15280 LENGTH OF WATER RESOURCE SIMULATION TESTS

KEY WORDS: **Correlation analysis;** Mathematical models; **Planning; Project feasibility;** Research; Sampling; **Simulation;** Statistical analysis; **Systems engineering; Water plans;** Water resources

ABSTRACT: The research procedure was to: (1)Explore the experimental nature of simulation; (2)outline criteria for making "sampling" decisions about the length of simulation experiments; and (3)develop test statistics for ranking alternative system designs. Lengths of simulation runs required to achieve desired levels of reliability are shown to depend on serial and cross-covariance properties of simulated net benefit time series. Application of the procedures suggested by this study to a set of simulations of alternative plans for the Rio Colorado, Argentina, shows that the longest runs are required for alternatives with largest variance of annual net benefits. Serial correlation tends to increase required run length whereas cross-correlation tends to reduce required run length. In need of further work are: procedures to rank many alternatives having correlated net benefit series; procedures for nonstationary series; procedures to deal with non-normal benefit series; and procedures to account for the consequences of making incorrect choices.

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Design of Length of Water Resource Simulation Experiments

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Introduction

Water resource system simulation models are used to evaluate alternative system designs and operating policies. Either historical or generated streamflow data are input to these models, and computed time series of annual benefits and other performance information are output(s). Systems designs and operating policies are evaluated and ranked on the basis of summary statistics from these output time series (5). For example, if alternatives were to be ranked by mean annual net benefits, the better of two alternatives would tend to have the greater simulated mean annual net benefits. But the latter will not always be greater because of random variations in simulation outputs caused by random variations in the streamflow inputs. There exists some risk, therefore, that alternatives may not be correctly ranked depending on lengths of simulation runs, serial and cross-correlation of annual benefits, variance of annual benefits, and required level of resolution of differences between alternatives.

Two types of uncertainty cause a risk that alternatives will not correctly be ranked. These are shown in Fig. 1. First, uncertainty is inherent in the simulation model and in input parameters estimated from historical data. Second, uncertainty is introduced by stochastic streamflow generation. This second type may be reduced to be arbitrarily small, but there is no similar way to reduce the first. Fig. 1 suggests computed means tend, as simulation durations are extended, to converge to expected values within limits of uncertainty inherent in the models and input parameters.

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Purpose

Each run of a simulation model constitutes an experiment. Design of simulation experiments involves: (1) Selecting alternatives to be simulated; (2) selecting functions for ranking the alternatives; and (3) determining durations of simulation runs or number of simulation runs, or both. Selecting alternatives and selecting ranking functions for simulation are beyond the scope of this paper. Only the duration of the simulation run is considered in this study.

Since cost of simulation is usually proportional to duration of simulation, the goal should be to minimize the total duration of all runs. The idea is to allocate simulation effort where it will be most productive in yielding information about the relative ranking of different alternatives. A method for doing this is presented.

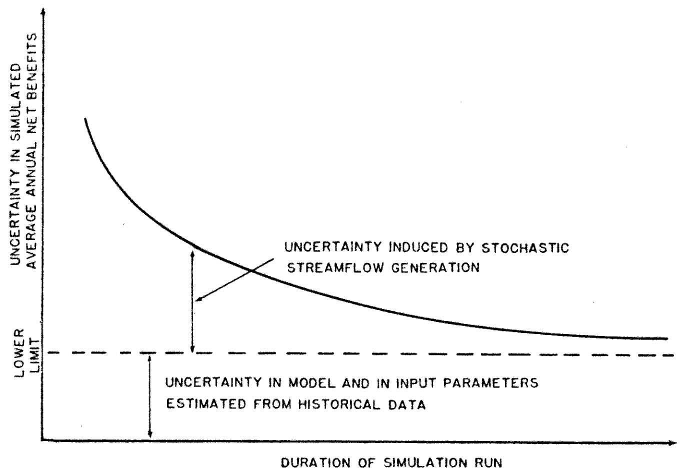

FIG. 1.—Relation between Uncertainty in Simulated Average Annual Net Benefits and Length of Simulated Run

Selecting Better of Two Alternatives

Sampling Theory.—As a specific example, suppose two alternatives are to be ranked by average annual net benefits. Each simulation run would give a time series of annual net benefits, and an average annual net benefit could be computed from this series.

The time series of annual net benefits is assumed to be generated by a stationary random process, throughout this work, although consideration of nonstationary time series is presented by the writers (16). It may be shown that the minimum variance estimator of the expected value of discounted total net benefits is a function of the mean of the simulation sample of net benefits for the stationary case. Thus, in the following analysis, the average annual net benefits are considered and are assumed to be discounted as a second step.

Different n-yr runs of the same alternative would give different time series of annual net benefits yielding different average annual net benefits. Therefore, the simulated average annual net benefit is a random variable, distributed according to a sampling distribution. The variance of this distribution may be controlled because it is a function of simulation duration.

If two different alternatives are simulated using the same historical or generated streamflow series, simulated average annual net benefits are jointly distributed according to a bivariate sampling distribution. Both variance and covariance properties of this distribution may be controlled.

Probability of Selecting Correctly.—The alternative having the largest simulated average annual net benefits would apparently be the better alternative, but this is not certain because of the random sampling variations in the simulated annual net benefits. Although a correct decision cannot be guaranteed absolutely, the chances of an incorrect decision can be controlled.

