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FLOOD ROUTING: A SYNOPSIS OF PAST, PRESENT, AND FUTURE CAPABILITY

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ABSTRACT

Flood routing may be defined as a mathematical method (model) for predicting the changing magnitude and celerity of a flood wave which propagates through a river, reservoir, or estuary. A brief review is presented of the development of flood routing models. Commencing with investigations as early as the 17th century, mathematical techniques to predict wave propagation have been continually developed and appear profusely in the engineering literature. The basic theory for the one-dimensional analysis of flood wave propagation was originally developed by Saint-Venant in 1871; however, due to the mathematical complexity of the theoretical equations, simplifications were necessary in order to attain feasible solutions of the salient properties of the wave. Such simplifications included: 1) purely empirical techniques, 2) linearized versions of the St. Venant equations, 3) hydrologic (storage) routing techniques based on the conservation of mass and an approximation of the relationship between flow and storage, and 4) simplified hydraulic routing techniques based also on the conservation of mass and a simplified form of St. Venant's conservation of momentum equation. During the last two decades, finite difference solutions of the complete St. Venant equations have become possible due to the advent of high-speed computers. Of the several complete solution techniques developed, the implicit finite difference method appears the most promising for many flood routing applications because of its desirable computational efficiency. An attempt is made to ascertain the future trends in flood routing model development. Such trends are proposed on the basis of the present deficiencies in flood routing and the tractability for their improvement. A significant area of model development will be the incorporation of the existing knowledge of sediment transport, ice hydraulics, groundwater hydraulics, flood plain and bridge hydraulics into modular-designed flood routing models suitable for a wide range of applications.

INTRODUCTION

Flood routing has long been of vital concern to man as he has sought to understand, construct, and improve the transport of water via such waterways as canals, rivers, reservoirs, and estuaries. Flood routing as treated herein is the mathematical prediction of the salient properties of a flood wave such as its shape, magnitude, and celerity. These are continually changing as the wave propagates along the waterway. The wave may emanate from precipitation runoff, tides, and reservoir releases.

Commencing with investigations by such eminent scientists as Newton (1687), Laplace (1776), and Lagrange (1783), and continuing into the 1800's by such as Poisson (1816), Cauchy (1827), Green (1837), Russell (1844), Bazin (1862), Boussinesq (1871), and culminating in the one-dimensional equations of unsteady flow derived by Saint-Venant (1871), the theoretical foundation for flood routing was essentially achieved. The St. Venant equations consist of a conservation of mass equation:

$$\frac{\partial(AV)}{\partial x} + \frac{\partial A}{\partial t} = 0 \tag{1}$$

and a conservation of momentum equation:

$$\frac{\partial V}{\partial t} + V\frac{\partial V}{\partial x} + g\left(\frac{\partial h}{\partial x} + S_f\right) = 0 \tag{2}$$

in which t is time, x is distance along the longitudinal axis of the waterway, A is cross-sectional area, V is velocity, g is the gravity acceleration constant, h is the water surface elevation above a datum, and S_f is the friction slope which may be evaluated using a steady flow empirical formula such as the Chezy or Manning equation. Eqs. (1-2) are quasi-linear hyperbolic partial differential equations with two dependent parameters (V and h) and two independent parameters (x and t). A is a known function of h and S_f is a known function of V and h. No analytical solutions, particularly for practical boundary conditions, exist for Eqs. (1-2).

Due to the complexities of the St. Venant equations their solution was not feasible, and various simplified approximations of flood wave propagation continued to be developed. Indeed such techniques appear profusely in the engineering literature. An excellent summary of such is presented by Miller and Yevjevich (1975). The simplified methods may be categorized as: 1) purely empirical, 2) linearization of St. Venant equations, 3) hydrologic, i.e., based on the conservation of mass and an approximate relation between flow and storage, and 4) hydraulic, i.e., based on the conservation of mass and a simplified form of the conservation of momentum equation. Within the last quarter of a century the advent of high-speed computers have made it possible to obtain solutions of the complete St. Venant equations. In fact, in recent years, flood routing models based on the complete St. Venant equations have become economically feasible as a result of advances in computing equipment and improved numerical solution techniques. Herein is presented a brief summary review of several one-dimensional flood routing models in each of the above categories. Also, proposals are made for future improvements in flood routing which are considered to be possible and which will effect important gains in the ability of hydrologists and engineers to predict flood wave propagation in natural waterways.

EMPIRICAL MODELS

Lag Models

Some flood routing models are based on intuition and observations of past flood wave motion. One category of empirical models is the lag models in which lag is the time difference between inflow and outflow within a routing reach. The successive average-lag method developed by Tatum (1940) assumes that there is some point downstream where the flow (I_2) at time (t_2) is equal to an average flow, i.e., $(I_1 + I_2)/2$. Tatum found that the number of successive averages occurring within a reach was approximately the time of travel of the wave divided by the reach length. Outflow at the end of the reach is computed by:

$$O_{n+1} = c_1 I_1 + c_2 I_2 + \cdots c_{n+1} I_{n+1} \tag{3}$$

where n is the number of sub-reaches (successive averages) within the

routing reach. The routing coefficients used in the method can be obtained via Tatum's approach or by trial and error using observed inflow and outflow hydrographs. Another similar lag model known as the progressive average-lag method (Harris, 1970) was developed by the Army Corps of Engineers (1935). The routing coefficients in Eq. (3) may also be obtained via a least-squares correlation of inflow and outflow hydrographs as described by Linsely, et al. (1949).

