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The Permeability of a Melting Snow Cover

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Data from snow lysimeters in California and Vermont are used to find the saturated permeability of a melting snow cover in the range of 10–40 × 10^{-10} m^2 depending on snow density. The unsaturated permeability increases as about the third power of liquid saturation. The gravity flow theory is shown to be an accurate representation of meltwater drainage from snow covers in two diverse areas even though the snow covers are treated as homogeneous units. The variation of saturated permeability with snow density occurs about as predicted by Shimizu's formula for dry snow, although ice layers decrease the permeability somewhat.

INTRODUCTION

Daily forecasts of snow cover runoff have long been needed because of flood potentials, water supply demands, and other water management requirements. There is a continual need to improve the procedures used to generate these forecasts by incorporating more physically based techniques to model the various parts of the snow accumulation and ablation processes. Among these are improved techniques to account for the movement of water through the snow cover.

Thinking of the snow cover as another porous material overlying the soil, it is clear that the snow's unsaturated permeability would have to be known for use with the usual methods of describing flow through an unsaturated medium. Unfortunately, the usual methods of measuring the intrinsic and unsaturated permeabilities are not applicable to snow for several reasons. First, the liquid content of snow at its melting temperature would be very difficult to control in the laboratory during testing. Second, meaningful measurements of the saturated case would be difficult because wet snow undergoes changes in its structure when saturated. The saturated state consists of well-rounded, cohesionless grains, while at low liquid contents (2–6% by volume is normal) the grains join in large, well-bonded clusters [*Colbeck*, 1979]. Third, the basic geometry of the snow grains changes in response to changes in capillary pressure. Thus methods such as *Mualem*'s [1976] are of little value with snow, even if high-quality information about capillary pressure versus liquid content were to become available.

Fortunately, snow is a large-grained material. During meltwater percolation through snow, gravity forces dominate tension gradients such that the flow is essentially noncapillary. Thus the flow can be described in a straightforward manner, and when good-quality surface input and discharge data are available, the unsaturated and intrinsic permeabilities can be determined from those data. The method of analysis, originally developed for columns of homogeneous snow by *Colbeck and Davidson* [1973], is reviewed and applied here to undisturbed seasonal snow.

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The method has also been applied on the small scale to a variety of snow types by *Denoth and Seidenbusch* [1978]. The gravity flow theory, which is the basis of this method, has been successfully applied on a large scale to deep glacial firn by *Ambach et al.* [1981] and to a melting snow cover by *Dunne et al.* [1976]. Nevertheless, the unsaturated permeability of a melting snow cover has never been directly determined on a scale larger than a few tens of centimeters. The determination is made here for snow that has experienced melt metamorphism.

Method

During the percolation of meltwater the conservation of the liquid phase requires that

$$\frac{\partial u}{\partial z} + \phi_e \frac{\partial S_e}{\partial t} = 0 \tag{1}$$

where the effective porosity is

$$\phi_e = \phi(1 - S_{wi}) \tag{2}$$

and the effective saturation is

$$S_e = \frac{S_w - S_{wi}}{1 - S_{wi}} \tag{3}$$

The noncapillary flow of water is described by

$$u = \alpha k_w \tag{4}$$

where we assume the usual power law relation between the saturated and unsaturated permeabilities,

$$k_w = k S_e{}^n \tag{5}$$

Mualem [1978] reviews other forms of this relation, but the power law form is most convenient for our purposes. Furthermore, we assume $n = 3$. Although this value is somewhat dependent on the types of snow [*Denoth et al.*, 1979] and soil [*Mualem*, 1978], the integral value is very convenient. As will be shown later, there is very little error in choosing this value.