Consider the computed mean annual net benefits for two alternatives:

$$\hat{X}_1 = \frac{\sum_{i=1}^{n_1} x_{1i}}{n_1}; \quad \hat{X}_2 = \frac{\sum_{i=1}^{n_2} x_{2i}}{n_2} \qquad (1)$$

in which $\hat{X}_1$, $\hat{X}_2$ = sample mean for alternatives 1 and 2; x_{1i}, x_{2i} = annual net benefits for alternatives 1 and 2 for year i of simulation; and n_1, n_2 = length of simulation run for alternatives 1 and 2.

As described before, annual net benefits x_{1i} and x_{2i} are random variables. For simplicity, the assumption that they are generated from an independent Normal process will be made. Therefore, sample means $\hat{X}_1$ and $\hat{X}_2$ are Normal also. Even if x_{1i} and x_{2i} are not Normal, $\hat{X}_1$ and $\hat{X}_2$ approach normality according to the Central Limit Theorem. The rate at which they approach normality is a function of their underlying probability distribution function (pdf). In summary, if

$$x_{1i} \sim N(\mu_1, \sigma_1^2); \quad x_{2i} \sim N(\mu_2, \sigma_2^2) \qquad (2)$$

then $\hat{X}_1 \sim N(\mu_1, \sigma_{\hat{X}_1}^2)$, $\hat{X}_2 \sim N(\mu_2, \sigma_{\hat{X}_2}^2)$ (3)

in which $\sigma_{\hat{X}_1}^2 = \frac{\sigma_1^2}{n_1}$, $\sigma_{\hat{X}_2}^2 = \frac{\sigma_2^2}{n_2}$ (4)

Eq. 4 shows that, as lengths of the simulation runs increase, variances of sample means decrease; and consequently, the reliability of the estimates increases. A "correct" decision would be to select the alternative having the largest *true* mean. Therefore, a correct decision will be made if, and only if, that alternative also has the largest sample mean. In other words

$$\hat{X}_1 > \hat{X}_2 \quad \text{when} \quad \mu_1 > \mu_2 \quad \text{or} \quad \hat{X}_2 > \hat{X}_1 \quad \text{when} \quad \mu_2 > \mu_1 \qquad (5)$$

Mathematically, this can be expressed with a *test statistic* θ, where

$$\theta = \frac{\hat{X}_1 - \hat{X}_2}{\mu_1 - \mu_2} \qquad (6)$$

A correct decision is made when θ is positive. The variable θ is a weighted sum of two random variables, and is thus a random variable itself with properties. Therefore

$$\theta \sim N(1,\sigma_\theta^2) \qquad (7)$$

in which $$\sigma_\theta^2 = \frac{\sigma_{\bar{X}_1}^2 + \sigma_{\bar{X}_2}^2}{(\Delta\mu)^2} \qquad (8)$$

and $$\Delta\mu = \mu_1 - \mu_2 \qquad (9)$$

is the net benefit resolution of the test.

The probability of making a correct decision is equal to the probability θ is greater than zero [i.e., $P(\theta > 0)$].

Criterion for Experiment Design.—If the costs of simulation are assumed to be proportional to the total number of years simulated, an efficient experiment would

$$\text{Minimize} \quad n_1 + n_2 \quad \text{subject to:} \quad \text{Prob (Correct decision)} \geq P^* \qquad (10)$$

The optimal solution for this problem will give the values of n_1 and n_2, which reduces the variance of θ to yield a probability of a correct decision equal to P^*. Transforming θ to a standard normal deviate, β, gives

$$\beta = \frac{\theta - 1}{\sigma_\theta} \qquad (11)$$

Now $$\beta \sim N(0,1) \qquad (12)$$

and the reliability statement becomes

$$P(\theta > 0) = P\left(\beta > -\frac{1}{\sigma_\theta}\right) = P^* \qquad (13)$$

Solution Technique.—Fishman (8) proposed using Lagrange Multipliers (10,17) to solve the optimization problem of Eq. 10 as follows:

$$\text{Minimize} \quad L = n_1 + n_2 + \lambda\,(V - \sigma_\theta^2) \qquad (14)$$

in which L = Lagrangian function; λ = Lagrange multiplier; and V = value of σ_θ^2 which will satisfy Eq. 13. Expanding Eq. 8 for σ_θ^2 gives

$$\sigma_\theta^2 = \frac{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}{(\Delta\mu)^2} \qquad (15)$$

Therefore, Eq. 14 can also be expressed as

$$\text{Minimize} \quad L = n_1 + n_2 + \lambda\left(V - \frac{\sigma_1^2}{n_1(\Delta\mu)^2} - \frac{\sigma_2^2}{n_2(\Delta\mu)^2}\right) \qquad (16)$$

Define new variables s_1 and s_2 as

$$s_1 = \frac{\sigma_1^2}{(\Delta\mu)^2}; \quad s_2 = \frac{\sigma_2^2}{(\Delta\mu)^2} \qquad (17)$$

The complete solution to Eq. 16 through the Lagrange method (7,16) results in

$$n_1^* = \frac{s_1}{V}\left[1 + \left(\frac{s_2}{s_1}\right)^{1/2}\right] \qquad (18)$$

$$n_2^* = \left(\frac{s_2}{s_1}\right)^{1/2} n_1^* \qquad (19)$$

in which n_1^* and n_2^* are the optimal run lengths. The value of V can be obtained from normal probability tables, since

$$V = \left(-\frac{1}{\beta}\right)^2 \qquad (20)$$

in which β = the value of the normal unit deviate that satisfies Eq. 13 for a specific reliability level, P^*.