Gage Relations

Other empirical techniques include gage relations (Linsley, et al., 1949, pp. 517-530) which relate the flow at a downstream point to that at an upstream station. Gage relations can be based on flow, stages, or a combination of each. The effect of lateral inflow is automatically contained in the empirical relation.

Empirical models are limited to applications with sufficient observations of inflows and outflows to calibrate the essential coefficients. They provide best results when applied to slowly fluctuating rivers with negligible lateral inflows and backwater effects. They are extremely economical in computational requirements; however, considerable effort may be required to derive the empirical coefficients.

LINEARIZED MODELS

The complexity of the St. Venant equations has caused many scientists and engineers to simplify them in order to obtain solutions. The simplifications have been to either totally ignore the least important nonlinear terms and/or to linearize the remaining nonlinear terms in the equations. Given a sufficiently simplified form of the equations, they can be integrated analytically to obtain solutions of velocity and water surface elevation for any pair of (x,t) values at a relatively small expenditure of computational effort. Usually the most common simplifying assumptions are: 1) ignore the second term in Eq. (2); 2) constant cross-sectional area, usually rectangular; 3) constant channel bottom slope, often assumed to be zero; 4) the friction slope term is linearized with respect to velocity and depth; 5) no lateral inflow; and 6) the routed flood wave has a simple shape that is amenable to an analytical expression. These simplifications usually invoke severe limitations on the conditions for which the solution is valid.

Classical Wave Models

Neglecting lateral inflow and frictional resistance and nonlinear terms $V\,\partial A/\partial x$ and $V\,\partial V/\partial x$ in Eqs. (1-2), the following classical linear wave equations may be obtained:

$$\frac{\partial^2 V}{\partial t^2} = g\bar{y}\,\frac{\partial^2 V}{\partial x^2} \tag{4}$$

$$\frac{\partial^2 h}{\partial t^2} = g\bar{y}\,\frac{\partial^2 h}{\partial x^2} \tag{5}$$

where $\bar{y}$ is the average depth. The analytical solutions of Eqs. (4-5) have the following form (Abbott, 1966):

$$V = C_1\,(x - \sqrt{g\bar{y}}\,t) + C_2\,(x + \sqrt{g\bar{y}}\,t)$$

where C_1 and C_2 are functions determined by initial flow conditions and the boundary conditions.

Assuming a rectangular cross section, zero bottom slope, linearized

resistance, and neglecting the V $\partial V/\partial x$ term, the following equation may be obtained after combining the resulting simplified forms of Eqs. (1-2) and eliminating h:

$$g\bar{y}\frac{\partial^2 V}{\partial x^2} = \frac{\partial^2 V}{\partial t^2} + gC_o\frac{\partial V}{\partial t} \qquad (6)$$

in which C_o is a constant depending on the linearized resistance term. Eq. (6) is in the form of the telegraph-equation which has been extensively studied (Dronkers, 1964).

Simple Impulse Response Model

Linear systems theory has also been used to develop routing techniques (Dooge, 1973). In this approach, the routing model is assumed to be composed of linear reservoirs connected by linear channels. According to linear systems theory, any linear system is completely and uniquely characterized by its unit impulse response. By knowing the unit impulse response, all possible system outputs may be determined for all possible inputs. The input-output relationship is defined by the convolution integral:

$$O(t) = \int_0^t I(t)\ H(t-\tau)d\tau$$

in which O(t) is the routed flow, I(t) is the inflow, and $H(t-\tau)$ is the unit impulse response. The unit impulse response for a distributed linear reservoir is given by Maddaus (1969) as:

$$H_N(x,t) = \frac{1}{N}\sum_{n=1}^{N}\frac{e^{-(t-n\tau)/k}}{k\,\Gamma(n)}\left(\frac{t-n\tau}{k}\right)^{n-1} \quad \ldots\ t>n\tau$$

where N is the number of linear elements, $\Gamma(\ )$ is the gamma function, k is the characteristic linear reservoir time constant, and τ is the time constant of the channel. The parameters k and τ are obtained by a fitting procedure described by Maddaus. A similar unit-response approach for routing through a single linear reservoir was reported by Sauer (1973). This approach is analogous to the unit hydrograph used by hydrologists to compute precipitation runoff. It is also somewhat related to the lag methods described previously.

Complete Linearized Model

Linearized models of the complete St. Venant equations were developed by Lighthill and Whitham (1955) and Harley (1967). If Eqs. (1-2) are rewritten for a unit-width channel and in terms of unit discharge (q) and depth (y), and then combined and linearized about a reference flow velocity ($V_o=q_o/y_o$), the following linearized equation is obtained (Harley, 1967):

$$(gy_o^3 - V_o^2)\frac{\partial^2 q}{\partial x^2} - 2V_o\frac{\partial^2 q}{\partial x\partial t} - \frac{\partial^2 q}{\partial t^2} = 3gS_o\frac{\partial q}{\partial x} + 2g\frac{S_o}{V_o}\frac{\partial q}{\partial t} \qquad (7)$$

in which S_o is the channel bottom slope. Harley obtained the following unit response function for Eq. (7):

$$H(x,t) = e^{-Px}\,\delta(t-x/C_1) + h\,(x/C_1 - x/C_2)\,e^{sx-rt}\ I[2hm]/m$$

where: $C_1 = V_o + \sqrt{gy_o}$

$$C_2 = V_0 - \sqrt{gy_0}$$

$$F = V_0/\sqrt{gy_0}$$

$$P = S_0(2-F)/[2y_0(F^2+F)]$$

$$r = S_0V_0(2+F^2)/(2y_0F^2)$$

$$s = S_0/(2y_0)$$

$$h = S_0V_0\sqrt{(4-F^2)(1-F^2)}/(4y_0F^2)$$

$$m = \sqrt{(t-x/C_1)(t-x/C_2)}$$

and I[ ] is a first order Bessel function of the first kind and δ is the delta function. This model is similar to the diffusion analogy model developed by Hayami (Chow, 1959, p. 601-604); however, it does not overattenuate the flood wave as much as the simpler diffusion analogy model. The accuracy of the model is very dependent on the reference flow (Bravo, et al. 1970).