Combining these equations and concepts into a single form, the flow is described by

$$3\alpha^{1/3}k^{1/3}u^{2/3}\frac{\partial u}{\partial z} + \phi_e \frac{\partial u}{\partial t} = 0 \tag{6}$$

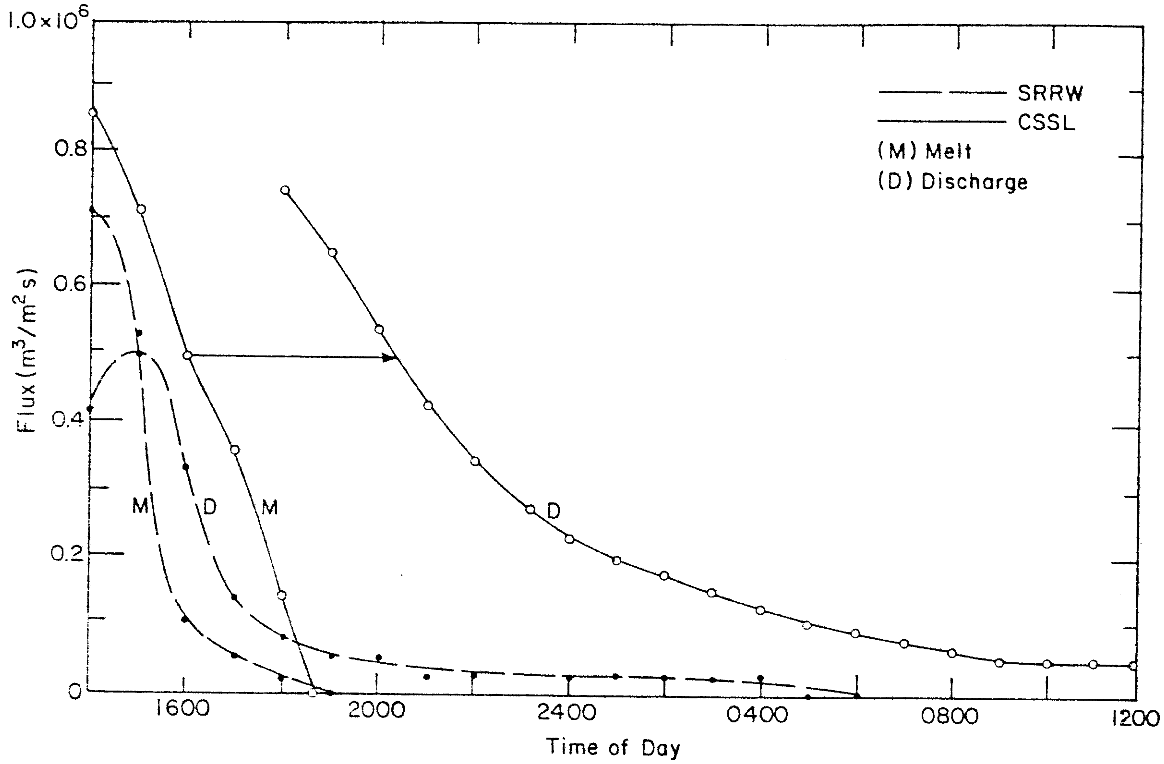

Fig. 1. The surface melt (M) and discharge (D) from two lysimeters, one at the Sleepers River Research Watershed in Vermont (March 29, 1973; snow depth of 0.20 m) and one at the Central Sierra Snow Laboratory in California (April 19, 1954; snow depth of 1.45 m). The travel time of a flux value equal to 0.5×10^{-6} m^3/m^2 s is shown by the arrow for the Sierra snow cover. Note the greater distance between melt and discharge for the deeper Sierra snow.

For our purposes this equation can be most readily solved by the method of characteristics. The slope of a characteristic line for a fixed value of flux is [*Colbeck and Davidson*, 1973]

$$\left.\frac{dz}{dt}\right|_u = 3\alpha^{1/3}k^{1/3}\phi_e^{-1}u^{2/3} \tag{7}$$

Physically, this slope in z-t space is the speed with which a given value of flux (u) moves downward through the snow. Since larger values of flux move faster, the characteristic lines intersect to form a shock if the surface flux is increasing with time but disperse without intersecting if the surface flux is decreasing with time. Thus during the afternoon when the rate of snowmelt decreases monotonically, the characteristic line for each value of flux extends from the surface to the base of the snow without crossing other lines. In this case, the characteristic slope of the value u, $dz/dt|_u$, equals the snow depth divided by the travel time between the surface and base. If the surface flux (melt plus rain and condensation) and snow discharge are known as functions of time, the travel time of a value of flux is the difference in time between when that value left the surface and when it reached the base (see Figure 1).

