UPWARD MIGRATION OF WATER UNDER COLD BUT ABOVE FREEZING SURFACE CONDITIONS

by

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Introduction

The objective of this paper is to estimate the order of magnitude of the upward migration of water from a water table to the surface due to a gradient of temperature directed toward the surface. The rigorous solution would require simultaneous solution of nonlinear partial differential equations for heat and mass transport. To size the importance of the phenomenon a very crude approach is taken in this paper which includes many assumptions. As the mass transport is the phenomenon of concern a steady-state temperature profile will be assumed, independent of moisture evolution. Initially the moisture profile is assumed in equilibrium at a uniform relatively warm temperature. Then a steady-state temperature profile is imposed on the system with colder temperatures towards the surface. It is assumed that there is a plentiful water supply at the water table and no evaporation from the surface. When a thermal gradient is imposed the following transient state results in an upward flow of water. The questions to be answered are: (1) how fast will this migration take place, and (2) ultimately when a new equilibrium is reached how much more water will be in storage in the upper part of the soil.

Background

It has been observed in the field (e.g., Ferguson et al. 1964, Willis et al. 1964, Peck, 1974) that appreciable upward water movement occurs during the winter time. For example Ferguson et al. (1964, p. 701) report an increase of (volumetric) water content from a value of 0.26

on Dec. 20, 1962 to a value of 0.32 on March 20, 1963, followed by a decrease to a value of 0.28 on April 26, 1963 at a soil depth of 24 in. On the other hand at a depth of 42 in. the reverse trend was noted (0.23 on Dec. 20, 1962, 0.20 on March 20, 1963 and 0.21 on April 26, 1963). The ground was frozen to a depth of 12 in. on January 4. Thus a relative increase of water content of the order of 23 percent i.e. $100\ (\frac{0.32 - 0.26}{0.26})$ occurred right around the depth of maximum penetration of the freezing front. The mechanism of attraction of liquid water to a freezing front and the growth of ice lens is well understood qualitatively and semiquantitatively (e.g., Morel-Seytoux, 1978). Peck (1974, p. 406) reports that in the spring of 1969 "the average soil moisture for the 18 stations was 44 percent" (by weight) "or approximately 12 percent greater than their average field capacity" (32 percent by weight). This corresponds to a relative increase of water content of 38 percent in the top soil. Similar water content changes were reported for the years 69-70, 70-71 and 71-72 between mid-November and February or March of the following year. However, there is a difference between these results and the spring 1969 observations. During the water years 70, 71 and 72 "frost was observed under the late season snow cover" (Peck, 1974, p. 408) whereas in "the spring of 1969 a large portion of the Rock River basin was found to be frost free." Could such a large upward water migration have been induced strictly by a temperature gradient without the added suction effect of ice?

Temperature Effects

It is known that capillary pressure (and therefore field capacity) and hydraulic conductivity are temperature dependent (e.g., Klock, 1972). Klock found that the variability of (saturated) hydraulic conductivity with temperature can be completely accounted for by the dependence of hydraulic conductivity on water viscosity. A change of temperature from $25^{o}C$ to $0^{o}C$ will double the water viscosity and halve the hydraulic conductivity. Klock also found that warming a previously soil column from $0.3^{o}C$ to $25^{o}C$ would result in additional drainage of a volume equal to 12 percent of the initially drained volume. Again this effect could be completely accounted by the dependence of surface tension on temperature. In the experiment the soil column was 123 cm long and a constant water table level was maintained at the bottom of the column.

The effect of temperature on unsaturated soil properties was investigated by Jensen et al. (1970). They found similar results.

In all these investigations however comparison was made between properties at two different but both equilibrium isothermal states. It is the objective of this study to investigate the effect of a temperature gradient on the equilibrium capillary state and on the rate of migration.