Multiple Linearized Models

Keefer and McQuivey (1974) present an improved method for linearized models in which they introduce the concept of multiple linearization. They applied the multiple linearization technique to both the complete linearized model of Harley and the diffusion analogy model; they concluded the latter was more practical.

The applicability of linearized models is limited by the assumptions in their derivation. The complete linearized model and the diffusion analogy model of Hayami are the least restricted, although neither is appropriate when backwater effects exist due to the presence of tides, significant inflows, dams, bridges, or cross-section irregularities.

HYDROLOGIC MODELS

Significant river improvement projects in the early 1900's provided the impetus for development of an array of simplified flood routing methods. These have been termed hydrologic models. They are based on the conservation of mass Eq. (1) written in the following form:

$$I - O = \Delta S/\Delta t \tag{8}$$

in which ΔS is the change in storage within the reach during a Δt time increment; the storage (S) is assumed to be related to inflow and/or outflow, i.e.,

$$S = K[XI + (1-X)O] \tag{9}$$

Reservoir Routing Models

One hydrologic model, variations of which are attributed to Goodrich (1931) and Puls (1928), was developed by letting X in Eq. (9) be assumed zero, i.e., storage is dependent only on outflow. Expressing Eq. (8) in centered finite difference form, the following reservoir routing model is obtained:

$$\frac{I_1 + I_2}{2} - \frac{O_1 + O_2}{2} = \frac{S_2 - S_1}{\Delta t} \tag{10}$$

which can be rearranged as:

$$\frac{2S_2}{\Delta t} + O_2 = I_1 + I_2 + \frac{2S_1}{\Delta t} - O_1$$

which can be solved step-by-step for the left-hand side since O_1 and S_1 are known at time t = 0. An S - O relationship obtained from observed inflow-outflow hydrographs, allows the outflow (O_2) to be determined.

Muskingum Model

If Eq. (9) with non-zero values for K and X is used for the storage relationship and this is substituted in Eq. (10), the following equation for computing O_2 is obtained:

$$O_2 = C_0 I_2 + C_1 I_1 + C_2 O_1 \qquad (11)$$

where: $C_0 = - (KX - \Delta t/2)/C_3$

$C_1 = (KX + \Delta t/2)/C_3$

$C_2 = (K - KX - \Delta t/2)/C_3$

$C_3 = K - KX + \Delta t/2$

Eq. (11) is the widely used Muskingum routing model first developed by McCarthy (1938). The parameters K and X are determined from observed inflow-outflow hydrographs using such techniques (Singh and McCann, 1980) as: 1) least-squares or its equivalent, the graphical method, 2) method of moments, 3) method of cumulants, and 4) direct optimization method. Singh and McCann conclude that there is no particular advantage of one method over another. Cunge (1969), has also provided a method for estimating K and X on the basis of channel characteristics. The Muskingum model continues to be popular, and papers treating it or its variations continually appear in the literature, e.g., Nash (1959), Diskin (1967), Cunge (1969), Ponce and Yevjevich (1978), Gill (1978), Ponce (1979), Koussis (1980), Singh and McCann (1980), and Strupczewski and Kundzewicz (1980).

Muskingum-Cunge Model

An important variation of the Muskingum model was developed by Cunge (1969). Cunge develops the Muskingum equation using kinematic wave theory including the assumption of a single-valued stage-discharge relation and a four-point implicit finite difference approximation technique. Eq. (11) remains the same, but the following expressions for K and X are determined:

$$K = \Delta x/c$$

$$X = \frac{1}{2}\left[1 - Q_0/(B_0 c\, S_0\, \Delta x)\right]$$

where: $c = dQ/dA$

in which c is the kinematic wave speed corresponding to a reference discharge Q_0, Δx is the reach length, S_0 is the channel bottom slope, and B_0 is channel width corresponding to Q_0. Ponce and Yevjevich (1978) expanded this method by using variable parameters c and B for temporally varying Q.

Kalinin-Miljukov Model

Another variation of the Muskingum model is the Kalinin-Miljukov method (Miller and Cunge, 1975, pp. 232-236) developed in the 1950's in the U.S.S.R. This model is:

$$O_2 = O_1 + (I_1 - O_1)\, K_1 + (I_2 - I_1)\, K_2 \qquad (12)$$

where: $K_1 = 1 - e^{-c\Delta t/\Delta x}$

$K_2 = 1 - K_1 \Delta x/(c\Delta t)$

$\Delta x = Q_o/S_o\ (\Delta h/\Delta Q)$

in which $\Delta h/\Delta Q$ is the slope of the stage-discharge rating curve. Eq. (12) is identical to the Muskingum model if in the latter $K = \Delta x/c$ and $X = 0$.