A set of values of wave speed ($dz/dt|_u$) versus flux (u) is constructed from the receding part of the surface flux and discharge data such as shown in Figure 1. The snow depth is the only additional information needed. When this data set is plotted on log-log scale such as shown in Figure 2, the slope of the line is given by the exponent of u in (7), and the position of the line on the graph is determined by the coefficient $3\alpha^{1/3}k^{1/3}\phi_e^{-1}$. Therefore the ratio $k^{1/3}\phi_e^{-1}$ can be calculated directly from the graph. Given the snow density, liquid water content, and irreducible water content (S_{wi} is about 0.07), the effective porosity can be determined from (2) and

$$\rho_s = \rho_i(1 - \phi) + \rho_l S_w \phi \tag{8}$$

With values of effective porosity and $k^{1/3}\phi_e^{-1}$, the intrinsic or saturated permeability is found directly. While the saturated permeability is desirable, the ratio

$$k^{1/3}\phi_e^{-1}$$

is actually sufficient to route the water with the gravity flow theory. The intrinsic or saturated permeability would be required for routing snowmelt runoff using other approaches [e.g., *Morris and Godfrey*, 1979].

Lysimeter Data

Snowmelt and runoff data are used from two lysimeters, one at the Central Sierra Snow Laboratory (CSSL) in California [see *U.S. Army Corps of Engineers*, 1956] and one at the NOAA-ARS Snow Research Station located within the Sleepers River Research Watershed (SRRW) in Vermont [*Anderson et al.*, 1977]. The CSSL device was 55.7 m^2 in area, which provides a large sample of undisturbed snow with a uniform slope, depth, and meteorological environment. The slope of 10% was sufficient to ensure that water movement to the drains along the sloping base was much faster than the percolation down through the snow; thus the time delay due to movement along the base could be ignored. The snow itself rested on a pervious subdeck within the lysimeter.

For the CSSL lysimeter, melt was computed from the

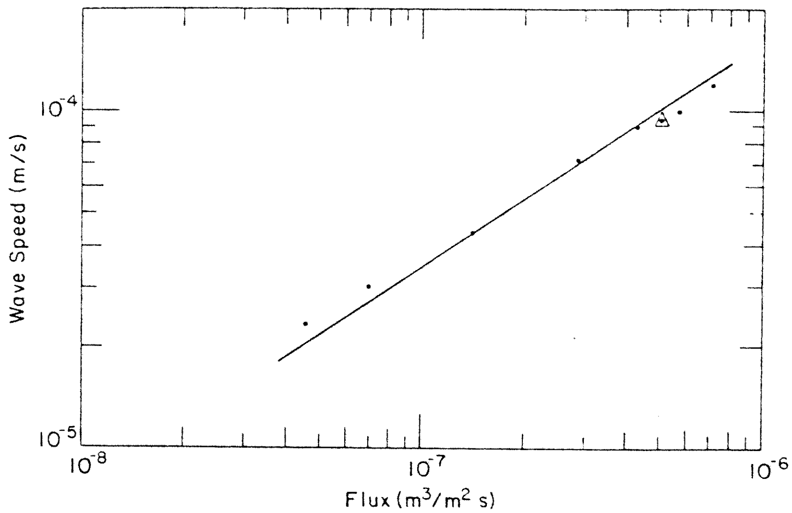

Fig. 2. The wave speed ($dz/dt|_u$) versus flux (u) for April 19, 1954, from the Central Sierra Snow Laboratory data shown in Figure 1. The line is assumed to have a slope of two thirds. The triangular data point corresponds to the arrow shown in Figure 1.

energy balance equations described by *Anderson* [1968]. Hourly melt was determined by applying a fixed time distribution, based on clear day melt conditions at CSSL, to the computed daytime melt. In order to arrive at the excess water available at the snow surface, nighttime heat deficits were subtracted from the first hours of melt, rain was added to the meltwater, and the result was adjusted so that inflow was equal to outflow on each day.

On the days analyzed at the CSSL the weather was clear, with melt caused principally by solar radiation. Thus daily melt consisted of monotonically increasing melt until just after noon followed by monotonically decreasing melt until the sun vanished. There was sufficient melt prior to the days analyzed that the snow was thoroughly wetted and was not undergoing rapid changes in density or grain structure during any of our test days. Thus the values of $k^{1/3}\phi_e^{-1}$ and k shown in Table 1 represent snow which has stable properties on the time scale over which the melting occurs.