Temperature Dependence

The capillary pressure is dependent upon temperature because the water-air surface tension is. Presuming that the temperature dependence is strictly reflected through the surface tension then the capillary pressure, h_c, expressed as a water height, at a uniform temperature T is deduced from that at a uniform *base* temperature T_B by the relation:

$$h_c = h_{cB}(\theta)\left(\frac{374-T}{374-T_B}\right) \qquad (1)$$

where temperature is expressed in °C. The h_c curve on Figure 1 is for a uniform temperature of 15°C. For a water table 2 m below the soil

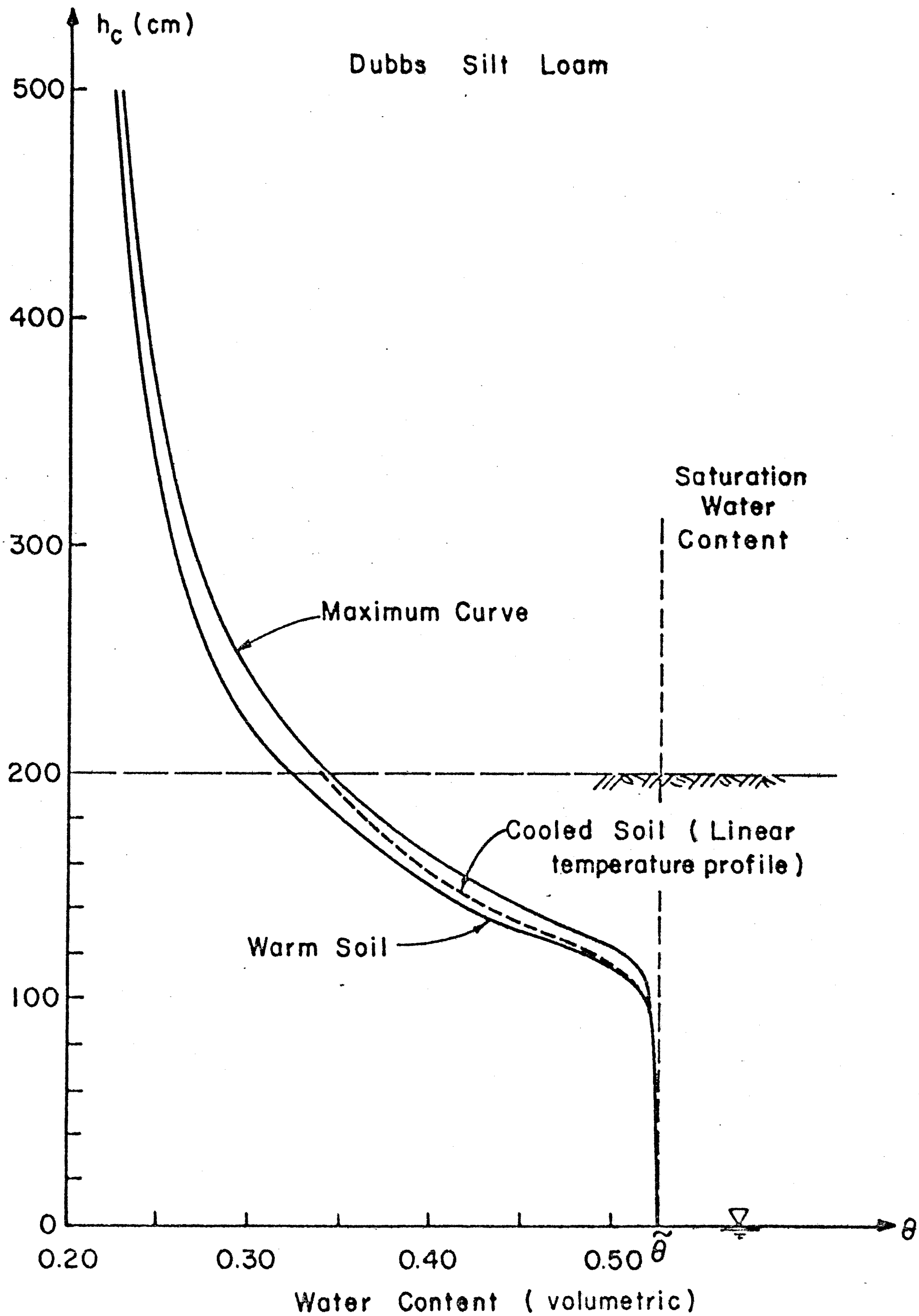

Figure 1. Isothermal water content profiles at different temperatures and steady-state equilibrium profile for a uniform temperature gradient (data after Jensen et. al., 1970).

surface under equilibrium conditions the moisture profile above the water table would be as shown on Figure 1 reading h_c as elevation above the water table. At the soil surface the water content would be 0.320 which is somewhat larger than field capacity estimated at 0.26, if field capacity is defined as water content at 0.3 atmosphere. However it can be seen from Figure 2 that at a water content of 0.26 the relative permeability is very small. If a value of $k_{rw} = 0.01$ is chosen as a definition of field capacity then water content at field capacity, θ_{fc} is more likely to be about 0.30.

Steady-State Profiles

At time zero a new temperature profile is imposed on the system and is maintained indefinitely. For simplicity it is assumed that the temperature gradient is uniform between the soil surface and the water table and the variation of temperature with depth is given by the relation:

$$T = T_f + (T_B - T_f)\,\frac{z}{D} \tag{2}$$

where T_f is the temperature at the soil surface, T_B is the initial uniform (base) temperature, z is the vertical coordinate oriented positive downward with origin at the soil surface, and D is the depth to a level where the temperature T_B is maintained.

Under this new maintained condition of temperature, ultimately a new equilibrium moisture profile will develop. Under equilibrium $dh_c + dz$ must be zero or explicitly using Eq. (1):

$$\left(\frac{374-T}{374-T_B}\right) h'_{cB}(\theta)\,d\theta - \frac{h_{cB}(\theta)\,dT}{374-T_B} + dz = 0 \tag{3}$$