Example Relationships between P^* and Length of Simulation Run

Independent Annual Benefits: Case I.—A desired reliability level, P^*, defines a required maximum standard deviation of θ, σ_θ, which consequently specifies the length of the simulation runs. Table 1(*a*) summarizes example data for Case I. Note in this example that $\sigma_1^2 = \sigma_2^2$.

The optimal solution requires $n_1 = n_2$ because Eq. 17 gives $s_1 = s_2$ and Eq. 19 gives $n_2 = n_1$.

A display of the length of the simulation runs versus P^* for Case I is shown in Fig. 2(*a*).

Serially Correlated Annual Benefits: Cases II and III.—There are numerous references in the water resources literature to the fact that streamflow data may be serially correlated. Similarly, there exist serial correlations in surface and ground storage levels. As a result, it is reasonable to assume that outputs from development alternatives will also be correlated (14). For example, benefits from irrigation regions are heavily dependent on the water available. Low flows followed by low flows will probably result in low benefits followed by the same. Therefore, it is crucial to consider the possibility of serial correlation in simulation outputs.

The structure and notation definéd in the preceding section allows us to include the correlation effects. Only Eq. 4 needs to change. The variance of the sample means has been defined by Fishman (8) as

$$\sigma_{\bar{X}_1}^2 = \frac{\sigma_1^2}{n_1}\left[1 + 2\sum_{\tau=1}^{n_1-1}\left(1 - \frac{\tau}{n_1}\right)\rho_1(\tau)\right] \qquad (21)$$

$$\sigma_{\bar{X}_2}^2 = \frac{\sigma_2^2}{n_2}\left[1 + 2\sum_{\tau=1}^{n_2-1}\left(1 - \frac{\tau}{n_2}\right)\rho_2(\tau)\right] \qquad (22)$$

in which $\rho_1(\tau)$ and $\rho_2(\tau)$ = the lag-τ serial correlation coefficients for the time

TABLE 1.—Parameters of Example Benefit Generating Processes for Two Alternatives

Parameter (1)	Alternative 1 (2)	Alternative 2 (3)
(*a*) Case I: Independent Samples		
μ	4.5×10^6	4.0×10^6
σ^2	3.24×10^{12}	3.24×10^{12}
(*b*) Case II: Markov Process		
μ	4.5×10^6	4.0×10^6
σ^2	3.24×10^{12}	3.24×10^{12}
$\rho(1)$	0.4	0.4
$\rho(\tau)$	0.4^{τ}	0.4^{τ}
(*c*) Case III: Fractional Gaussian Process[a]		
μ	4.5×10^6	4.0×10^6
σ^2	3.24×10^{12}	3.24×10^{12}
H	0.8	0.8
(*d*) Case IV-A: Cross-Correlated Samples[b]		
μ	4.5×10^6	4.0×10^6
σ^2	3.24×10^{12}	3.24×10^{12}
ρ	0	0
(*e*) Case IV-B: Cross-Correlated Samples[b]		
μ	4.5×10^6	4.0×10^6
σ^2	3.24×10^{12}	3.24×10^{12}
ρ	0.4	0.4

[a]The autocovariance of the fractional Gaussian process is 0.5 $[|s+1|^{2H} - 2|s|^{2H} + |s-1|^{2H}]$.

[b]The cross correlation coefficient between the net benefit series for the two alternatives is equal to 0.3.

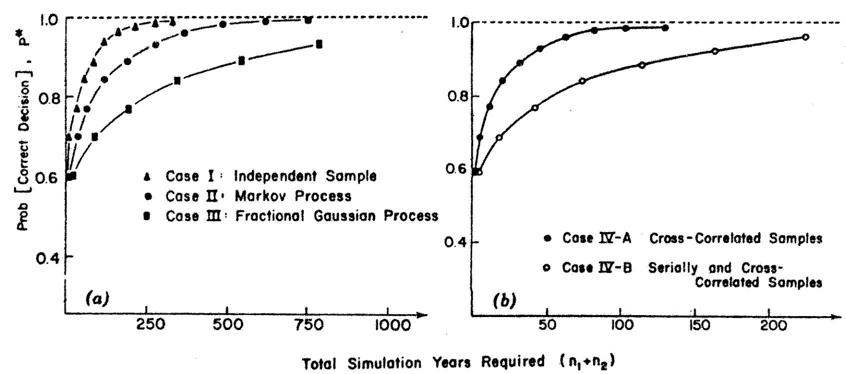

FIG. 2.—Required Length of Simulated Runs: (*a*) Cases I, II, III; (*b*) Cases IV-A and IV-B

series. These can now be introduced in Eq. 8. The variables s_1 and s_2 are now

$$s_1 = \frac{\sigma_1^2}{(\Delta\mu)^2}\left[1 + 2\sum_{\tau=1}^{n_1-1}\left(1 - \frac{\tau}{n_1}\right)\rho_1(\tau)\right] \quad (23)$$

$$s_2 = \frac{\sigma_2^2}{(\Delta\mu)^2}\left[1 + 2\sum_{\tau=1}^{n_2-1}\left(1 - \frac{\tau}{n_2}\right)\rho_2(\tau)\right] \quad (24)$$

These can be rearranged and used in Eq. 16 to solve the optimization problem in the same manner as for independent samples.