Conditions at SRRW in Vermont are quite different from those at CSSL. The snow is shallower in Vermont and experiences more rapidly changing weather patterns during the melt period. The days analyzed were chosen to ensure conditions as similar to those in California as possible; the snow was thoroughly wetted prior to the days analyzed, the melt occurred primarily due to solar radiation, and the melt rate decreased smoothly to zero (see Figure 1). The lysimeter in Vermont has a smaller area of 0.7 m^2, which is still large enough to sample the undisturbed snow cover as a unit.

For the SRRW lysimeter, energy and mass balance computations were carried out at hourly intervals to determine the excess water at the snow surface [*Anderson*, 1976]. The computed excess water was not adjusted; thus on a daily basis there are slight differences between the excess water entering at the surface and the outflow measured by the lysimeter.

Results

The results are shown in Figure 3 and Table 1 for the 8 days analyzed. The data in Figure 3 are scattered, but the results generally support the use of the gravity flow theory for snow and the use of the exponent 3 in (5). *Ambach et al.* [1981] found the value of 2.8 rather than 3, based on their work with glacial firn. A visual inspection of our data does confirm their result, although the difference is probably not worth sacrificing the convenience of the integer. This is especially important for routing the leading edge of a meltwater wave through snow.

Some of the scatter in the data in Figure 3 can be explained by the day-to-day variations in the original data and between inflow and outflow at SRRW. The variations in reported snow density undoubtedly contain sampling errors, which cause some of the scatter but do not have a large effect on our results, especially when k is calculated from $k^{1/3}\phi_e^{-1}$. Any error in ϕ_e, which is calculated from ρ_s, is magnified by cubing the derived quantity to get k. Nevertheless, these calculated values of k should be close to the true values. In

TABLE 1. Data Summary

Date	Depth, m	$k^{1/3}/\phi_e$, $m^{2/3}$	k, m^2	ρ_s, mg/m^3	ϕ_e
		CCSL			
April 17, 1954	1.57	2.39×10^{-3}	25.9×10^{-10}		0.47
April 19, 1954	1.45	3.01×10^{-3}	28.8×10^{-10}	0.486	0.47
April 21, 1954	1.32	2.87×10^{-3}	25.3×10^{-10}	0.484	0.47
April 25, 1954	1.07	2.53×10^{-3}	13.9×10^{-10}	0.517	0.44
May 4, 1954	0.914	2.80×10^{-3}	21.7×10^{-10}	0.496	0.46
May 8, 1954	0.61	2.73×10^{-3}	18.5×10^{-10}	0.509	0.45
		SSRW			
March 12, 1973	0.41	2.54×10^{-3}	22.9×10^{-10}	0.44	0.52
March 29, 1973	0.20	2.74×10^{-3}	38.1×10^{-10}	0.39	0.57

