KINEMATIC PARAMETER ESTIMATION FOR TRANSPORT OF POLLUTANTS IN OVERLAND FLOW

L. E. Brazil*, T. G. Sanders†, D. A. Woolhiser†

*Research Hydrologist, National Weather Service, NOAA - USDC
†Assistant Professor of Civil Engineering, Colorado State University
†Hydraulic Engineer, Agricultural Research Service - USDA

INTRODUCTION

Pollutants transported by storm water runoff from non-point sources are major contributors to the degradation of the nation's rivers. Studies have revealed that at certain times urban storm water runoff is a more serious threat to water quality than raw sewage discharges. During some storm events, runoff from street surfaces has been found to contribute significantly more BOD (Biochemical Oxygen Demand) to area waters than the effluent from sewage treatment plants (Ref. 1). Urban non-point pollution results primarily from debris and contaminants on the streets, contamimants from open land areas, publicly used chemicals, and dirt and contaminants washed from vehicles. Water soluble chemicals and pollutants have been found in urban storm water runoff at environmentally significant concentrations (Ref. 2).

Much attention has been given in recent years to the development of models which can simulate storm water runoff from watershed surfaces. Theoretical equations have been developed to describe shallow flow in open channels and several adequately approximate overland flow. These models or simplifications of them have been used to simulate the accumulation and removal of insoluble and partially soluble non-point source pollutants (Refs 3, 4, 5). Although these models have been tested for their ability to describe unsteady, free-surface flow, they have not been adequately tested as chemical transport models. The purpose of this research was to develop a transport model for conservative soluble pollutants on an impervious watershed and to test it with data from a carefully controlled experimental watershed.

KINEMATIC WAVE THEORY

One set of equations used to describe overland flow is based on kinematic wave theory. First introduced by Lighthill and Whitham in 1955 (Ref. 6), the theory has been applied to surface runoff by Henderson and Wooding (Ref. 7), Wooding (Ref. 8), and Woolhiser (Ref. 9).

Derivation of the kinematic equations is based upon the governing equations for unsteady free-surface flow introduced by DeSaint Venant (Ref. 10). The equation for continuity on a plane surface is:

$$\frac{\partial h}{\partial t} + \frac{\partial (uh)}{\partial x} = q \qquad (1)$$

and the simplified momentum equation is:

$$S_o = S_f \qquad (2)$$

where: h = actual depth of flow,
q = lateral inflow rate,
x = distance from upstream boundary,
u = local average velocity,
t = time,
S_o = bed slope,
S_f = friction slope.

Substitution of the Darcy-Weisbach equation or the Chezy equation into Eq. (2) will produce the following parametric relationship for depth of flow and velocity:

$$u = \alpha h^{m-1} \qquad (3)$$

where: α = roughness and slope parameter,
m = flow regime parameter.

The parameter m has a value of approximately 1.5 for turbulent flow. Equation (3) has been used by researchers as the basic relationship in numerous watershed models (Refs 11, 12). Models by Harley et al. (Ref 13) and Schaake (Ref. 14) have demonstrated the applicability of the kinematic wave theory in prediction of runoff hydrographs.

FORMULATION OF THEORETICAL MODEL

The water quality model presented herein is derived to predict the convective movement of soluble, conservative pollutants in overland flow. Dispersion is ignored. The basic prediction model is derived by Brazil (Ref. 15) from Eq. (3). The water particle velocities are represented by:

$$u = \left(\frac{dx}{dt}\right)_p \qquad (4)$$

Prior to the time of equilibrium (within the shaded zone in Fig. 1) the theoretical depth of the water (h_*), is a function of the intensity of the rainfall (i) and the time (t) since the start of rainfall input. If intensity of rainfall is assumed to be constant and the surface is initially dry, water depth at time, t, after rainfall begins is given by:

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$$h_* = i\ t \qquad (5)$$

where: h_* = theoretical depth,
i = rainfall intensity.

