SNOWMELT MODELING BY THE U.S. NATIONAL WEATHER SERVICE

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INTRODUCTION

The National Weather Service (NWS), National Oceanic and Atmospheric Administration (NOAA) of the U.S. Department of Commerce has participated in snowmelt and related studies for many years. During the late 1940's, the NWS (then the U.S. Weather Bureau) cooperated with the U.S. Army Corps of Engineers on snow investigation studies at the Central Sierra, Upper Columbia, and Willamette Basin Snow Laboratories in the Western United States. The results of these studies were summarized in the report "Snow Hydrology" (Corps of Engineers 1956).

Anderson (1968) reported on a snow cover energy balance model that was developed and tested using the data collected at the Central Sierra Snow Laboratory during the snow investigations (Corps of Engineers 1955). These data included all of the necessary input variables for one season but only under one general set of meteorological conditions.

No other set of high-quality data was available that included measurements of all the necessary input and verification variables required for development and evaluation of snowmelt energy balance models.

SNOW RESEARCH STATION

In the mid-1960's, the NWS established a snow research station to acquire all data required for testing snow cover energy exchange models. In cooperation with the Agricultural Research Service (ARS) of the U.S. Department of Agriculture, the snow research station was established on the ARS's Sleepers River Research Watershed near Danville, Vermont (Johnson and Anderson 1968). Actual data collection at the NOAA-ARS snow research station began in December 1968. A paper on the publication of complete data from this research location will be presented in the workshop of this meeting (Anderson, et al. 1977).

TEMPERATURE INDEX METHOD

An operational snow accumulation and ablation model (Anderson 1973) using temperature as the sole index to energy exchange across the snow-air interface was developed using the early data from the NOAA-ARS snow research station and data from the earlier cooperative snow

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For presentation at the Snow Accumulation and Ablation Models Symposium, Northern Research Basin Symposium-Workshop, Fairbanks, Alaska, August 15-19, 1977.

laboratories in the Western United States. This model was described at the Edefors, Sweden, Northern Research Basin Symposium. A summary of field applications will be presented during the present symposium.

LATEST STUDIES

The primary purpose of this report is to describe briefly a point energy and mass balance model recently developed by Anderson (1976). The basic purpose for this model was to aid in the understanding of snow cover energy exchange. The model combines the results of recent and past theoretical studies of energy transfer processes and has been tested using the data collected at the NOAA-ARS snow research station.

Since the data necessary for a complete energy exchange model are available only from research locations such as near Danville, Vermont, the model is essentially a research tool rather than an operational tool.

ENERGY AND MASS BALANCE MODEL

The new model described by Anderson (1976) was developed using energy transfer information from snow evaporation and turbulent transfer studies. The coefficients used in the model were kept within the range suggested by experimental results. The model results were verified on observed water equivalent, snow cover outflow, total pack and snow surface densities, and snow surface temperature. The results were quite good; thus, indicating that, if exceptionally good data are available, the energy exchange theory is adequate to obtain satisfactory simulations.

The snow cover albedo, density, and roughness are subject to fairly rapid changes and therefore snow is a more dynamic medium that most others to which energy transfer theory has been applied. Nevertheless, the success of simulating the observed pack conditions verifies the adequacy of the Danville data as well as the theory used in developing the model. The model should hold for other locations if the data are sufficient.

SNOW COVER ENERGY BALANCE

The complete derivation of the equations for the energy balance of snow cover are described by Anderson (1976). In his work, Anderson began with a general equation representing the various interchanges of energy and the energy balance as:

$$Q_n + Q_e + Q_h + Q_g + Q_m = \Delta Q \qquad (1)$$

where: Q_n = net radiation transfer,

Q_e = latent heat transfer,

Q_h = sensible heat transfer,

Q_g = heat transfer across the snow-soil interface,

Q_m = heat transfer by mass changes (advected heat), and

ΔQ = change of heat storage of the snow cover.

Introducing specific relations for all of the terms with the exception of Q_g produced the following equation for the energy balance of the entire snow cover:

$$\overbrace{Q_{ir}}^{1} - \overbrace{\Delta t \cdot \varepsilon \cdot \sigma \cdot T_o^{\,4}}^{2} + \overbrace{\frac{L_s \cdot \rho_w}{10} \cdot f(U_a) \cdot \left[(e_a - e_o) + \gamma \cdot (T_a - T_o) \right]}^{3} + \overbrace{\frac{c_w \cdot \rho_w}{10} \cdot P_x \cdot (T_w - 273.16)}^{4}$$

$$+ \overbrace{Q_g}^{5} - \overbrace{(d \cdot \rho_s)^t \cdot \left[(c_i \cdot T_s)^{t+\Delta t} - (c_i \cdot T_s)^t \right]}^{6} - \overbrace{\frac{L_f \cdot \rho_w}{10} \cdot (W^{t+\Delta t} - W^t)}^{7} = 0.0 \qquad (2)$$