Yet another variation of the Muskingum model is the SSARR routing model (Rockwood, 1958) which Miller and Cunge (1975, pp. 237-241) show is similar to the Muskingum model with $X = 0$.

Lag and Route Model

Another storage routing model is the Lag and K model (Linsely, et al., 1958, pp. 230-232). The inflow is first lagged and then the outflow (O_2) at time (t_2) is determined by substituting the relation $\Delta S = K\ (O_2 - O_1)$ in Eq. (10) and solving for O_2, i.e.,

$$O_2 = [I_1 + I_2 - O_1\ (1-2K/\Delta t)]/(1+2K/\Delta t)$$

The lag factor and the K factor may be constant, or they can be functions of the inflow and outflow, respectively. Actually, this model is derived via a combination of storage routing principles and empiricism introduced through the lag factor. Another recently proposed lag and route model is reported by Quick and Pipes (1975).

It appears that the Cunge version of the Muskingum model is the most versatile and physically relevant of the hydrologic models; however, it along with the other storage routing models are restricted to applications where the stage-discharge relation is single-valued. Thus, backwater effects from tides, significant tributary inflow, dams, and bridges are not considered by these models nor are they well-suited for very mild sloping waterways where looped stage-discharge ratings may exist. The other storage routing models are also limited to applications where observed inflow-outflow hydrographs exist. It must be remembered that when using the observed hydrographs to calibrate the routing coefficients, variations in flood wave shapes within the observed data set are not considered, and only the average wave shape is reflected in the fitted routing coefficients.

SIMPLIFIED HYDRAULIC MODELS

Simplified hydraulic models are based on the conservation of mass Eq. (1) and a simplified form of Eq. (2). These models were developed as the high-speed computer became available.

Kinematic Models

One type of simplified hydraulic model is the kinematic wave model. Interest in this model was sparked by the work of Lighthill and

Whitman (1955). The essence of the kinematic model is the use of the following simplified form of the conservation of momentum Eq. (2), i.e.,

$$S_f - S_o = 0 \tag{13}$$

where $\partial h/\partial x = \partial y/\partial x - S_o$ in Eq. (1). Eq. (13) essentially states that the momentum of the unsteady flow is assumed to be the same as that of steady uniform flow as described by the Chezy or Manning equation or some other similar expression in which discharge is a single-valued function of stage, e.g.,

$$A = \alpha Q^{\beta} \tag{14}$$

in which A is the cross-sectional area, $\alpha = [B/(C^2 S_o)]^{1/3}$, $\beta = 2/3$, C is the Chezy coefficient. Combining Eqs. (13-14) and Eq. (1) results in the following nonlinear kinematic wave model (Li, et al., 1975, 1976):

$$\frac{\partial Q}{\partial x} + \alpha\beta Q^{\beta-1} \frac{\partial Q}{\partial t} = 0$$

which can be solved by explicit or implicit finite difference methods, the latter being more efficient in most river applications. The kinematic wave model is limited to applications where single-valued stage-discharge ratings exist, and where backwater effects are insignificant since in kinematic models flow disturbances can only propagate in the downstream direction. Also, the kinematic model modifies the flood wave through attenuation and dispersion via the errors inherent in the finite difference solution technique. The phenomenon of numerical damping merely mimics the actual physical damping of a flood wave since there is no mechanism in the basic kinematic equation to cause such damping. The kinematic wave models are very popular in applications to overland flow routing of precipitation runoff, e.g., Wooding (1965), Woolhiser and Liggett (1967), and Gburek and Overton (1973). Kinematic wave models have been used in streamflow applications by Harley, et al. (1970) in the MIT catchment model and in the Hydrocomp model (Linsely, 1971).

Diffusion Models

Another simplified hydraulic model is the diffusion model which utilizes Eq. (1) and the following simplified form of Eq. (2):

$$S_f - \partial h/\partial x = 0 \tag{15}$$

Eq. (15) may be expressed in terms of channel conveyance K_c which is a single-valued function of elevation h, i.e.,

$$Q = -K_c (h_x)^{1/2} h_x / |h_x| \tag{16}$$

where $h_x = \partial h/\partial x$. Eq. (15) allows for upstream directed flows. Brakensiek (1965) solved Eqs. (15-16) with a four-point centered implicit finite difference solution technique for reasons of computational efficiency. Harder and Armacost (1966) used an explicit finite difference solution technique for the diffusion routing model used by the Army Corps of Engineers (Harrison and Bueltel, 1973) on the Missouri River. This model is restricted to small Δt time steps due to the numerical stability constraint given by:

$$\Delta t \leq B\, S_o^{1/2}\, \Delta x^2 / (K_c + S_o \Delta x\, \Delta K_c / \Delta h)$$

The nonlinear diffusion wave model is a significant improvement over the kinematic model because of the inclusion in Eq. (15) of the water sur-

face slope term ($\partial h/\partial x$) of Eq. (2). This term allows the diffusion model to describe the attenuation (diffusion effect) of the flood wave. It also allows the specification of a boundary condition at the downstream extremity of the routing reach to account for backwater effects. It does not use the inertial terms (first two terms) of Eq. (2) and, therefore, is limited to slow to moderately rising flood waves in channels of rather uniform geometry. Of significant interest, Sevuk (1973, p.22) found that inclusion of the inertial terms in an implicit (finite difference) diffusion model resulted in only a 20% increase in computational effort.

A third type of simplified hydraulic model is the quasi-steady dynamic wave hydraulic model in which Eq. (1) is used along with Eq. (2) with all its terms except $\partial V/\partial t$. This simplification saves very little in computational effort and introduces more error than the simpler diffusion model. The quasi-steady model has been infrequently used and its further use is not recommended.