If in the previous example, the data were serially correlated, longer simulation run lengths would be required. This is illustrated by Case II which is the same as Case I except the data are taken from a Markov process with a lag-one correlation coefficient equal to 0.4 for both series. Required run lengths for the serially correlated case are shown in Fig. 2(*a*).

An exponential autocorrelation function is typical of processes possessing relatively short "memory." Other processes with longer "memory" would require longer simulation runs. Such is the case for a fractional Gaussian noise process (12). Table 1(*c*) contains the data for this case. Required run lengths are plotted on Fig. 2(*a*). They show that processes with longer "memory" require the simulation runs to be longer.

Cross-Correlated Annual Benefits.—If the same streamflow data are used to simulate two different alternatives, the annual benefits will be cross-correlated. This cross-correlation may help in reducing the length of run required.

To consider the reduction in σ_θ^2 due to cross-correlation, Eq. 8 must be expanded. With cross-correlation, but without serial correlation, the expression is

$$\sigma_\theta^2 = \frac{\sigma_{\hat{X}_1}^2 + \sigma_{\hat{X}_2}^2}{(\Delta\mu)^2} - \frac{2\,\mathrm{Cov}(\hat{X}_1, \hat{X}_2)}{(\Delta\mu)^2} \quad (25)$$

in which the Cov $(\hat{X}_1, \hat{X}_2)$ is approximately

$$\mathrm{Cov}(\hat{X}_1, \hat{X}_2) \simeq \frac{\sigma_1\sigma_2}{\max(n_1, n_2)}\left\{1 + \sum_{\tau=1}^{r}\left(1 - \frac{\tau}{r}\right)[\rho_{12}(\tau) + \rho_{21}(\tau)]\right\} \quad (26)$$

and $\rho_{12}(\tau)$ and $\rho_{21}(\tau)$ = the lag-τ cross-correlation coefficients. If s_3 is defined as

$$s_3 = \frac{\mathrm{Cov}(\hat{X}_1, \hat{X}_2)}{(\Delta\mu)^2} \quad (27)$$

then the Lagrangian function, Eq. 14, becomes

$$\text{Minimize} \quad L = n_1 + n_2 + \lambda\left[V - \frac{s_1}{n_1} - \frac{s_2}{n_2} + 2\frac{s_3}{\max(n_1, n_2)}\right] \quad (28)$$

Fishman (8) solved this problem, and this derivation is described in detail by the writers (16). The results will be only summarized here. For

$$s_2 \le s_1 - 2s_3 \quad (29)$$

$$n_1^* = \frac{1}{V}\{s_1 + [s_2(s_1 - 2s_3)]^{1/2} - 2s_3\} \quad (30)$$

$$n_2^* = \left[\frac{s_2}{(s_1 - 2s_3)}\right]^{1/2} n_1^* \quad (31)$$

For $s_2 \geq s_1 + 2s_3$ (32)

$$n_1^* = \left[\frac{s_1}{(s_2 - 2s_3)}\right]^{1/2} n_2^* \quad (33)$$

$$n_2^* = \frac{1}{V}\{[s_1(s_2 - 2s_3)]^{1/2} + s_2 - 2s_3\} \quad (34)$$

And for $s_1 - 2s_3 < s_2 < s_1 + 2s_3$ (35)

$$n_1^* = n_2^* = \frac{s_1 + s_2 - 2s_3}{V} \quad (36)$$

Fig. 2(*b*) displays the effect of cross-correlation. The same data as in previous examples were used [see Table 1(*d*)], except cross-correlation was added. Two cases are shown; one in which there was only cross-correlation; and a second in which both serial and cross-correlation existed. Comparison of Figs. 2(*a*) and 2(*b*) will show cross-correlation reduces the number of samples required even when serial correlation is present.

Selecting Best of Several Alternatives

Although very simple in structure, the run length selection procedures described in the preceding section cannot directly be extended to selection from more than two alternatives. In such cases a more general procedure must be derived. Some procedures applicable to this task are the so-called identification or ranking procedures described in Bechofer, et al. (1). One of these procedures will be described.

Problem Statement.—Suppose there are $k(\geq 2)$ alternatives to be simulated. The goal is to select the one with the largest expected annual net benefits. The results from the simulation are assumed to be independent observations from normally distributed populations, i.e.

$$x_{jt} \sim \mathrm{IN}(\mu_j, \sigma_j^2),\ j = 1, 2, ..., k \quad (37)$$

in which x_{jt} = an observation from year t of the run for alternative j. The variances σ_j^2 are unknown and unequal. All serial and cross-correlations between the x_{jt} are assumed, in this case, to be zero. A mathematical expression for the goal is

$$P\,[\text{Correct selection}] \geq P^* \quad (38)$$

whenever the difference between the largest and next to largest true mean is at least δ^*. Both P^* and the desired net benefit resolution, δ^*, are specified by the experimenter in advance.