A description of each symbol in the above equation is given in the appendix. The unknowns in equation 2 are: (a) the snow surface temperature (T_o), (b) the state or mean temperature (T_s) of the solid portion of the snow cover, and (c) the amount of water (w) in the snow cover. The seven terms of the equation represent the following:

(1) incident minus reflected all-wave radiation,

(2) long-wave radiation remitted by the snow,

(3) latent and sensible heat transfer assuming the transfer coefficients are equal (this does not assume the eddy transfer coefficient for momentum is the same),

(4) heat from rain water,

(5) heat transfer from the ground,

(6) change in heat storage of the pack due to a change in the temperature of the snow, and

(7) energy required to change the amount of liquid water in the snow cover.

For an isothermal case terms 6 and 7 are known. If Q_g is regarded as negligible for short periods of time compared to the energy exchange at the snow-air interface, the amount of melt can be determined directly for the entire pack.

HEAT TRANSFER WITHIN SNOW COVER

When the temperature of the snow cover is changing, additional computations are required to account for heat transfer exchanges within the snow cover. The heat transfer within the snow cover must be estimated so that the change in heat storage for the entire snow cover can be determined.

The heat transfer within a snow cover can be expressed as:

$$\underbrace{\frac{\partial T}{\partial t}}_{1} + \underbrace{\frac{L_f \cdot \frac{\partial w}{\partial t}}{c_i \cdot \rho_s}}_{2} - \underbrace{\frac{\frac{\partial I}{\partial z}}{c_i \cdot \rho_s}}_{3} - \underbrace{\left(\frac{k_e + L_s \cdot D_e \cdot f'}{c_i \cdot \rho_s}\right) \cdot \frac{\partial^2 T}{\partial z^2}}_{4} - \underbrace{\left(\frac{\frac{\partial k_e}{\partial z} + L_s \cdot f' \cdot \frac{\partial D_e}{\partial z}}{c_i \cdot \rho_s}\right) \cdot \frac{\partial T}{\partial z}}_{5}$$

$$\underbrace{- \left(\frac{L_s \cdot D_e \cdot f''}{c_i \cdot \rho_s}\right) \cdot \left(\frac{\partial T}{\partial z}\right)^2}_{6} = 0.0 \qquad (3)$$

The six terms of the above equation represent the following:

(1) change in temperature with time,

(2) heat required to change the amount of liquid water (w),

(3) heat received from penetrating solar radiation (I)

(4) and (6) heat transfer by conduction and water vapor diffusion with no change in density with depth, and

(5) accounts for the effect of changes in density with depth on the thermal conductivity and diffusion coefficients.

DENSITY CHANGES

Other equations are used in the model to take into account how density changes with time. Some of these changes in density with time are as follows.

a. Compaction of the snow. The rate of compaction is the main factor affecting density changes in a snow cover, except for the surface layer.

b. Destructive metamorphism. This is the result of moisture moving from sharp points to central part of crystals. The change from star shape to round causes the pack to increase in density. This is the most important factor affecting the density of the surface layer.

c. Constructive metamorphism. Accounts for transfer of moisture by vapor transfer in the snow cover. The vapor moves from warm to colder areas. This changes the densities of the layers. In Alaska, this can be important since a deep hoar frost formation is a common occurrence.

It is interesting to note that for different climates the density of ripe snow varies. For example, ripe snow occurs with densities of 35-38 percent in Northeastern United States, as low as 30 percent in the Midwest, and as high as 45-50 percent in the mountainous West. This variation in ripe snow densities is probably due to the fact that the typical maximum depth of snow varies from one region to another. With an increase in depth, more compaction occurs, resulting in higher densities. Variations about a mean depth-ripe snow density relationship are the result of climatic conditions that control the density of new snow and the number of freeze-thaw cycles from location to location and from year to year.

The model also includes mathematical representations of other processes that affect the energy and mass balance. These processes include the addition of new snow and the retention and transmission of liquid water.

FINITE-DIFFERENCE FORMULATION

The energy and mass balance model is built around equation 2 for snow cover energy exchange and equation 3 for heat transfer within the snow cover. The snow cover is divided into layers in order to combine and solve these equations. The energy balance equation for each layer is expressed in an implicit finite-difference form and solved by using the Newton-Raphson iteration technique. The time steps used may vary from 1 to 6 hours.