The applicability of the kinematic and diffusion models has recently been treated by Ponce, et al. (1978) who utilized a linear stability analysis of the finite difference form of the St. Venant equations to examine the applicability of kinematic and diffusion models. They compared wave attenuation factors and celerities and concluded that bottom slope and wave shape determine the range of their suitable applicability. In general, the steeper slopes associated with overland flow or steep streams with slow-rising floods were amenable to the use of kinematic models. The diffusion models had a wider range of applicability and could accommodate milder bottom slopes. However, there still remain many practical combinations of mild sloping channels and flood wave shapes that are not suitable for either diffusion or kinematic approximations and should be treated with the complete St. Venant equations.

COMPLETE (DYNAMIC WAVE) HYDRAULIC MODELS

If the complete St. Venant equations are used, the model is known as a dynamic wave model. With the advent of high-speed computers Stoker (1953) and Isaacson, et al. (1954) first attempted to use the complete St. Venant equations for flood routing on the Ohio River. Since then, much effort has been expended on the development of dynamic wave models, and the literature contains many dynamic models. They can be categorized according to direct and characteristic methods. In the direct methods, finite difference approximations are substituted directly into Eqs. (1-2) and solutions are obtained for incremental times (Δt) and incremental distances (Δx) along the waterway. In the method of characteristics, the partial differential Eqs. (1-2) are first transformed into an equivalent set of four ordinary differential equations which are then approximated with finite differences to obtain solutions. Dynamic models can be classified further as either explicit or implicit, depending on the type of finite difference scheme that is used. Explicit schemes transform the differential equations into a set of easily solved algebraic equations. However, implicit schemes transform the differential equations into a set of algebraic equations which must be solved simultaneously; the set of simultaneous equations may be either linear or nonlinear, the latter requiring an iterative solution procedure.

Characteristic Models

Several method of characteristic models (Abbott, 1966) were developed in the 1960's. Most were explicit, e.g., Liggett and Woolhiser

(1967), Streeter and Wylie (1967), Baltzer and Lai (1968), and Ellis (1970). Implicit characteristic models were reported by Amein (1966) and Wylie (1970). Characteristic models can have a curvilinear grid or a rectangular grid in the x-t solution domain. The former is not practical for application in natural waterways of irregular geometry. The latter, known as the Hartree method, requires interpolation formulae meshed within the finite difference solution procedure. These restrictions have tended to discourage the application of characteristic models for flood routing. The method of characteristic models for prismatic channels are based upon the following four total differential equations:

$$dx/dt - V - \sqrt{gA/B} = 0 \tag{17}$$

$$dV/dt + \sqrt{gB/A}\, dy/dt + g(S_f - S_o) + q(V - v_x)/A - \sqrt{gB/A}\, q/B = 0 \tag{18}$$

$$dx/dt - V + \sqrt{gA/B} = 0 \tag{19}$$

$$dV/dt - \sqrt{gB/A}\, dy/dt + g(S_f - S_o) + q(V - v_x)/A + \sqrt{gB/A}\, q/B = 0 \tag{20}$$

Eqs. (17-20) are equivalent to the St. Venant partial differential Eqs. (1-2) except that lateral inflow (q) has been included. The term v_x is the velocity of the lateral inflow in the x-direction of the waterway, A is cross-sectional area, B is cross-sectional top width, and y is depth.

Explicit Models

Explicit finite difference models advance the solution of the St. Venant equations point by point along one time line in the x-t solution domain until all the unknowns associated with that time line have been evaluated. Then, the solution is advanced to the next time line. In an explicit scheme, the spatial derivatives and non-derivative terms are evaluated on the time line where the values of all variables are known. Only the time derivatives contain unknowns. Thus, in an explicit model, two linear algebraic equations are generated from the two St. Venant equations at each net point (node). Since the two equations can be solved directly for the unknowns, the equations are described as "explicit."

The development of explicit models began with the pioneering work of Stoker (1953) and Isaacson, et al. (1954, 1958) who applied an explicit scheme to route floods in the Ohio River. Among those who have reported on explicit models are Liggett and Woolhiser (1967), Martin and DeFazio (1969), and Strelkoff (1970). Also, Dronkers (1969), Balloffet (1969), Kamphuis (1970) and Thatcher and Harleman (1972) applied explicit models to analyze tidal movement in estuaries. Garrison, et al. (1969) and Johnson (1974) applied the explicit models for flood routing in rivers and reservoirs. Many variations of the explicit method have been developed. Some were developed specifically for rapidly varying unsteady flow in which bore formation was likely, e.g., the Lax-Wendroff two-step scheme reported by Richtmyer (1957). Other popular schemes include the Stoker scheme, which was used by Isaacson, et al. (1958), the diffusion scheme, and the leap-frog scheme which were developed for gradually varying flows. These explicit finite difference schemes have been described and analyzed by Liggett and Cunge (1975).

In explicit models, Eqs. (1-2) are usually expressed in the following form to allow an explicit solution of their finite difference approximations:

$$A \frac{\partial V}{\partial x} + V \frac{\partial A}{\partial x} + B_T \frac{\partial y}{\partial t} - q = 0 \tag{21}$$

$$\frac{\partial V}{\partial t} + V \frac{\partial V}{\partial x} + g \left(\frac{\partial y}{\partial x} - S_o + S_f \right) + \frac{(V - v_x)q}{A} = 0 \tag{22}$$

in which B_T is the wetted top width of the total cross-sectional area (active and inactive or off-channel storage areas). Also, the effect of lateral inflow (q) is included in Eqs. (21-22).