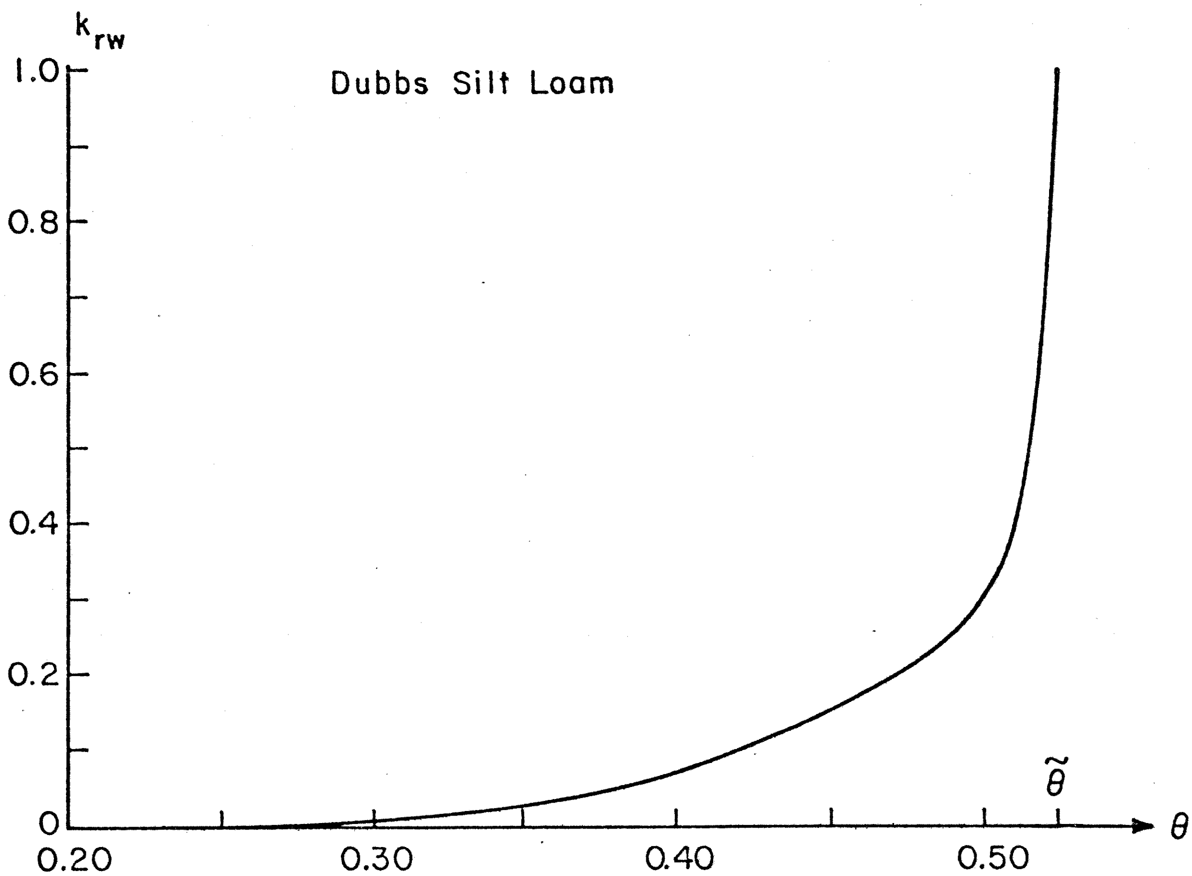

Figure 2. Relative permeability to water for Dubbs silt loam at 15°C (data after Jensen et. al., 1970).

where the prime indicates differentiation with respect to θ. Expressing dz in terms of dT from Eq. (2) and rearranging one obtains the separable differential equation:

$$\frac{h'_{cB}(\theta)}{h_{cB}(\theta) - \dfrac{D(374-T_B)}{T_B-T_f}} = -\frac{dT}{T-374} \qquad (4)$$

which can be integrated to yield the equilibrium profile in the implicit form:

$$h_{cB}(\theta) - h_{cBD} = \frac{D(374-T_B)}{(T_B-T_f)} \cdot \frac{(T_B-T)}{(374-T)} \qquad (5)$$

where h_{CBD} is the capillary pressure existing at depth D or in terms of z:

$$h_{cB}(\theta) - h_{cBD} = \frac{D(374-T_B)(D-z)}{(374-T_f)D - (T_B-T_f)z} \qquad (6)$$

or in terms of elevation above the water table, h:

$$h_{cB}(\theta) - H_{cBD} = \frac{D(374-T_B)(h-h_{cBD})}{D(374-T_B) + (T_B-T_f)(h-h_{cBD})} \qquad (7)$$

The new equilibrium profile for values of $T_B = 15°C$, $T_f = 0°C$, and $D = 200$ cm is shown on Figure 1. It was calculated from Eq. (7) reqritten in the form:

$$h - h_{cBD} = \frac{D(374-T_B)(h_{cB}-h_{cBD})}{D(374-T_B) - (T_B-T_f)(h_{cB}-h_{cBD})} \qquad (8)$$

For the selected numerical values h_{cBD} is equal to zero. The difference between the two curves h_{cB} and h (linear gradient) represents the increase in moisture storage in the soil due to its cooling. In this particular instance the effect is small. The water content at the soil surface changes from 0.32 to 0.34, i.e. a relative change of 6 percent. The reduction in pore space available for infiltration is 10 percent.