MODEL LIMITATIONS

The model does not take into account some factors that could affect results in some areas under certain circumstances. For example, the exchange of air between the snow cover and its environment is not explicitly accounted for. This could result in changes in the thermoconductivity and may therefore be lumped to some extent in empirically determined conductivity. Changes in grain size are not modeled but are assumed to be related to changes in density. Unusual ice layers may also not be accounted for.

RESULTS OF SIMULATIONS

A sample of the ability of the model to simulate snow cover density and water equivalent is shown in figure 1 for the 1970-71 accumulation season. Figure 2 illustrates the simulation of water equivalent during the melt season. Table 1 is a comparison of computed versus observed snow surface temperature for the data used for the original testing of the model. Recently, atmospheric long-wave radiation measurements have been obtained with a new CSIRO-type radiometer and have resulted in improved correlations between simulated and observed snow surface temperature (increased in correlation from 0.90 to near 0.97).

Simulations using the energy balance model and the earlier temperature index model (Anderson 1973) were compared with observed water equivalent values (see fig. 2). For the point measurements at the Danville snow research site, the energy balance model produced the better simulations. However, this would not necessarily be true for locations where the basic data are not available or of lesser quality.

In general, with high quality data, the energy balance model produces the best results in open and partly forested areas where there may be considerable variability in meteorological conditions during the snowmelt period. The index model is recommended for heavily forested areas where meteorological conditions are not as variable and for locations where the available data are not adequate or of sufficient quality for use with the energy balance model. For research studies of snowmelt processes, the energy balance model is much superior and provides an excellent tool for test and elevation.