Selection Procedure.—Dudewicz and Dalal (4) presented the following solution.

First, take an initial sample of size $n_0(\geq 2)$ from each population. From these observations compute the sample means and variances as follows:

$$\hat{X}_j = \frac{1}{n_0}\sum_{t=1}^{n_0} x_{jt} \quad (39)$$

$$\hat{S}_j = \left[\frac{1}{n_0 - 1}\sum_{j=1}^{n_0}(x_{jt} - \hat{X}_j)^2\right]^{1/2} \quad (40)$$

From these estimate

$$n_j = \max\left\{n_0, \left[\left(\frac{\hat{S}_j h}{\delta^*}\right)^2\right]\right\} \quad (41)$$

in which h uniquely satisfies

$$\int_{-\infty}^{\infty}[F_{n_0}(z + h)]^{k-1} f_{n_0}(z)\,dz = P^* \quad (42)$$

and $F_{n_0}(\cdot)$ and $f_{n_0}(\cdot)$ = respectively, the cumulative distribution function and density function of a Student's-*t* random variable with $n_0 - 1$ degrees-of-freedom (2). Dudewicz and Dalal (4) present tables to solve for h given k, n_0, and P^*. Take $n_j - n_0$ additional observations from alternative j and redefine. Thus

TABLE 2.—Parameters of Example Benefit Generating Processes for Multiple Alternatives[a]

Alternative j (1)	μ_j (2)	σ_j^2 (3)	n_0 (4)
1	4.5×10^6	3.24×10^{12}	30
2	4.0×10^6	2.56×10^{12}	30
3	3.5×10^6	7.84×10^{12}	30
4	3.0×10^6	5.76×10^{12}	30

[a]Example benefits are independently distributed.

TABLE 3.—Statistics Computed from Observed Sample of Example Benefits and Require Simulation Run Length[a]

Alternative j (1)	$\hat{X}_j$ (2)	$\hat{S}_j^2$ (3)	n_j (4)	$n_j - n_0$ (5)
1	4.40×10^6	3.00×10^{12}	58	28
2	3.61×10^6	2.23×10^{12}	43	13
3	3.50×10^6	7.03×10^{12}	136	106
4	2.77×10^6	3.18×10^{12}	61	31

[a]$\delta^* = 1.0 \times 10^6$; $P^* = 0.995$; and $h = 4.40$.

$$\hat{X}_j = \frac{1}{n_j} \sum_{t=1}^{n_j} x_{jt} \quad \ldots \quad (43)$$

Select that alternative which yields the largest sample mean as defined by Eq. 43.

Example.—The alternative with the largest mean of four possibilities is to be selected. Table 2 presents the "true" (but unknown to the investigator) statistics on each of the alternatives. Samples of n_0 observations were randomly generated from these distributions, and the corresponding sample means and variances were computed. Next, Eq. 42 was used to select h; and finally, Eq. 41 was used to compute n_j. Table 3 summarizes the results. The final column in Table 3 shows that $n_j - n_0$ additional observations would be required from alternative j. As expected, those processes with higher variances require more samples than the others. Figs. 3 and 4 display the effects of the desired net

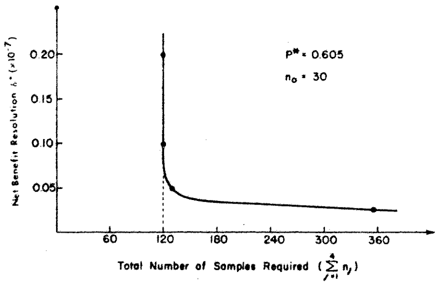

FIG. 3.—Relation between Net Benefit Resolutions δ* and Required Run Length to Rank Multiple Alternatives

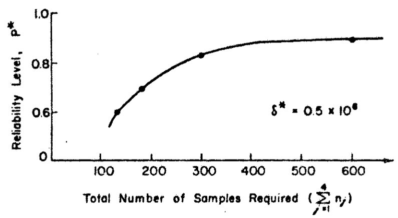

FIG. 4.—Relation between Reliability and Required Run Length to Rank Multiple Alternatives

benefit resolution δ* and selected values of P^* on the total number of samples required n_j.

Serially Correlated and Cross-Correlated Annual Benefits.—At present there is no available procedure to account for the effects of serial correlation or cross-correlation among the x_{jt}. Modification of the procedure described in the previous section to account for serial correlation in a manner parallel to the bivariate procedure would only be approximately correct. In fact, such a modification would underestimate the required run length for highly positively correlated processes. Further work in this area is necessary.

Application in Water Resources Planning

The statistical theory presented in previous sections forms part of the intellectual basis for the application of simulation techniques as a tool in water resources planning. Another part of this intellectual basis derives from multiple objective planning theory (11). In this section, important elements of these theories are brought together and illustrated in an application to planning for the Rio Colorado, Argentina.

Review of Multiple Objective Theory.—In multiple objective planning theory, each alternative plan j makes net contributions b_{ijt} towards each of several relevant objectives i during year t.

If a simulation model is programmed both to simulate the operation of a given alternative and to apply multiple objective accounting rules, a statistical analysis of the net contributions, b_{ijt}, toward each objective can be obtained. Among the statistics would be, assuming a stationary process, the average, the standard deviation, and the cross-correlation coefficients of the b_{ijt}.