Inspection of Eq. (7) rewritten for emphasis in the form:

$$h_{cB}(\theta) - h_{cBD} = \frac{h - h_{CBD}}{1 + \left(\frac{T_B - T_f}{374-T_B}\right)\left(\frac{h-h_{cBD}}{D}\right)} \qquad (9)$$

indicates that the effect is more pronounced when T_B is low and $T_B - T_f$ is large, that it is maximum for $h = D$ i.e. at the point of lowest temperature and for $h_{cBD} = 0$ that is when the uniform warm base temperature region does not extend upward beyond the water table. Under these most favorable conditions the ratio of $\frac{h_{cB}}{h} = \frac{374-T_B}{374-T_f}$. In other words the maximum relative decrease in capillary pressure cannot exceed $100\left(\frac{T_B-T_f}{374-T_B}\right)$ percent. Realistically T_B is at most of the order of 20°C and the difference $T_B - T_f$ might be of the order of 30°C (allowing some degree of supercooling because water is under suction and would not freeze so readily and ice is not present). Thus under these circumstances the ratio h_{cB}/h is about 0.92. This ratio was applied at all elevations h, the corresponding $h_{cB}(\theta)$ was calculated (0.92h) and θ determined from the h_{cB} curve. Then a point of coordinate θ,h was obtained. The locus of all these points is the capillary pressure at a uniform temperature 30°C less than the original curve and is shown on Figure 1. The maximum absolute increase in water content at a given elevation (i.e., warm capillary pressure) occurs at a capillary pressure of 130 cm. The absolute change in water content is 0.03 and the relative increase in water content is 7 percent. The maximum relative increase occurs at a capillary pressure of 230 cm where it is 23 percent. The maximum absolute effect will occur at pressures where the moisture retention curve is flatest and the maximum relative effect at pressures where the moisture retention curve has highest curvature, which is usually around the field capacity value. It appears doubtful therefore that the high water contents in the Rock River basin in the spring 1969, were solely due to a water migration induced by a temperature gradient. However as much as half of it, but no more, could have been.

The calculations performed for the Dubbs silt loam were repeated for the Moody silt loam in Rock County although the capillary pressure curve is not as well defined experimentally (Figure 3). At 1/3 of an atmosphere the absolute increase in water content is 1 percent by weight and the relative water content increase is 3 percent.

Rate of Migration

As has been shown previously (Morel-Seytoux and Khanji, 1974) the total velocity (i.e., algebraic sum of water and air velocities) is given by the relation:

$$v = \frac{\tilde{K}\,[\int_1^2 f_w dh_c + \int_1^2 f_w\, dz]}{\int_1^2 \mu_{rT}\, dz} \tag{10}$$

where v is the total velocity, $\tilde{K}$ is the hydraulic conductivity at natural saturation, f_w is the *little fractional flow* function defined by the relation:

$$f_w = \frac{k_{rw}}{k_{rw} + (\frac{\mu_w}{\mu_a})\, k_{ra}} \tag{11}$$

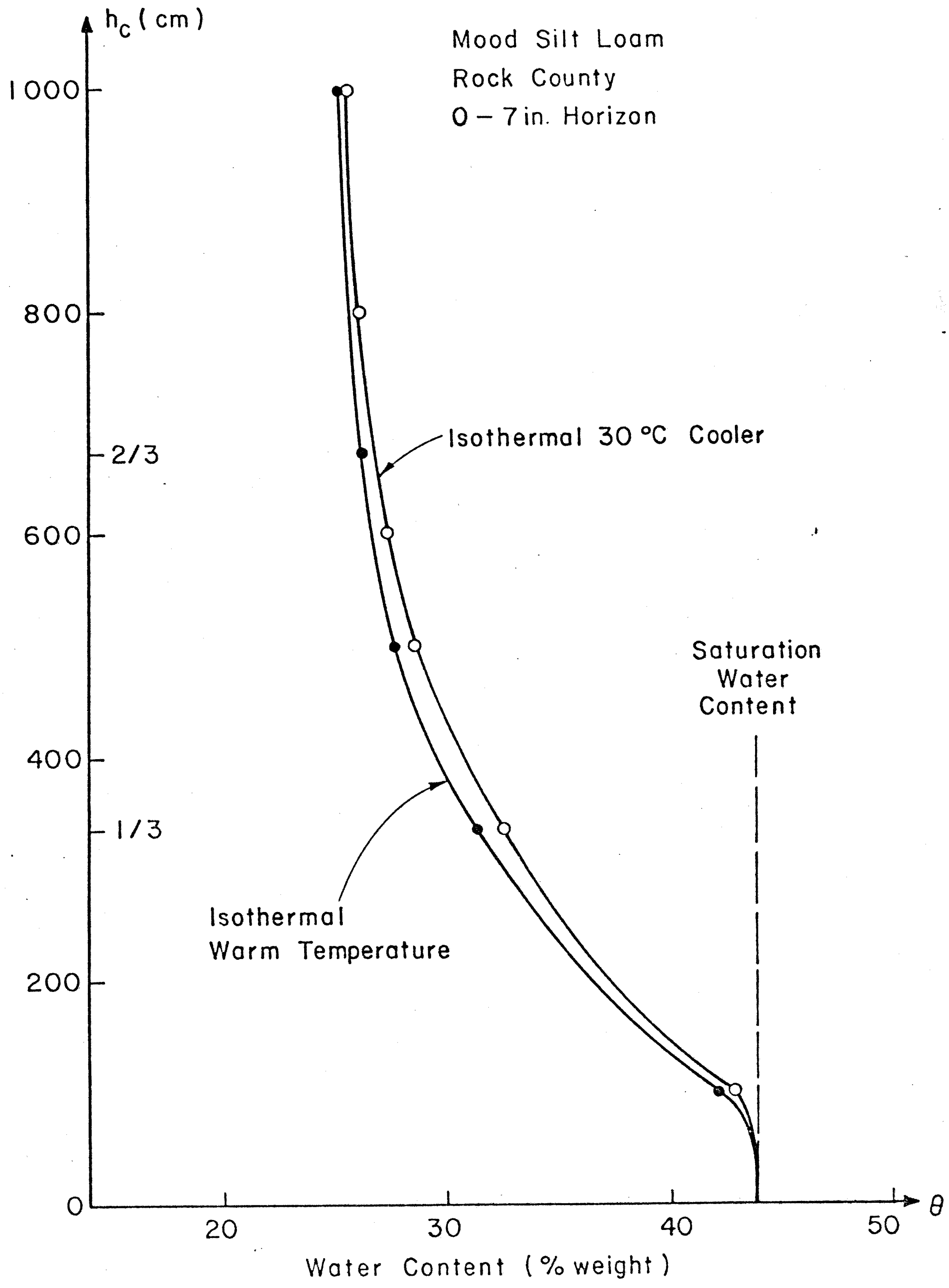

Figure 3. Isothermal water content profiles for Moody silt loam (data after Jensen et. al., 1970).

where k_{ri} and μ_i are respectively relative permeability and viscosity for phase i (water or air), μ_{rT} is the total relative viscosity defined at:

$$\mu_{rT} = \frac{f_w}{K_{rw}} \qquad (12)$$

and where the indices 1 and 2 refer to 2 arbitrary levels in the soil. For the soil defined in Figure 1 and *assuming* a first order power curve for k_{ra} as a function of air content, the curves of k_{rw} and f_w versus h_c are shown on Figure 4. Curves of k_{ra} are rarely measured and the location of the intersection of the curves k_{rw} and f_w is somewhat uncertain. In Eq. (10) the area under the f_w curve is needed (i.e., the first integral in the numerator). For practical purposes and due to the uncertainty about the f_w curve, one actually uses the area under the k_{rw} curve. The error is not large as has been estimated for different soils in previous studies (Morel-Seytoux and Khanji, 1974). A curve of μ_{rT} versus h_c is also shown on Figure 4.