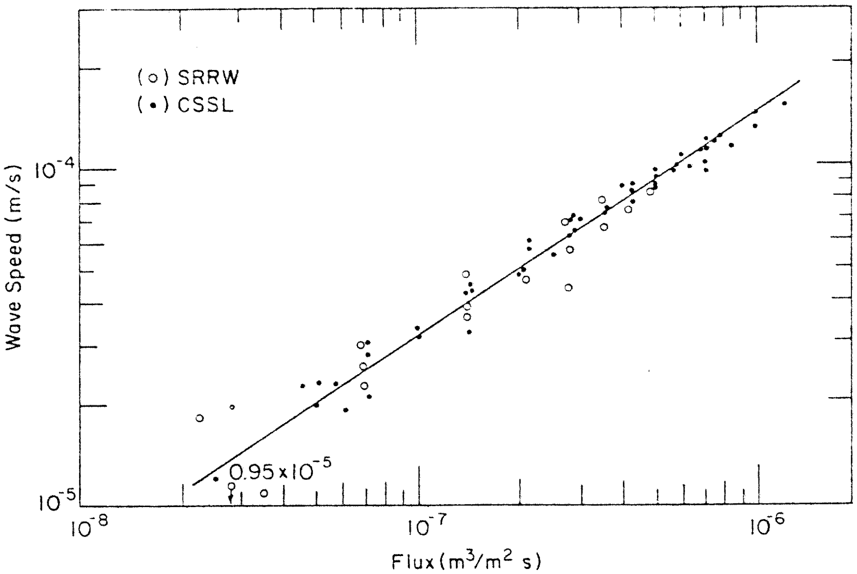

Fig. 3. The wave speed ($dz/dt|_u$) versus flux (u) from data sets from 6 days of melting at the Central Sierra Snow Laboratory and 2 days at Sleepers River Research Watershed. The line is assumed to have a slope of two thirds.

general, values obtained by this method agree with values calculated from permeability formulae [*Colbeck and Davidson*, 1973; *Denoth and Seidenbusch*, 1978]. The calculated values of saturated permeability are compared with *Shimizu*'s [1970] formula in Figure 4.

The derived values of $k^{1/3}\phi_e^{-1}$ from the CSSL data vary from 2.53 to 3.01 × 10^{-3} $m^{2/3}$, without any apparent cause. In Figure 4 it is clear that permeability decreases with increasing snow density as expected. There are definite indications of differences between the snow at the Vermont and California sites. A difference is suggested in Figure 4 but is even more apparent in the values of $k^{1/3}\phi_e^{-1}$ versus density shown in Table 1. For a given snow density the permeability seems to be higher in California, possibly because of the presence of more ice layers in the snow cover in Vermont. In California during long periods of sunny weather, the ice layers tend to break down [*Gerdel*, 1954].

Conclusions

The intrinsic or saturated permeability of a well-metamorphosed, melting snow cover lies in the range of 10–40 × 10^{-10} m^2. This figure is 1 order of magnitude lower than the value assumed by *Bengtsson* [1981] and refutes his conclusion about the very rapid drainage of a melting snow cover.

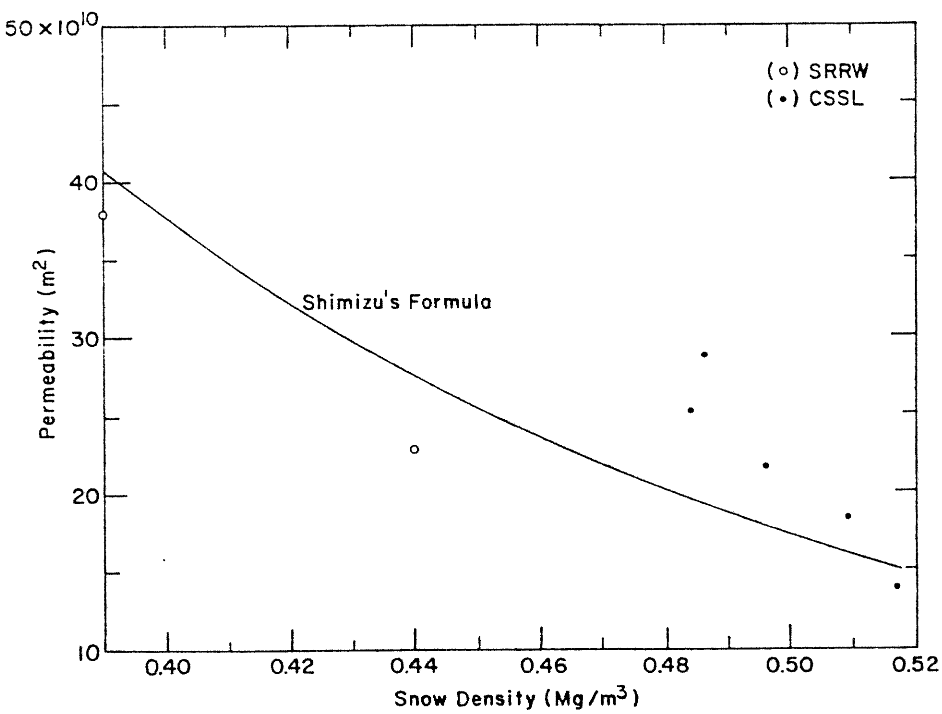

Fig. 4. The intrinsic or saturated permeability versus density and the permeability from *Shimizu*'s [1970] formula for an arbitrary grain size of 0.9 mm.