If only the average net contribution, $\bar{b}_{ij}$, were considered, the subset of alternatives could be found that lie at or near the net benefit transformation surface. This is the subset of "noninferior" alternatives, one of which is "optimal" under a different set of weights. The role of systems techniques in multiple objective planning is to separate this partial ordering of alternatives into two sets: a noninferior set and an inferior set. The specific role of simulation models is to provide more precise evaluations of the alternatives than given by optimization models.

According to statistical theory, it is not possible to perform this partial ordering with certainty. It is only possible to estimate the probability that the correct partial ordering has been made. This is because the annual net benefits depend on the stochastic streamflow events and, probably, on other random events as well. It follows that the annual net benefits, b_{ijt}, form a multiple time series that may be regarded as a sample function from a multivariate stochastic process.

The intellectual basis for choosing *among* stochastic processes is not well developed at this time. For the purpose of this study, it is assumed that the mean value is the only measure of concern and that alternatives can be compared in terms of average present values of contributions toward the weighted relevant objectives.

Statistical Properties of Annual Net Benefits.—The annual net benefits, b_{ijt}, are given by the simulation for N yr of alternative j. If the system configuration and operating policy parameters remain fixed over the entire N-yr period, the stochastic process that generated the b_{ijt} may be regarded as a stationary process, unless there is an initial transient (due to the fixed initial conditions) that persists for a significant part of the N yr. On the other hand, variations with time of the system configuration or operating policy parameters implies that the b_{ijt} are generated by a nonstationary process. It is assumed here that for convenience the b_{ijt} are generated by a stationary process. Modifications to the theory due to nonstationarity are briefly reviewed by the writers (16).

The present value of the weighted net benefits is

$$z_j = \sum_{i=1}^{NO} w_i \left[\sum_{t=1}^{T} \frac{b_{ijt}}{(1 + r_i)^t} \right] \quad \ldots \quad (44)$$

in which w_i = the relative weight on objective i [Major (11)]; NO = the number of objectives being considered; r_i = the discount rate for objective i; and z_j = a random variable. The expected value of z_j is

$$E[z_j] = \sum_{i=1}^{NO} w_i \left\{ \sum_{t=1}^{T} E\left[\frac{b_{ijt}}{(1 + r_i)^t} \right] \right\} \quad \ldots \quad (45)$$

If the process generating the random variable b_{ijt} is stationary, then

$$E[z_j] = \sum_{i=1}^{NO} w_i \left[\sum_{t=1}^{T} \frac{1}{(1+r_i)^t} \right] E[b_{ijt}] \quad (46)$$

Let $$\bar{b}_{ij} = \frac{1}{T} \sum_{t=1}^{T} b_{ijt} \quad (47)$$

and $$\mathrm{PVF}(r_i, T) = \sum_{t=1}^{T} \frac{1}{(1+r_i)} \quad (48)$$

It may be shown that (17) the estimator

$$\hat{z}_j = \sum_{i=1}^{NO} w_i \,\mathrm{PVF}(r_i, T) \bar{b}_{ij} \quad (49)$$

is the minimum variance estimator for z_j, since it keeps the constant factors "outside" of the estimator $\bar{b}_{ij}$. (In other words, it is statistically "better" to calculate the average annual net benefit and then discount to present value than to discount to present value the individual annual net benefits directly.)

Definition of Net Benefit Transformation Curve.—A given set of weights, w_i, leads to a vector of values $\hat{\mathbf{z}}$ with elements $\hat{z}_j$. The largest expected value of the $\hat{z}_j$

$$z_{\max} = \max E[z_j] \quad \text{over all} \quad j \quad (50)$$

corresponds to the optimal alternative, j^*, for the given weights, w_i. The largest element of $\hat{\mathbf{z}}$, $\hat{z}_{\max}$, is an estimate of $z_{\max}$, and the corresponding $\hat{j}^*$ is an estimate of j^*. The probability that $\hat{j}^*$ is the correct value of j^* can be estimated according to the procedures given earlier.

The probability that $\hat{j}^*$ is the correct value of j^* applies to a particular set of weights, w_i. A much more difficult problem is to consider all possible sets of weights, w_i, estimate the optimal alternative, $\hat{j}^*$, for each set i; and then, assess the probability that the set of selected alternatives correctly contains the set of noninferior alternatives. This more difficult problem has not been addressed in this study and remains an important subject for further study.

The approach taken in the next section is to select a set of weights, w_i, apply the procedures given previously, and assume that the optimal alternative is a member of the noninferior set, lying on or near the net benefit transformation curve.

Application to Rio Colorado, Argentina

The design principles of simulation experiments examined in the previous sections were applied to HISIM, one of two simulation models developed at the Massachusetts Institute to Technology as part of a major river basin planning study for the Rio Colorado, Argentina (13,16). It was necessary to determine whether 50 of 500 yr of simulation were required as a basis for reliable project comparisons or rankings.

Simulation Model HISIM.—A simulation model was used to evaluate alternatives proposed by a screening model (13). The basic equation in the simulation was continuity; a detailed accounting of the flow of water through alternative systems was carried out and benefits were computed.

The inputs to HISIM were the specific values for all the decision variables, such as: reservoir locations and storage capacities; powerplant locations and generating capacities; irrigation site locations and sizes. In addition, operating rules were specified and benefit functions were given.