Since initially the soil was in equilibrium at a temperature of 20°C it follows that under these conditions:

$$\int_0^D k_{rw}\, dh_{cB} + \int_0^D k_{rw}\, dz = 0 \qquad (13)$$

Thus the *instantaneous extra* capillary drive when the linear temperature gradient is imposed is:

$$\Delta H_c = \int_0^D k_{rw}\, (dh_c - dh_{cB}) \qquad (14)$$

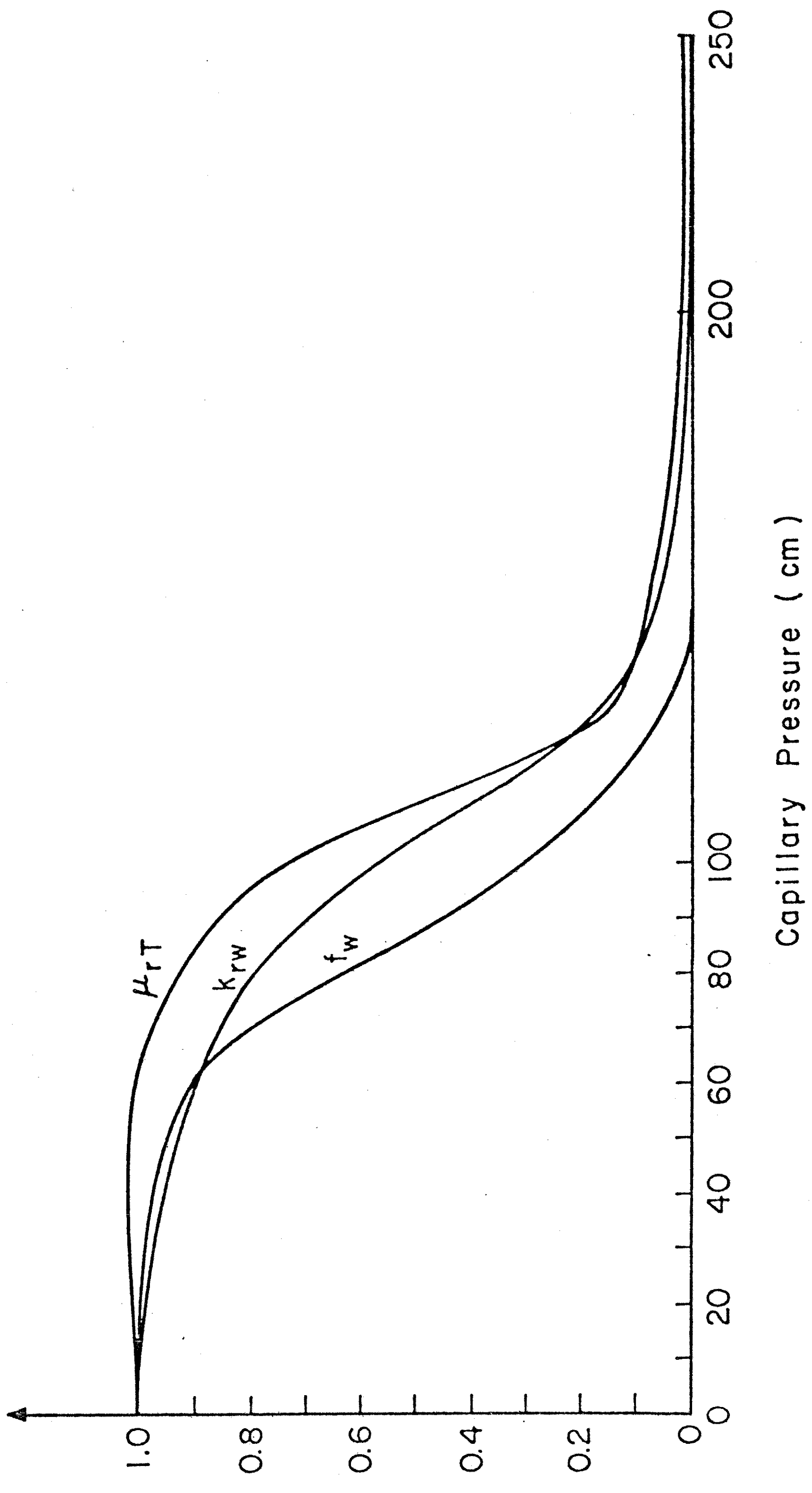

Figure 4. Hydraulic properties of Dubbs silt loam as function of capillary pressure.