Explicit models, although relatively simple compared to implicit models, have a restriction in the size of the computational time step due to reasons of numerical stability. In the diffusion scheme the restriction in Δt is given by the lesser of the following two inequalities (Terzidis, 1968):

$$\Delta t \leq \frac{\Delta x}{V + \sqrt{g\, A/B}} \tag{23}$$

$$\Delta t \leq \frac{c_1 R^{4/3}}{g n^2 |V|} \tag{24}$$

in which n is the Manning roughness coefficient, $c_1 = 1.0$ in metric units and $c_1 = 2.21$ in English units, and R is the hydraulic radius. Eq. (23) is known as the Courant condition. It is derived for a frictionless flow and is dependent upon the flow velocity and the celerity of small disturbances. When friction is considered, Eq. (24) is also a restriction. Eqs. (23-24) or some slight modification are applicable to most explicit models.

Inspection of the stability criteria of Eq. (23) indicates that the computational time step is substantially reduced as the hydraulic depth (A/B) increases. Thus, in large rivers, it is not uncommon for time steps on the order of a few minutes or even seconds to be required for numerical stability even though the flood wave may be very gradual, having a duration in the order of weeks. Such small time steps cause the explicit method to be very inefficient in the use of computer time.

Another disadvantage of explicit schemes is the requirement of equal Δx distance steps. Although this can be relaxed somewhat by using weighting factors, it can be quite disadvantageous for modeling flows in natural waterways.

Implicit Models

Implicit finite difference schemes advance the solution of the St. Venant equations from one time line to the next simultaneously for all points along the time line (i.e., along the x-axis of the waterway). Thus, in an implicit model, a system of 2N algebraic equations is generated from the St. Venant equations applied simultaneously to the N cross sections along the x-axis. Depending upon the type of implicit finite difference scheme chosen, the system of algebraic equations so generated may be either linear or nonlinear.

Implicit models were developed because of the limitations on the size of the time step required for numerical stability of explicit models. The use of implicit models was suggested by Isaacson, et al. (1956) and first appeared in the literature in the early 1960's with the

work of Preissmann (1961), and Vasiliev, et al. (1965). Later, Abbott and Ionescu (1967), Baltzer and Lai (1968), Dronkers (1969), Amein and Fang (1970), Gunaratnam and Perkins (1970), Kamphuis (1970), Contractor and Wiggert (1972), Quinn and Wylie (1972), Fread (1973), Chaudhry and Contractor (1973), Greco and Panattoni (1975), Amein (1975), Amein and Chu (1975), Chen and Simons (1975), Muir (1975), Bennett (1975), and Fread (1976) were among those reporting their research with implicit methods.

Analysis of the numerical stability and accuracy of various implicit schemes have been reported by Cunge (1966), Abbott and Ionescu (1967), Dronkers (1969), Gunaratnam and Perkins (1970), Fread (1974), Liggett and Cunge (1975, pp.157-163), and Ponce and Simons (1977). Within the simplifications required in making the numerical stability analyses, the various implicit methods were found to be unconditionally linearly stable, i.e., the simplified linearized versions of the St. Venant equations were numerically stable independent of the size of the time or distance steps. However, Chaudhry and Contractor (1973), Fread (1974) and Cunge (1975, pp. 539-586) found that instability could occur for the implicit schemes if the time steps were too large and the x-derivative terms were not sufficiently weighted towards the future time line when modeling rapidly varying transients. Also, time steps are restricted in size for reasons of accuracy; Δt is found to depend upon the shape of the wave, the Courant condition, the Δx step size, and the type of implicit scheme used. Nonlinearities due to irregular cross sections having widths that vary rapidly in the x-direction along the waterway or in the vertical direction can also cause numerical instabilities.

Implicit models are computationally more complex than explicit models. Depending on the type of implicit scheme (linear or nonlinear), the number of computations during a time step increases by a factor of approximately 2 to 4 compared to the requirements of an explicit scheme. This increase is very much greater if the method of solving the system of simultaneous equations is not an efficient method such as: 1) a compact quad-diagonal elimination method described by Fread (1971) which makes use of the banded structure of the coefficient matrix of the system of equations, or 2) the double sweep method developed in Europe (Liggett and Cunge, 1975, pp. 149-156). If the implicit scheme is linear, only one solution of the system of equations is required at each time step. However, if the implicit scheme is nonlinear, an iterative solution is necessary, and this requires one or more solutions of the system of equations at each time step. The use of the Newton-Raphson iterative method for nonlinear systems of equations (Amein and Fang, 1970) provides a very efficient solution if selected convergence criteria are practical. If the Newton-Raphson method is applied only once, the nonlinear implicit model is essentially equivalent to the linearized implicit models with respect to computational effort and performance.

Nonlinear implicit methods can be based on the conservation form of the St. Venant equations including lateral flow q (inflow is positive, outflow is negative) and off-channel (inactive flow) storage area A_o, i.e.,

$$\frac{\partial Q}{\partial x} + \frac{\partial (A+A_o)}{\partial t} - q = 0 \tag{25}$$

$$\frac{\partial Q}{\partial t} + \frac{\partial (Q^2/A)}{\partial x} + gA\left(\frac{\partial h}{\partial x} + S_f\right) + L = 0 \tag{26}$$

where $L = -qv_x$ for lateral inflow $L = -qQ/A$ for bulk lateral outflow, $L = -qQ/(2A)$ for seepage lateral outflow, and Q is discharge. An important advantage of Eqs. (25-26) when they are expressed in finite difference form is their ability to describe steep-fronted waves. Also, the dependent variables Q and h are more convenient and useful than V and y of Eqs. (21-22). Eqs. (25-26) apply to waterways of nonprismatic geometry.