The theoretical depth (h_*) will vary from the actual depth (h) by a factor determined

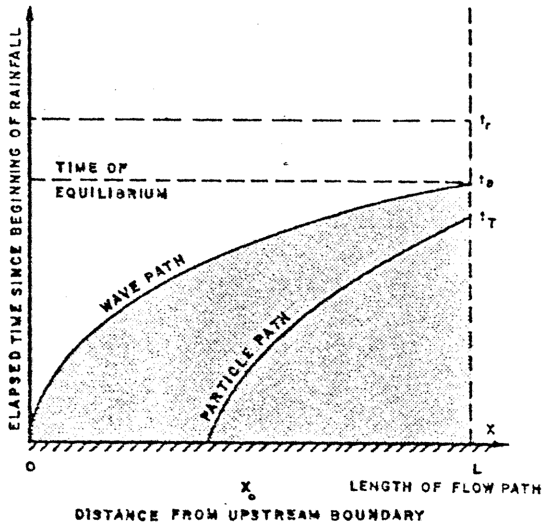

Fig. 1. Characteristic wave and particle paths in the x-t plane.

from the concentration of objects blocking flow paths on the flow plane. For most practical situations, however, the use of theoretical depths causes no problems in model applications. Substituting Eqs. (4) and (5) into Eq. (3) yields:

$$\left(\frac{dx}{dt}\right)_p = \alpha(it)^{m-1} \qquad (6)$$

Integration and rearranging terms gives:

$$t_T = \left(\frac{m(L-x_o)}{\alpha(i)^{m-1}}\right)^{1/m} \qquad (7)$$

where: t_T = travel time (seconds),
i = intensity of rainfall (meters/second),
L = length of plane (meters),
x_o = distance from top of plane (meters).

Equation (7) is applicable only on the rising limb of the hydrograph. After equilibrium is reached on the watershed, the depth no longer increases with time and the flow is considered to be steady and nonuniform. Therefore, it is necessary to compare the observed and predicted t_T with time to equilibrium (t_e) and with the time at which rainfall stops (t) for partial equilibrium hydrographs. Time to equilibrium is dependent upon the characteristic wave velocities, which can be determined with an expression derived from the continuity equation and Eq. (6). Substituting Eq. (3) into Eq. (1) yields:

$$\frac{\partial h}{\partial t} + \alpha m h^{m-1}\frac{\partial h}{\partial x} = q \qquad (8)$$

Solving Eq. (8) and the total differential of h(x,t) in matrix form yields a solution to the characteristic velocity in the x-t plane:

$$\left(\frac{dx}{dt}\right)_w = \alpha m h^{m-1} \qquad (9)$$

Equation (9) represents the velocity of the characteristic wave down the plane. The notation $\left(\frac{dx}{dt}\right)_w$ is used to denote the wave velocity as opposed to the particle velocity $\left(\frac{dx}{dt}\right)_p$. Note that the wave velocity is m times greater than the particle velocity. The characteristics paths for both the wave and particle velocities are shown in Fig. 1.

By substituting Eq. (5) into Eq. (9) and integrating we obtain an expression for time to equilibrium, t_e.

$$t_e = \left(\frac{L}{\alpha(i)^{m-1}}\right)^{1/m} \qquad (10)$$

It should be noted that the time to equilibrium is determined by the kinematic parameters α and m. A similar expression can be derived for the time at which recession begins for a partial equilibrium hydrograph.

VERIFICATION OF THEORETICAL MODEL

An experimental procedure was designed to collect data which could test the model's accuracy in predicting time of travel for soluble, conservative pollutants. Because water particle movement was investigated using a fluorescent dye, Rhodamine WT, its travel time across a watershed could be accurately recorded using a fluorometer.

Experimental Facility

The Rainfall-Runoff Experimental Facility at Colorado State University was used as the model watershed. Having an area of approximately 25,000 square feet and being impervious, the facility simulates a watershed without infiltration which is an intermediate size between laboratory models and natural watersheds. The general arrangement of the Rainfall-Runoff Facility is shown in Fig. 2. Characteristics such as surface roughness, imperviousness, and geometry can be changed to represent a wide variety of natural catchments. In addition, simulated rainfall can be generated at rates of 13, 28, 58, and 108 millimeters per hour. Holland (Ref. 16) gives further details of the facility.

To simplify the conditions on the watershed, one plane section of the surface, two meters wide by thirty meters long, was partitioned from the rest of the facility for use in the experimental runs. Like the plane section of the facility, the experimental section was oriented so as to have a five percent slope. Discharge was measured through a 0.6 foot HS flume and the concentration of the fluorescent dye in the water was analyzed with a continuous flow Turner Model 111 fluorometer.