The outputs were of two types. First, measures of effectiveness were estimated such as the annual net benefits towards each objective, seasonal irrigation deficits, and seasonal power produced. Second, detailed engineering data were provided, such as: head fluctuations at reservoirs; evaporation volumes at reservoirs; and variations in irrigation levels.

TABLE 4.—Average Annual Net Benefits[a]

Alternative (1)	Average annual national income net benefits (2)	Average annual regional income net benefits (3)
WT1	56.5	4.7
WT1-N	58.9	.5
WT1-R	42.5	5.4

[a] Benefits are in 10^9 Argentine Pesos.

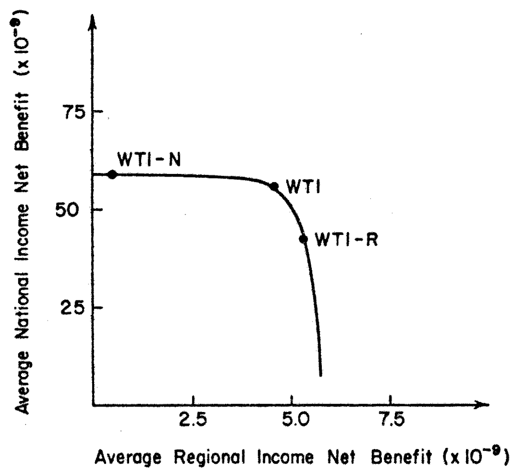

FIG. 5.—Net Benefit Transformation Curve

Presampling Decisions.—Sampling from the simulation experiments should be a two-stage process. First, n_0 yr of observations are taken to estimate the parameters. These are then used to compute the length of run required to attain a certain reliability level. Then, the additional observations are taken. In some cases no additional samples would be required. An initial sample of 50 yr was used in this study. This choice was based on a study of the correlation structure of the simulation outputs (16).

Example Simulation Results.—Three preliminary alternatives for development

TABLE 5.—Relative Weights on Objectives

Weight set (1)	National income weight (2)	Regional income weight (3)
1	1.0	0
2	1.0	1.0

TABLE 6.—Weighted Annual Net Benefits[a]

Weight set	Alternative j	μ_j	σ_j^2	$\rho_j(1)$
1	WT1	56.5×10^9	9.66×10^{18}	0.696
	WT1-N	58.9×10^9	3.46×10^{18}	0.618
2	WT1	61.2×10^9	10.5×10^{18}	0.692
	WT1-N	59.4×10^9	3.51×10^{18}	0.612

[a] Weighted benefits have no units.

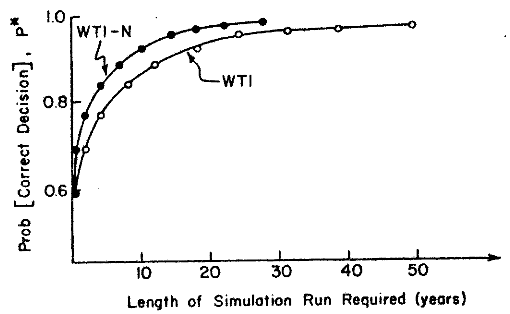

FIG. 6.—Required Length of Simulation Runs (Weight Set No. 1, Using Sample Autocorrelation Function)

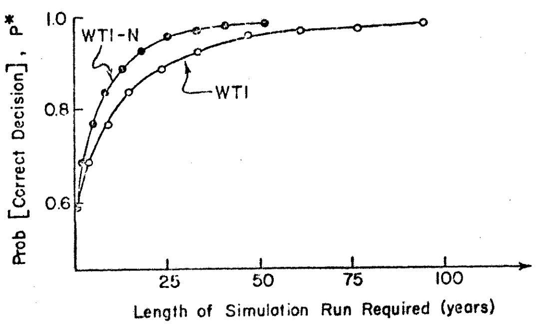

FIG. 7.—Required Length of Simulation Runs (Weight Set No. 2, Using Sample Autocorrelation Function)

of the Rio Colorado have been selected for this example. These alternatives differ in their emphasis of one or the other of the planning objectives. Specifically, these alternatives were evaluated under two potentially conflicting objectives: (1) Increase of national income; and (2) increase of the income to a particular region. One alternative favors national income, another regional income, and the third lies in between. Under different sets of weights for the contributions to each objective a different alternative would be selected as optimal. If these were the only alternatives available, they would in fact define the net benefit transformation curve. Table 4 shows the average annual net benefits for the three alternatives for each of the two objectives. Fig. 5 displays the expected value net benefit transformation curve.

Pair-Wise Comparisons.—As an example, consider the comparison of two alternatives, WT1 and WT1-N, under two different sets of weights on the objectives. The two sets of weights used are presented in Table 5. Table 6 shows the parameters estimated from a 50-yr run of each alternative. For each set of weights, the length of simulation runs required are presented in Figs. 6 and 7. Only one case is shown for each weight set. It uses the sample autocorrelation function. A second case that uses an exponential autocorrelation functions with a sample lag-one autocorrelation coefficient yields similar results (16).