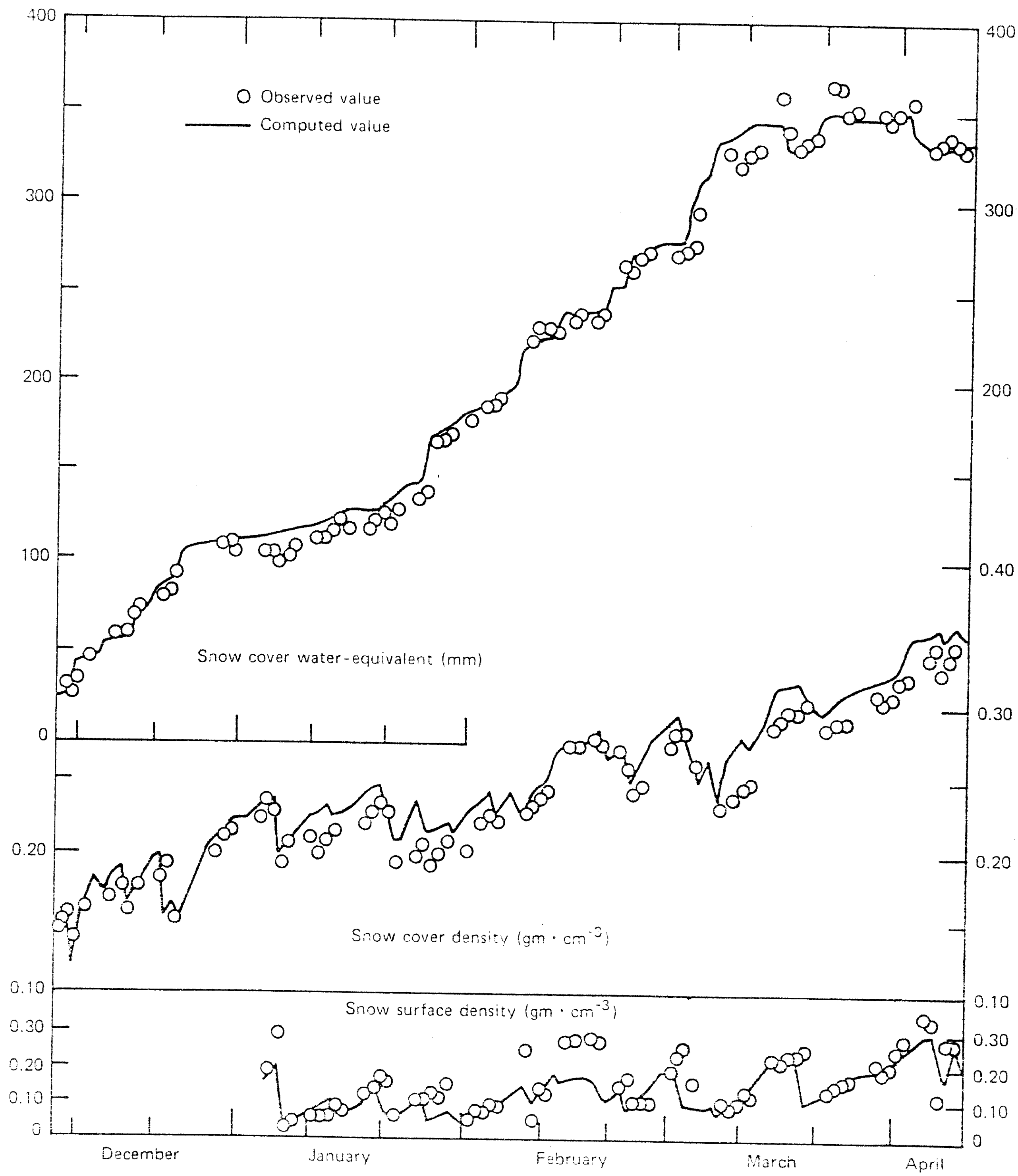

Figure 1.--Snow cover density and water-equivalent comparisons during the 1970-71 accumulation season.

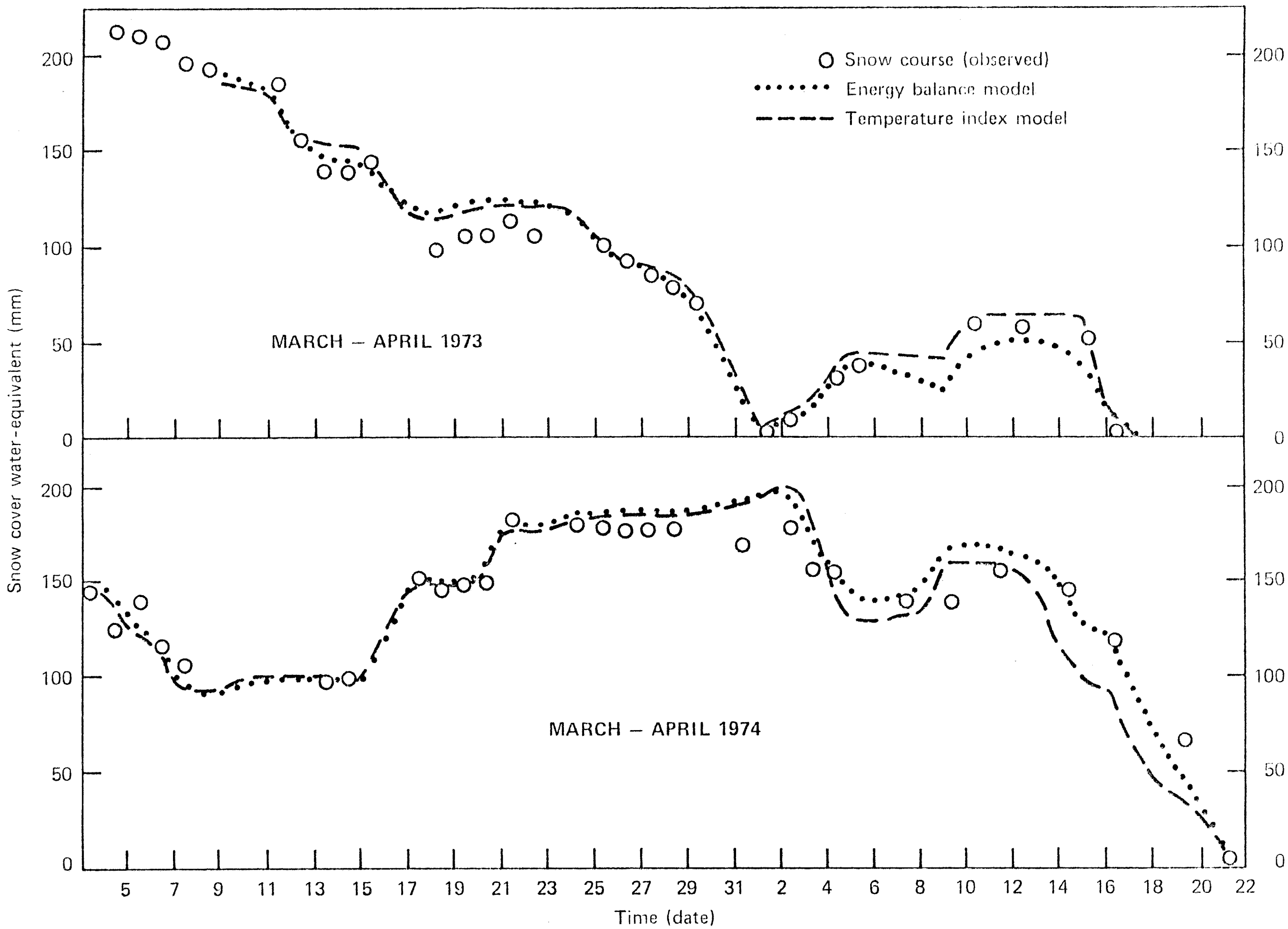

Figure 2.--Comparison of energy balance and temperature index models during the 1973 and 1974 melt seasons.