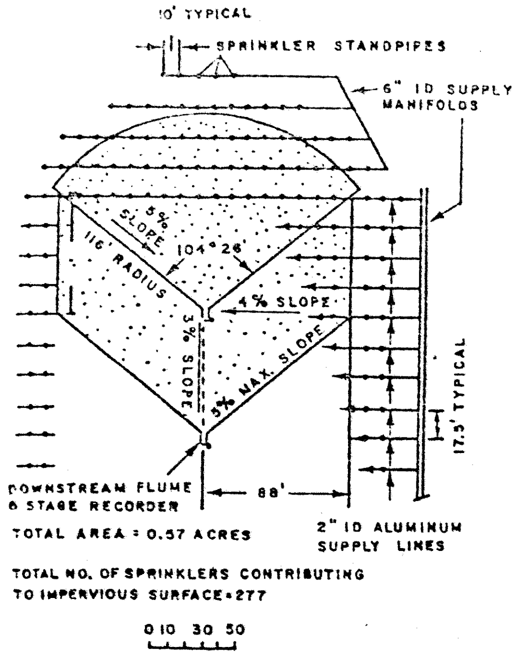

Fig. 2. Experimental Rainfall-Runoff facility-general arrangement.

Experimental Procedure

All experimental runs were conducted with the dye injected at a point or line on the plane surface simultaneous with the start of rain. Thus, the dye represented a pollutant already on the surface at the beginning of a rainstorm as well as being immediately soluble in the runoff. During each run, a stopwatch was used to determine the length of time the dye was actually traveling on the exposed plane. This time was equal to the time interval between tracer injection and peak concentration readout minus the time the tracer was pumped and transmitted to the fluorometer.

Variables investigated included surface roughness, rainfall intensity, rainfall duration, and the quantity and location of tracer input. Four surface roughnesses were included in the tests. These were:

- Set A - bare butyl surface which covers the watershed.
- Set B - 2 kg/m^2 of 1-1/2 inch diameter gravel spread on the surface.
- Set C - 10 kg/m^2 of 1-1/2 inch diameter gravel spread on the surface.
- Set D - artificial rilled surface (a smooth surface of a wet sand and cement mixture was eroded by simulated rainfall and allowed to harden for two days).

All four of the available rainfall intensities were utilized for each set of runs. Rainfall was applied for three durations: approximately one-half the time before the watershed runoff reaches equilibrium, a time approximately equal to equilibrium time, and a time approximately 1.3 to 1.5 greater than the equilibrium time. Injections were made at two locations: distances of six meters and fifteen meters from the upper boundary. The four roughnesses, four intensities, three durations, and two locations made a total of ninety-six combinations of travel time runs.

A few experiments were also conducted to collect data to examine the variables under conditions different from the normal travel time runs. One series included tracing the dye down the plane with the Set D roughness while the intensity of the rainfall was varied. These runs allowed the prediction model to be tested for rainfall patterns more characteristic of nature. Another set of runs was made with the dye injected as a line source across the flow plane to examine the effects of nonuniform cross sections. Approximately six hundred runs were made to test the model under various conditions.

RESULTS

Statistical Analysis

A statistical regression analysis was performed on the travel time data to make sure it was predictable and could be correlated with the variables in the model. The variables examined included intensity, initial tracer location, rainfall duration, time of concentration, peak time, amount of dye injected, and dye concentration. The analysis indicated that the most important variables were rainfall intensity and the injection location and that the correlation coefficient was 0.90 or greater for combinations of these two variables alone. The theoretical model, Eq. (7), uses both intensity and injection location as input variables.

Theoretical Analysis

The theoretical analysis was divided into three phases. The first phase of the study used a technique for finding the best estimate of the kinematic parameters α and m. Next, the travel time data were compared with the predicted travel times using the best estimates of α and m in Eq. (7) and finally, an analysis was also conducted to see how well the model could predict travel times when rainfall intensity was varied during a single run.