In fact, as shown in Figure 1, the vertical part of the drainage is quite slow. It is shown to be much slower for the deeper Sierra snow cover and for lower values of flux. The drainage of an undisturbed snow cover can be explained by the gravity flow theory, even in the presence of ice layers and drainage channels. For a deep, natural snow cover, we think of the 'equivalent permeability' of the snow cover as a whole. This is necessary because snow covers are layered and the layering, as well as the average density, affects the permeability.

NOTATION

$dz/dt\|_u$	wave speed or rate of propagation of a given value of u.
k	intrinsic or saturated permeability, m^2.
k_w	unsaturated or relative permeability to water, m^2.
n	exponent assumed to equal 3.
S_e	effective saturation.
S_w	water saturation as fraction of pore volume.
S_{wi}	irreducible water saturation.
t	time, s.
u	volume flux of water, m^3/m^2 s.
z	depth, m.
α	water density times gravity divided by viscosity, equal to 5,470,000 m^{-1} s^{-1}.
ρ_i	ice density, equal to 0.917 Mg/m^3.
ρ_l	water density, equal to 1 Mg/m^3.
ρ_s	snow density, equal to Mg/m^3.
ϕ	porosity.
ϕ_e	effective porosity.

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REFERENCES

Anderson, E. A., Development and testing of snow pack energy balance equations, *Water Resour. Res., 4,* 19–37, 1968.

Anderson, E. A., A point energy and mass balance model of a snow cover, *NOAA Tech. Rep. NWS 19,* U.S. Dep. of Commerce, Silver Spring, Md., 1976.

Anderson, E. A., R. Z. Whipkey, H. J. Greenan, and C. T. Machell, NOAA-ARS Cooperative Snow Research Project—Watershed hydro-climatology and data for water years 1960–1974, report, U.S. Dep. of Commerce, Silver Spring, Md., 1977.

Ambach, W., M. Blumthaler, and P. Kirchlechner, Application of the gravity flow theory to the percolation of meltwater through firn, *J. Glaciol., 27,* 67–75, 1981.

Bengtsson, L., Snowmelt generated run-off from small areas as a daily transient process, *Geophysica, 17,* 109–121, 1981.

Colbeck, S. C., and G. Davidson, Water percolation through homogeneous snow, in *The Role of Snow and Ice in Hydrology,* vol. 1, p. 242–256, UNESCO-WMO-IAHS, Geneva, 1973.

Colbeck, S. C., Grain clusters in wet snow, *J. Colloid Interface Sci., 72,* 371–384, 1979.

Denoth, A., and W. Seidenbusch, A method for determination of the hydraulic conductivity of snow, *Z. Gletscherk. Glazialgeol., 14,* 209–213, 1978.

Denoth, A., W. Seidenbusch, M. Blumthaler, P. Kirchlechner, W. Ambach, and S. C. Colbeck, Study of water drainage from columns of snow, *CRREL Rep. 79-1,* Cold Regions Res. and Eng. Lab., Hanover, N. H., 1979.

Dunne, T., A. G. Price, and S. C. Colbeck, The generation of runoff from subarctic snowpacks, *Water Resour. Res., 12,* 677–685, 1976.

Gerdel, R. W., The transmission of water through snow, *Eos Trans. AGU, 35,* 475–485, 1954.

Morris, E. M., and J. Godfrey, The European Hydrologic System snow routine, in *Proceedings Modeling of Snow Cover Runoff* edited by S. Colbeck and M. Ray, pp. 269–278, Cold Regions Research and Engineering Laboratory, Hanover, N. H., 1979.

Mualem, Y., A new model for predicting the hydraulic conductivity of unsaturated porous media, *Water Resour. Res., 12,* 513–522, 1976.

Mualem, Y., Hydraulic conductivity of unsaturated porous media: Generalized macroscopic approach, *Water Resour. Res., 14,* 325–334, 1978.

Shimizu, H., Air permeability of deposited snow, *Low Temp. Sci., Ser. A, 22,* 1–32, 1970.

U.S. Army Corps of Engineers, *Snow Hydrology: Summary Report of Snow Investigations,* North Pacific Division, Portland, Oreg., 1956.

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