IMPLICIT FLOOD ROUTING IN NATURAL CHANNELS[a]

Discussion by Danny L. Fread

DANNY L. FREAD,[6] M. ASCE.—The authors state that the 2N × 2N coefficient matrix associated with Eq. 18 has a maximum of only four non-zero el-

[a] December, 1970, by Michael Amein and Ching S. Fang (Proc. Paper 7773).

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ements in any one row and that these elements are banded around the main diagonal. This feature enables a fast matrix solution technique to be devised.

The writer has encountered this type of matrix previously and developed a direct solution technique similar to the Gauss elimination method. The technique offers the following advantages:

1. The computations do not involve any of the many zero elements in the coefficient matrix; this saves considerable computation time, and
2. The required computer core storage is reduced significantly from that required for a $2N \times 2N$ matrix to that required for a $2N \times 4$ matrix; this results in a 100 $(N - 2/N)$ percentage reduction of storage.

The authors state that the first advantage is to be sought when devising a matrix solution procedure; however, they do not mention the second advantage. The writer has found that the reduction of storage requirements is also a desirable feature of the matrix solution technique when the matrix is large and the computer storage capacity is limited.

Using matrix notation, Eq. 18 takes the form

$$AX = R \qquad (24)$$

in which A = the coefficient matrix with components a_{ij} and X, R are column vectors having components x_i and r_i, respectively, i.e.

$$A = \begin{bmatrix} a_{11} & a_{12} & & & & & & & & \\ a_{21} & a_{22} & a_{23} & a_{24} & & & & & & \\ a_{31} & a_{32} & a_{33} & a_{34} & & & & & & \\ & & a_{43} & a_{44} & a_{45} & a_{46} & & & & \\ & & a_{53} & a_{54} & a_{55} & a_{56} & & & & \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ & & & & & & a_{2N-2,2N-3} & a_{2N-2,2N-2} & a_{2N-2,2N-1} & a_{2N-2,2N} \\ & & & & & & a_{2N-1,2N-3} & a_{2N-1,2N-2} & a_{2N-1,2N-1} & a_{2N-1,2N} \\ & & & & & & & & a_{2N,2N-1} & a_{2N,2N} \end{bmatrix} \qquad (25)$$

$$\text{and } X = \begin{bmatrix} x_1 \\ x_2 \\ \cdot \\ \cdot \\ \cdot \\ x_{2N} \end{bmatrix} \text{ and } R = \begin{bmatrix} r_1 \\ r_2 \\ \cdot \\ \cdot \\ \cdot \\ r_{2N} \end{bmatrix} \qquad (26)$$

If the components of A are shifted horizontally such that the relative positions of the components in any one row remain the same, A takes the form of A' with components a'_{ij}. Thus

$$A' = \begin{bmatrix} & & a'_{13} & a'_{14} \\ a'_{21} & a'_{22} & a'_{23} & a'_{24} \\ a'_{31} & a'_{32} & a'_{33} & a'_{34} \\ a'_{41} & a'_{42} & a'_{43} & a'_{44} \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ a'_{2N-2,1} & a'_{2N-2,2} & a'_{2N-2,3} & a'_{2N-2,4} \\ a'_{2N-1,1} & a'_{2N-1,2} & a'_{2N-1,3} & a'_{2N-1,4} \\ a'_{2N,1} & a'_{2N,2} & & \end{bmatrix} \quad \ldots\ldots (27)$$

This requires a relatively simple change of the jth index of the components of A.

The following technique efficiently solves the system of linear equations, denoted by Eq. 18, and now described by

$$A'X = R \quad \ldots\ldots (28)$$

The recurrent formulae, applicable to even-numbered rows, i.e. (i = 2, 4, 6, . . . 2N) are

$$m_{i,2} = - a'_{i,1} \frac{m_{i-1,4}}{m_{i-1,3}} + a'_{i,2} \quad \ldots\ldots (29a)$$

$$z_i = - a'_{i,1} \frac{z_{i-1}}{m_{i-1,3}} + r_1 \quad \ldots\ldots (29b)$$

in which $m_{1,3} = a'_{1,3}$ $m_{1,4} = a'_{1,4}$ and $z_1 = r_1$. The recurrent formulae, applicable to the odd-numbered rows, i.e. (i = 3, 5, 7, . . . 2N - 1) are:

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$$z_i = - m_{i,2} \frac{z_{i-1}}{m_{i-1,2}} - a'_{i,1} \frac{z_{i-2}}{m_{i-2,3}} + r_i \quad \ldots\ldots (30d)$$

The computations proceed sequentially from i = 2 to i = 2N. The components of the solution vector, X, are obtained by back-substitution commencing at i = 2N and proceeding sequentially to i = 1. Thus

$$x_{2N} = \frac{z_{2N}}{m_{2N,2}} \quad \ldots\ldots (31)$$

and the recurrent formula for (i = 2N - 1, 2N - 3, . . . 5, 3, 1) is

$$x_i = \frac{[z_i - m_{i,4}\, x_{i+1}]}{m_{i,3}} \quad \ldots\ldots (32)$$

while that for (i = 2N - 2, 2N - 4, . . . 6, 4, 2) is

$$x_i = \frac{[z_i - a'_{i_1}\, x_{i+2} - a'_{i_3}\, x_{i+1}]}{m_{i_2}} \quad \text{. . . . .} \quad (33)$$

When programing the preceding solution technique, it is not necessary to introduce the new components m_{ij} and z_i as these may be expressed as a'_{ij} and r_i, respectively.

Acknowledgement.
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