TABLE 7.—Relative Weights on Objectives for Multiple Alternative Comparisons

Weight set (1)	National income weight (2)	Regional income weight (3)
3	1.0	0
4	0	1.0
5	1.0	0.1

TABLE 8.—Weighted Annual Net Benefit Statistics for Multiple Alternative Comparisons

Weight set (1)	Alternative j (2)	$\hat{X}_j$ (3)	$\hat{S}_j^2$ (4)
3	WT1	56.5×10^9	9.88×10^{18}
	WT1-N	58.9×10^9	3.55×10^{18}
	WT1-R	42.5×10^9	0.63×10^{18}
4	WT1	4.7×10^9	0.02×10^{18}
	WT1-N	0.5×10^9	0.0008×10^{18}
	WT1-R	5.4×10^9	0.00001×10^{18}
5	WT1	61.1×10^9	10.74×10^{18}
	WT1-N	59.4×10^9	3.61×10^{18}
	WT1-R	47.9×10^9	0.63×10^{18}

In almost all cases, a 50-yr run of the simulation model was sufficient to reach a high level of reliability, i.e., P^* larger than 90%. Moreover, the positive cross-correlation of the observations was not considered. Inclusion of this factor would have reduced the required run length significantly. Therefore, the first stage sampling was sufficient for comparing these particular alternatives under those particular weights on the objectives.

Multiple Alternatives.—In analogous manner, all three alternatives may be compared at the same time for a specific set of weights using the procedures for multiple alternatives. The sets of weights are presented in Table 7. The weighted annual net benefit statistics estimated for each of the alternatives from 50-yr simulation runs are shown in Table 8. The required simulation run lengths are in Table 9 for each set of weights. Again, there are two types of cases. In one, at least one of the alternatives is clearly superior to the others and therefore no more observations are required. For the other case, the choice is not so clear-cut. Longer runs are needed.

TABLE 9.—Required Length of Simulation Runs for Multiple Alternative Comparisons

Weight set (1)	δ^* (2)	P^* (3)	Run Length for Alternative: WT1 (4)	WT1-N (5)	WT1-R (6)
3	5.0 × 10	0.950	50	50	50
		0.999	50	50	50
	1.0 × 10	0.950	52	50	50
		0.991	114	50	50
		0.999	218	78	50
	0.5 × 10	0.815	56	50	50
		0.950	208	75	50
		0.991	456	164	50
4	1.0 × 10	0.999	50	50	50
	0.5 × 10	0.999	50	50	50
	0.1 × 10	0.999	55	50	50
5	5.0 × 10	0.950	50	50	50
		0.999	50	50	50
	1.0 × 10	0.950	56	50	50
		0.991	124	50	50
		0.999	237	79	50
	0.5 × 10	0.815	72	50	50
		0.950	227	76	50
		0.991	496	196	50

Summary and Conclusions

Summary.—Emphasis on the use of simulation models in water resource planning has been more on "model building" aspects than on experimental aspects that have been largely ignored. Questions asking how long simulation runs should be or how reliable the decisions based on a specific run length are have not been considered in detail. This study has attempted to deal with these questions, to apply some of the principles of design of experiments to simulation experiments, and to consider some of the problems and characteristics of water resources simulations.

Two procedures were described. The first allows pair-wise comparison of alternatives. The effect of correlated samples, both serially and across, may be included. The results are the reliability levels obtainable for various simulation run lengths. The second procedure allows comparison of more than two alternatives. This procedure does not consider correlated samples. Both techniques can compare weighted net benefits towards more than one objective.

Through the use of the experimental design procedures described the experimenter can assess the reliability of the information he presents to the decision-makers. For a specific set of weights on the objectives, the reliability of selecting an alternative as optimal can be evaluated. In addition, the number of additional observations required to attain a higher level may be determined. By using a variety of sets of weights the net benefit transformation curve may be defined and its reliability specified.

Computer programs were developed and used in a study of the Rio Colorado, Argentina to design the simulation experiments. These programs were quite useful in estimating the simulation run lengths.

Further Research.—This study has shown the need for further investigation in a number of areas as follows: (1) The sensitivity of the procedures to several factors such as different synthetic streamflow generators or large storage components in the alternative designs; (2) the effect of correlated observations in the sequential identification or ranking procedures for multiple alternatives; (3) the possibility of using either of these procedures within the simulation models themselves so as to evaluate the reliability *during* the runs; (4) the use of other procedures that explicitly consider the potential losses from incorrect decision rather than reliability criteria; (5) procedures for nonstationary net benefit series that arise when plan configurations or operating policies change over time; (7) the annual net benefits may not be normally distributed (the effects of non-normality and how this problem is handled should also be considered; and (8) procedures for estimating the probability that the proper partial ordering of alternatives has been achieved in multiple objective planning.

Conclusions.—The effect of serially correlated samples was to increase the required number of observations. Long memory generating processes require even longer runs than short memory processes. On the contrary, cross-correlated observations tend to reduce the length of runs required. In fact, cross-correlation in the observations tends to overcome the effect of the serial correlation.

The synthetic streamflow generator used may have an effect on the net benefit record obtained as output. Although this effect was not tested for in this study, it would appear that synthetic records generated by long-term memory processes such as those proposed by Mandelbrot (12) or Rodriguez-Iturbe, et al., (15), would result in the need for longer simulation runs.

A somewhat similar effect resulted from simulation runs of alternatives with large surface storage. Large reservoirs tend to "damp-out" the random variations in the streamflow inputs and result in stable and serially correlated benefit outputs.

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