All test runs in the first phase were initiated with the flow on the watershed at equilibrium. For this condition the discharge at any unit width of cross section was only a function of rainfall intensity and distance from the top of the plane. The plane was divided into several reaches and during subsequent runs dye injections were made at a different section to determine the average velocity for each section. This information was used along with equilibrium discharge quantity for each section to calculate average depth of flow, the principal variable in Eq. (3). A theoretical expression for discharge per unit width, Q, can be derived by substituting h_* into Eq. (3) and by multiplying through by h_*. The new equation is:

$$Q = \alpha h_*^{\,m} \tag{11}$$

The log form of this equation plots as a straight line on log-log paper with a dependent variable intercept of log α and a slope of m. Steady-state data were taken for each surface roughness and showed that Q was highly correlated to h_*. In every case the slope of the line, m, was approximately 1.5 indicating that the flow was turbulent. Equation (7) indicates that travel time is inversely proportional to α and that in general a rougher surface will have a smaller value of α. An analysis of the best estimates of α and m from the steady-state runs showed that because of the intercorrelation of α and m, and the fluctuation that can exist in the actual value of m, the roughest surface will not always have the smallest value of α.

Best estimates of α and m obtained from steady-state tracer studies were used in the model Eq. (7) to predict travel times for the ninety-six combinations of input variables for unsteady flow. The model equations and observed data points were plotted on log-log paper with travel times as a function of rainfall intensity and location of injection. Figure 3 shows a graphical representation of the runs for Set C. A mean error analysis of the predicted travel times was determined

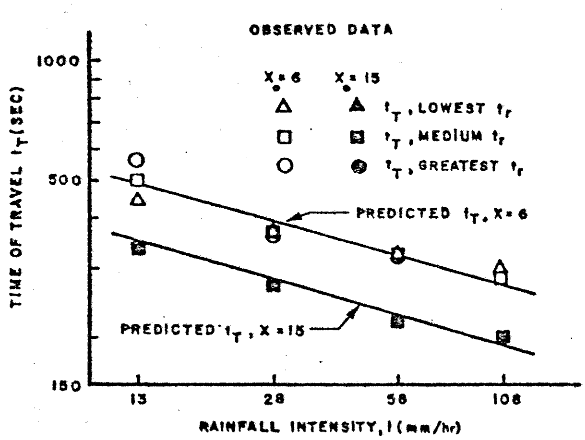

Fig.3. Predicted and observed travel times versus rainfall intensities for Set C.

using a sum of the errors square criterion for the observed data. The mean error for all four sets was 21 seconds using the best estimate of α and m from the steady-state runs. The observed data were generally within 10% of the predicted times and shorter travel times had lower deviations so that the percentage difference between the predicted and observed times was fairly consistent.

A form of the model equation was used to predict times for the variable intensity runs on the rilled surface. Substitution of

$$it = h + i_1 t_1 \tag{12}$$

into Eq. (6) gives an expression which can be used to route the water particles down the plane. The mean error of the predicted travel times using the observed data from the variable intensity runs was 16 seconds. Equation (7) was also used to predict tim with the average intensity calculated fro total rainfall accumulation used as the i put. The mean error for these travel time was 27 seconds, 11 seconds higher than th times using the routing technique.

Equation (7) was also used to predict time for the pollutants injected as a lin source. The mean error was 13 seconds, in ting that the model was even more accurat for the line source data examined than fo the point source data.

Estimation Techniques for Kinematic Param

Several techniques were examined to estimate the parameters in the kinematic tion. Equation (7) was solved with a pair simultaneous equations to isolate the var ables α and m.

The equation can be transformed into:

$$m = \frac{\ln (L-x_1) - \ln (L-x_2)}{(\ln t_1 - \ln t_2)}$$

$$\alpha = \frac{(L-x_1)m}{i^{m-1} t_1^m}$$

Two travel time data points, t_1 and t were used as input. The data were from ru having injections of dye made at differen distances, x_1 and x_2 from the top of t watershed but having equal rainfall inten ties. The prediction model with estimated values of α and m was tested against d from the ninety-six travel time runs. A m error of 28 seconds was calculated showin that the pair of parameters could be accu ately estimated with a small amount of data.

In an attempt to further simplify the estimating technique, the parameter m w held constant and Eq. (14) was solved to α. The technique was based on the Chezy turbulent flow relationship, giving a val of 1.5 for m. This method requires only dye injection and observed travel time to solve for α. The model equation with m e to 1.5 and α estimated from the simplif technique gave predicted travel times wit mean error of 20 seconds. For the four su face roughnesses, Sets A, B, C, and D, an assuming m = 1.5; the values of α were 8.62, 4.71, 1.68 and 4.03 respectively. T method assuming a constant m value prov more accurate for the observed data than technique which allowed both parameters t vary.

