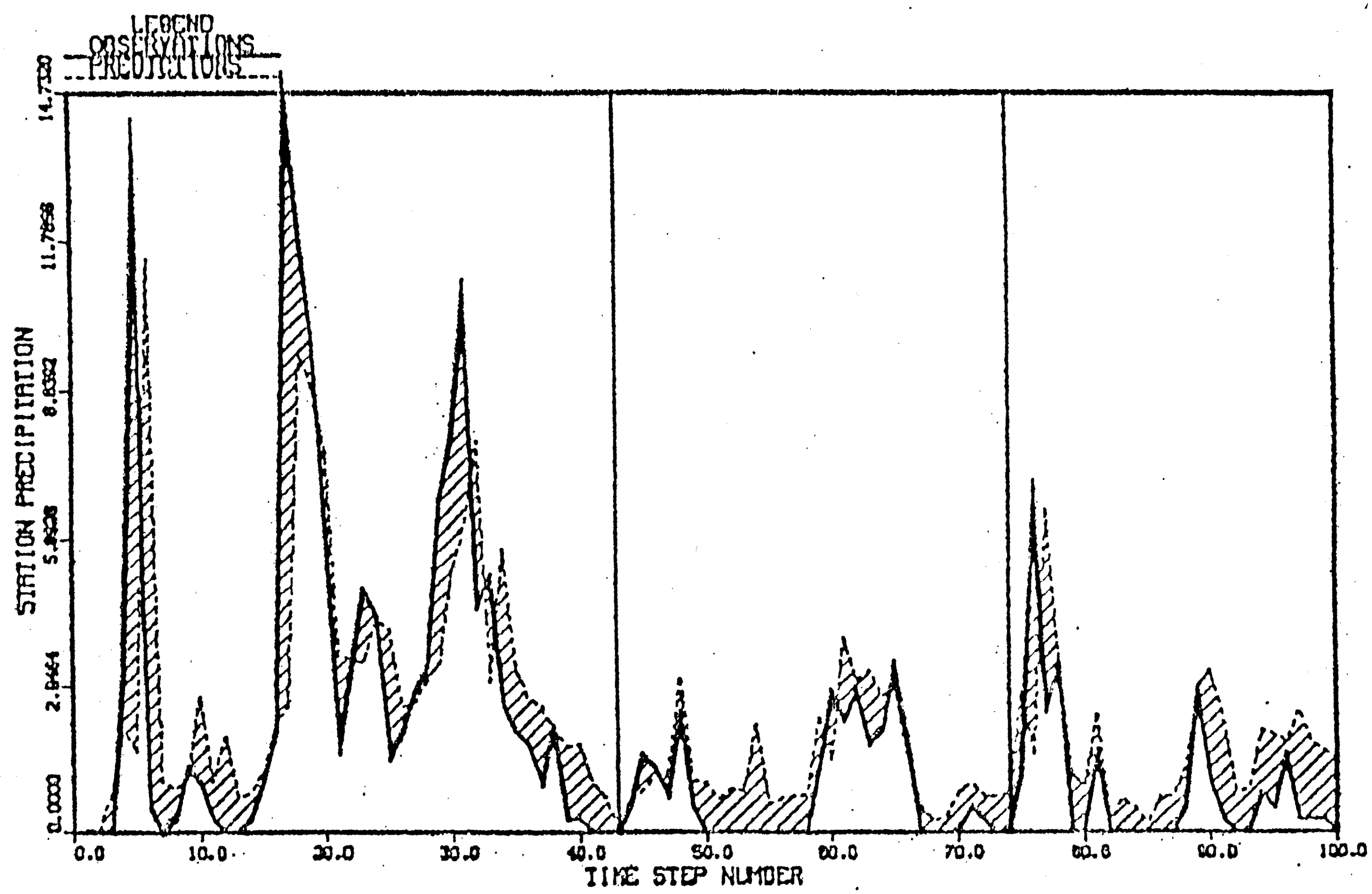

FIGURE 15 Forecasts (dashed line) vs. observations (thick solid line) for the verification storm group (TULSA, OK). One hour forecast lead time.

CONCLUSIONS

The contour maps of the parameter space of the stochastic model proposed by Georgakakos and Bras (1982) indicated robust model structure in the forecast of hourly precipitation rates for several performance criteria and different storm types. The physically based model components took physically realistic values in all cases. A verification run with hourly storm data from a different location (Tulsa, Oklahoma, vs. Boston, Massachusetts) indicated that the model parameters are reasonably location independent as well. Therefore, the model does not require recalibration for different storms and locations. This is especially convenient for real-time forecasting uses.

Currently, efforts are concentrated in the examination of different model structures as they are realized by non-uniform height profiles of updraft velocity and cloud layer-averaged diameter.

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