Table 1.—Comparison of computed versus observed snow surface temperature. (All temperatures are in degrees Celsius)

Period	Δt^1	Number of cases	Mean T_o		RMS error	Avg.abs. error	Correlation coefficient	Best fit line	
			Obs.	Comp.				Intercept	Slope
4/1/70 to 4/26/70[2]	1	212	- 2.4	-2.7	2.0	1.3	0.85	0.3	0.98
12/7/70 to 4/10/71	3	762	- 9.0	-8.3	2.7	1.9	0.90	-0.3	1.04
4/1/71 to 5/8/71[2]	1	763	- 2.3	-2.3	1.8	1.0	0.90	0.2	1.07
12/24/71 to 4/7/72	3	697	- 7.4	-8.0	2.7	1.9	0.90	0.3	0.97
4/6/72 to 5/8/72[2]	1	729	- 2.9	-2.5	2.1	1.4	0.87	-0.5	0.96
3/9/73 to 4/15/73[2]	1	743	- 4.0	-4.0	2.1	1.3	0.89	-0.2	0.94
1/3/74 to 4/12/74	3	710	-11.4	-9.8	3.9	2.6	0.93	-0.5	1.11
1/3/74 to 4/12/74	1	2136	-11.4	-9.8	4.0	2.6	0.93	-0.5	1.10
3/1/74 to 4/22/74[2]	1	1150	- 6.7	-5.8	3.4	2.1	0.89	-0.6	1.06

[1] Δt is the computational time interval in hours.

[2] Melt season values were computed using a theoretically based wind function with $z_o = 0.15$ cm and $Ri_{cr} = 0.4$.

APPENDIX

LIST OF SYMBOLS

c_i	Specific heat of ice ($cal \cdot gm^{-1} \cdot {}^\circ K^{-1}$)
c_w	Specific heat of water (1.0 $cal \cdot gm^{-1} \cdot {}^\circ K^{-1}$)
d	Depth of snow cover (cm)
D_e	Effective diffusion coefficient for water vapor in snow ($cm^2 \cdot sec^{-1}$)
e_a	Vapor pressure (mb) of the air 1 m above snow surface
e_o	Vapor pressure at snow surface (mb)--assumed equal to saturation vapor pressure at the snow surface temperature
f'	First partial derivative of C with respect to T (where C is the concentration of water vapor--grams of vapor per cm^3 of air)
f''	Second partial derivative of C with respect to T (where C is the concentration of water vapor--grams of vapor per cm^3 of air)
$f(U_a)$	Wind function (a function of the wind speed) ($mm \cdot mb^{-1}$)
I	Shortwave (solar) radiation flux ($cal \cdot cm^{-2} \cdot sec^{-1}$)
k_e	Effective thermal conductivity of snow ($cal \cdot gm^{-1} \cdot {}^\circ K \cdot sec^{-1}$)
L_f	Latent heat of fusion (79.7 $cal \cdot gm^{-1}$ at 0°C)
L_s	Latent heat of sublimation (677 $cal \cdot gm^{-1}$)
P_x	Water equivalent of precipitation (mm)
Q_g	Heat transfer loss across snow-soil interface
Q_{ir}	Incident minus reflected all-wave radiation

T	Snow cover temperature (°K)
T_a	Air temperature (°K) at 1 m above snow surface
T_o	Snow surface temperature (°K)
T_s	Temperature of solid portion of snow cover (°K)
T_w	Wet-bulb temperature (°K)
t	Time (sec)
Δt	Computational time interval (sec)
W	The amount of liquid water in the snow cover, expressed as a depth (mm)
w	Amount of liquid water ($gm \cdot cm^{-3}$)
z	Vertical distance
ε	Emissivity in long-wave portion of energy spectrum (0.99)
ρ_s	Density of solid (ice) portion of snow cover ($gm \cdot cm^{-3}$)
ρ_w	Density of water (1.0 $gm \cdot cm^{-3}$)
σ	Stefan-Boltzmann constant (1.355×10^{-12} $cal \cdot cm^{2} \cdot °K^{-4} \cdot sec^{-1}$)

<u>Superscripts</u>

t	Time (sec) at beginning of computational time interval
$t+\Delta t$	Time at end of computational time interval

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