As a rough estimate the extra capillary drive can be approximated to be:

$$\Delta H_c = \frac{1}{2}\left(\frac{T_B - T_f}{374 - T_B}\right) H_{cB} \tag{15}$$

where H_{cB} is the usual maximum effective capillary drive:

$$H_{cB} = \int_0^{\infty} k_{rw} \, dh_{cB} \tag{16}$$

This term can be estimated graphically as the area under the k_{rw} curve on Figure 4. Its value is about 110 cm and consequently from Eq. (15) one finds the value of the extra capillary drive:

$$\Delta H_c = \frac{1}{2} \frac{20}{354} 110 = 3 \text{ cm} \tag{17}$$

It remains to calculate the viscous resistance term in the denominator of Eq. (10). It is the area under the curve of μ_{rT} on Figure 4 in the range of h_c from 0 to 250 cm. It is of the order of 120 cm. Thus with a saturated hydraulic conductivity of the order of 3 cm/hour the total velocity v is of the order of 1.8 cm/day. Thus the sudden temperature gradient will result in a water flux at the water table of the order of 2 cm per day. Naturally as moisture migrates upward toward a new equilibrium, the extra capillary drive decreases. Also the water viscosity increases by a factor of two. Thus a more reasonable migration rate is about 0.4 cm per day. Nevertheless one can conclude that in the presence of a water table the migration rate will not be a limiting factor for the accumulation of water in the upper cold part of the soil. The water table will rarely be so close to the surface. Presuming that in the upper soil capillary pressure decreased from 250 cm (θ = 0.275) to 150 cm (θ = 0.40) and then remained at that value further down, a more realistic profile, then the warm isothermal effective capillary drive is only:

$$H_c = \int_{150}^{250} k_{rw}\, kh_c \simeq 1 \text{ cm} ,$$

value obtained from Figure 4. The extra capillary drive due to the temperature gradient is about 0.03 cm, which is quite small. However the resistance term in the denominator has decreased considerably and from Figure 4 it can be estimated at about 1.5 cm, so that the final migration rate will be of the order of 0.3 cm per day. This value confirms the previous conclusion for the case when the water table was close to the surface.

Conclusions

The study confirms the opinion that during *severe cold* weather yet *without frost* in the soil temperature gradients can produce significant upward migration of water in liquid form. The migration rate however is not limiting. What limits the water accumulation in the soil is the lack of retention capability of the soil even for a temperature drop of 30°C. Under the most favorable conditions it appears that at most half of the observed water accumulation in the spring of 1969 could be due to the phenomenon investigated. Nevertheless the magnitude of the temperature gradient induced flow is too large to be neglected in the soil moisture accounting module of existing hydrologic models such as the NWS River Forecasting System model.

Now one could speculate that the large accumulation was due to the presence of frost (ice) in the soil below the snow cover which attracted water and froze it. However since no ice was found when samples were taken in the soil during the two weeks prior to the snowmelt, the ice would have to have formed and melted before the sampling program. Even if no frost developed below the soil, there is ice (snow) right at the surface and an upward flow will be induced, not as strongly as if frost had penetrated the soil, but it will exist. In a sense the capillary pressure property in the snowpack is more akin to that of a coarse sand rather than to that of a silty loam. The hypothesis is particularly plausible because during the winter the snowpack was very dry.

The problem of this spring 1969 large water accumulation has only been partially resolved. Further studies are needed and will be done first on the basis of the hypothesis of no frost penetration.

Acknowledgments

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