Linear implicit methods often utilize an expanded form of Eqs. (25-26) such as that used by Chen, et al. (1975, pp. 316-319), i.e.,

$$\frac{\partial Q}{\partial x} + B_T \frac{\partial y}{\partial t} - q = 0$$

$$\frac{\partial Q}{\partial t} + 2\frac{Q}{A}\frac{\partial Q}{\partial x} - \frac{Q^2}{A^2}\left(B\frac{\partial y}{\partial x} + \left.\frac{\partial A}{\partial x}\right)_{y=c}\right) + gA\left(\frac{\partial y}{\partial x} - S_o + S_f\right) + L = 0$$

in which B_T is the total top width (active and inactive), $\partial A/\partial x)_{y=c}$ is the variation of A with respect to x with the depth (y) held constant and S_f is expanded in a Taylor series in order to linearize this highly nonlinear term, i.e.,

$$S_f^{t+\Delta t} = S_f^t + \left(\frac{\partial S_f}{\partial Q}\right)^t (Q^{t+\Delta t} - Q^t) + \left(\frac{\partial S_f}{\partial y}\right)^t (y^{t+\Delta t} - y^t)$$

in which the superscripts t and $t+\Delta t$ indicate at which time line the term is evaluated. In linear methods, the accuracy of the solution is very dependent on the size of Δt if the flow is rapidly changing with time due to the assumption of linearity of flow throughout a time step.

Implicit schemes have generally been four-point, i.e., the conservation of mass and momentum have been applied to the flow existing between two adjacent cross sections. The weighted four-point scheme allows a convenient flexibility in the placement of x-derivative and non-derivative terms between two adjacent time lines in the x-t solution domain. The weighting factor must be equal to or greater than 1/2 to provide unconditional linear stability with respect to time step size, and the accuracy of the scheme generally decreases as the weighting factor approaches unity, i.e., when the terms are expressed entirely at the forward time line. A few six-point schemes have been proposed, e.g., Abbott and Ionescu (1967) and Vasiliev, et al. (1965), but they have the disadvantage of requiring regular Δx intervals whereas the four-point schemes allow variable Δx spacing. Also, the six-point schemes treat the boundary conditions in a more complicated and less desirable manner than the four-point schemes.

Finite Element Models

The method of finite elements (Gray, et al. 1977) can also be applied to the complete St. Venant equations, e.g., Cooley and Moin (1976). Although this method of solution is popular in two-dimensional unsteady flow models, it does not appear to offer any advantages over the four-point nonlinear implicit models for the St. Venant one-dimensional equations of unsteady flow. Also, the mathematical basis for finite element solution schemes is not as easily understood as the finite difference approach. At this time it seems that the personal

preference of the model developer is the determining factor in selection of finite element or finite difference solution methods for the St. Venant equations.

Two-Dimensional Models

Two-dimensional hydraulic models such as the complete models described by Hinwood and Wallis (1975a,b) and Abbott (1976), Abbott and Cunge (1975, pp. 763-812), Grupert (1976), and the simplified models described by Cunge (1975, pp. 705-762) and Vicens, et al. (1975), are beyond the intended scope and have been omitted from consideration herein. They are generally much more expensive to calibrate and execute on high-speed computers than the one-dimensional models discussed herein. They are often considered as an alternative modeling approach whenever a large amount of flow information is desired in complex unsteady flows associated with estuarial networks and bays.

A VIEW TOWARD FUTURE IMPROVEMENTS

During the next few years, several improvements in flood routing models are anticipated. An attempt is made herein to delineate the general trend of such improvements and to suggest some specific deficiencies in present models on the basis of their tractability for improvement and their importance to hydrologists and hydraulic engineers.

It appears that the trend for increasing computational speed and storage capabilities of both large and small computers will be sustained throughout the 1980's. Also, the accessibility to such computational resources will become more commonplace and economically feasible to both large and small agencies, universities, and engineering consulting firms. For these reasons, flood routing models based on the complete St. Venant equations will continue to receive much attention from model developers and increasing use in the engineering community. Since the implicit dynamic models are the most promising of the complete hydraulic models for many flood routing applications due to their superior computational efficiency, many future improvements will likely be associated with this type of model.

However, the simplified models will continue to be much used, particularly as components of precipitation-runoff catchment models for routing overland flow and channel flow associated with the network of headwater streams which feed larger, more mild sloping collecting streams. Therefore, it is important that the strengths and limitations of the simplified models be set forth and their relationship to other routing models, especially the complete models, be understood through analyses similar to those by Cunge (1969), Miller and Cunge (1975, pp. 183-248), Ponce, et al. (1978), and Koussis (1978, 1980). The analysis should quantify a model's characteristics in terminology familiar to hydraulic engineers and devoid as much as possible of the terminology associated with other disciplines as electrical engineering, oceanography, etc. where similar analysis techniques were used prior to their application to flood routing models.