Two estimating methods were also test with m assumed to be 1.67, the value ob tained from Manning's equation. The model m = 1.67, had a greater mean error indica that the Chezy relationship better repres ed the data. The one injection parameter estimation techniques were also used with variable intensity rainfall. Results were similar to the runs with constant intensi

A nomograph was prepared to quickly estimate α with data from one observed travel time run and is shown in Fig. 4. T

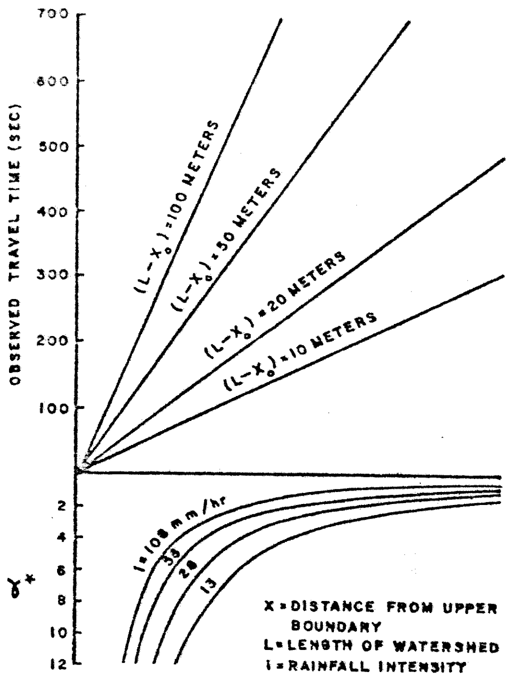

Fig. 4. Nomograph for predicting α, assume $m = 1.5$.

graph displays a solution to Eq. (14) with input variables of travel time, length of flow plane, and rainfall intensity. Because the model is designed to predict only time for flows on the rising limb of the hydrograph, dye injections must be made at the location within the watershed in order that the kinematic wave from the farthest boundary reaches the bottom of the watershed after the dye concentration peak arrives. To insure that this condition is met, the dye must be injected at a distance greater than

$L(\frac{m-1}{m})$ from the top of the plane (Ref. 5).

For $m = 1.5$, the injection must be made within the lower two-thirds of the watershed.

Applications of the Theoretical Model

Numerous papers have been written on the value of kinematic wave theory in synthesizing hydrographs and on methods of calculating the kinematic parameters. The parameter estimation techniques described herein can be useful for selecting parameter values for the hydrograph equations as well as for the pollutant travel time model.

The correlation of pollutant strength and sediment combined with the simulation of sediment transport in a watershed has been recently proposed as a methodology for assessing the movement of insoluble and partially soluble non-point pollutants (Ref. 5). Curtis has presented a model that uses the kinematic wave formulation and a set of relationships describing soil detachment and transport processes to simulate the discharge of sediment from an urban area (Ref. 17). The experimental evidence presented herein suggests that the kinematic model is appropriate for pollutant transport as well as runoff hydrograph prediction.

One specific application of the model would be its use in the prediction of salt transport in urban runoff. Deicing salts, which are soluble in water, are applied to roads in the United States during the winter when snow and ice accumulate. The use of road salts has been increasing over the past three decades and has been linked to increases in chloride content of streams (Ref. 18). The kinematic wave and chemical transport equations in finite difference form may be a useful tool in further understanding the processes involved in salt transport.

SUMMARY

A water quality time of travel model was developed based on the theory that soluble pollutants in overland flow travel at velocities consistent with the relationships governed by the kinematic wave equations. Estimation techniques were presented for calculating the parameters in the model. The model equation was used with estimated parameters to predict pollutant times and was proven to be quite accurate when compared with the observed data.

CONCLUSIONS

This study further verifies the validity of the kinematic wave equations for overland flow and also shows that the kinematic wave equations can be used to describe convective transport of pollutants. Data from the analyses indicate that the kinematic form for turbulent flow derived using Chezy's relationship is valid and that the presented model equations can be useful tools in the estimation of kinematic parameters as well as to predict movement of soluble pollutants on watersheds.

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