Future improvements in implicit dynamic models include the following: 1) Develop an efficient solution algorithm for flow which changes from subcritical to supercritical, and vice versa, with both time and distance along the waterway (this is especially important in the application of implicit dynamic models to routing dam-break waves). 2) Develop an efficient solution algorithm for flows subject to significant backwater effects in channel networks of dendritic and/or bifurcated

configurations; during the past decade some effort has been made in this area although an optimally efficient and versatile algorithm is still needed; previous work in this area of model development include Kamphuis (1970), Wood, et al. (1972), Fread (1973), Bennett (1975), and Yen and Osman (1976). 3) Develop improved one-dimensional modeling of meandering rivers with short-circuiting flood-plain flow and large differences between channel and flood-plain properties such as hydraulic roughness and wave celerity; some effort in this area has been made by Radojkovic (1976), Fread (1976), Tingsanchali and Ackermann (1976), and Weiss and Midgley (1978). 4) Analysis of effects of nonlinear terms in the St. Venant equations on the stability and accuracy of implicit solution algorithms. 5) Develop manual and/or automatic smoothing techniques to overcome nonlinear instabilities due to rapid variations of cross-sectional properties with elevation and distance along the waterway.

A significant area of general improvement consists of expanding flood routing models to account for significant effects of bridges, breached or over-topped levees, ice covers, ice jams, flow exchanges with groundwater aquifers due to bed and bank seepage and flood plain infiltration, and bed elevation and bed roughness changes caused by sediment transport. There exists a large body of knowledge in each of these areas; however, the incorporation of this into flood routing models has not received enough attention. Some work in this area has been done, e.g., Chen and Simons (1975) and Ponce, et al. (1979) concerning bed elevation changes due to sediment transport; Pinder and Sauer (1971), Freeze (1972), Hall and Moench (1972), Cooley and Westphal (1974), and Pogge and Chiang (1977) concerning the flow exchange between the waterway and adjacent aquifer; Uzner and Kennedy (1976) concerning ice jams; and Balloffet (1969), Cunge (1975, pp. 712-714), Fread (1978, 1980) concerning effects of levees, bridge/embankments, and other man-made structures.

Development of updating techniques to improve real-time simulation of unsteady flows such as in flood forecasting are needed. Approaches include the use of filter theory, e.g., the Kalman filter technique (Chiu and Isu, 1978).

Calibration of flood routing models is most essential for good results. The calibration process for diffusion and dynamic hydraulic models when applied to complex systems of waterways is often time-consuming and requires considerable experience. There is a need for the development of objective calibration methodologies which may be trial-error and/or automatic, e.g., Yeh and Becker (1973), and Fread and Smith (1978).

Flood routing models should be developed having a modular design. This will permit convenient selection of various combinations of external and internal boundary conditions permitting the same model to be used for a wide range of applications.

CONCLUDING REMARKS

Flood routing has in the past and will continue to be an important engineering endeavor, and this importance along with its inherent complexity have been the reasons for the proliferation of routing models. The literature abounds with a wide spectrum of useable and reasonably accurate mathematical models for flood routing when each is used within the bounds of its limitations.

Among the many models reviewed herein, the hydraulic models based

on the complete St. Venant equations have the capability to correctly simulate the widest spectrum of wave types and waterway characteristics. Since the hydraulic models contain only one parameter (the roughness coefficient), they are very amenable to the calibration process. Also, since the roughness coefficient can be estimated with some degree of accuracy from inspection of waterways, or better still from minimal stage-discharge data, the hydraulic flood routing method is preferred when there is a scarcity of pertinent inflow-outflow observations such as in the case of ungaged rivers or proposed man-made changes to waterways. The hydraulic method is also preferred for routing floods which extend beyond the range of the floods for which the model is calibrated. The dynamic wave models are preferred over all other models when the downstream backwater effect is important such as that produced by tides, significant tributary inflows, dams, and/or bridges, or when upstream propagation of waves can occur from large tides and storm surges or very large tributary inflows. The implicit dynamic wave model is the most efficient and versatile although also the most complex of the complete hydraulic models.

In the absence of significant backwater effects, the hydrologic storage routing models offer the advantage of simplicity. The hydrologic models have two calibration parameters which can be calibrated to effectively reproduce the simple characteristics of a flood wave such as its celerity and crest attenuation. The Muskingum model continues to be one of the more popular hydrologic models, and the Muskingum-Cunge model has the advantage of identifying its two parameters with wave and channel characteristics when there is insufficient inflow-outflow observations. The linearized impulse response models, particularly the multiple linearized models, also appear to be capable alternatives to the Muskingum-type models. It should be remembered, however, that insignificant backwater effect alone does not always justify the use of hydrologic or the linearized models, since combinations of gently sloping channels and rapidly varying flood waves may also require the complete hydraulic models for best results.

The final choice of a routing model is also influenced by other factors such as the required accuracy, the type and availability of data, the available computational facilities, the computational costs, the extent of flood wave information desired, and the familiarity of the user with a given model. Taking all factors into consideration results in the reality that there is no universally superior routing model.

At this time, with the demonstrated results of the many practical and theoretical applications of flood routing models, it can be accepted that the general principles of flood wave propagation in waterways with unchanging characteristics are well understood. It remains for flood routing models to be developed which can effectively and efficiently consider the interactions of the flood wave propagation phenomenon with sediment transporation, groundwater aquifers, ice, and man-made structures. In addition, the more complex implicit dynamic models which promise economical feasibility for complex routing problems require further development for efficient and convenient application to channel networks, very irregular channel geometry, and mixed subcritical-supercritical unsteady flows.

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