MODERN EVAPORATION FORMULAE ADAPTED TO COMPUTER USE

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[Manuscript received July 26, 1961; revised November 1, 1961]

ABSTRACT

Several graphical solutions for computing pan and lake evaporation have been published, but because of the advantages of using a high-speed digital computer in processing masses of data, the graphical representations by Kohler et al. are reduced to mathematical expressions adaptable to computer use and in terms of readily available input data.

A measure of the evaporation rate has long been recognized as a pertinent factor to any quantitative appraisal involving the hydrologic cycle. While recognition of the need, of itself, does not assure adequate measure or estimate, it is a stimulant toward this goal. Of recent years, notable gains have been made through the work associated with the Lake Hefner and Lake Mead water-loss investigation studies [1, 2]. Improved methods for estimating lake evaporation have been a principal objective of the Weather Bureau in conducting its evaporation studies. Kohler, Nordenson, and Fox [3] describe the development of two such methods:

1. Lake evaporation estimated from meteorological factors—(a) air temperature; (b) dewpoint temperature; (c) wind movement; (d) solar radiation.
2. Lake evaporation estimated from meteorological factors and observed pan evaporation—(a) air temperature; (b) pan water temperature; (c) wind movement; (d) observed Class A pan evaporation.

Since their development, the techniques have had extensive application with satisfactory results. The results of one such project which relied heavily upon the two techniques have been presented by Kohler, Nordenson, and Baker [4]. Computed lake evaporation has also been used as a measure of potential evapotranspiration in conjunction with basin accounting techniques [5, 6] in attempts to improve the Weather Bureau river-forecasting procedures.

The graphical relation to compute lake evaporation from meteorological factors, figure 1, is based on the Penman [7] formula:

$$E=\frac{Q_n\Delta+E_a\gamma}{\Delta+\gamma} \tag{1}$$

where E is evaporation rate; Q_n is net radiation exchange; Δ is slope [1] of saturation vapor pressure curve at T_a; γ is a factor defined by equation for Bowen's ratio, $R=\gamma(T_o-T_a)/(e_o-e_a)$; E_a is pan evaporation assuming $T_o=T_a$; T_a, T_d, T_o are air, dewpoint, and water-surface temperatures; e_s, e_a, e_o are saturation water vapor pressures corresponding to above temperatures.

For Class A pan evaporation (E_p), equation (1) becomes

$$E_p=\frac{Q_n\Delta+E_a\gamma_p}{\Delta+\gamma_p} \tag{2}$$

where $\gamma_p=0.025$ inch Hg/°F., and for lake (E_L) or open water evaporation

$$E_L=0.7\left(\frac{Q_n\Delta+E_a\gamma}{\Delta+\gamma}\right) \tag{3}$$

where $\gamma=0.0105$ inch Hg/°F.

It may be noted that, while equation (3) has a factor of 0.7, the lake-to-pan coefficient does not remain fixed at this ratio. It is only under conditions where actual pan water temperature would equal air temperature that the 0.7 coefficient holds—that is, under conditions where there would be no sensible heat transfer into or out of pan and $Q_n=E_a=E_p$.

If figure 1 is examined with the terms of equation (3) in mind, the graphical pattern of the equation is easily followed. The upper left-hand insert provides the E_a values, with entering data in terms of air and dew point temperatures and wind movement. The top half of this insert is for reduction of air and dew point temperatures to vapor pressure deficit. The second half combines the deficit and wind movement (u_p) and is of the generalized form:

$$E=(e_0-e_a)^n(a+bu_p). \tag{4}$$

When fitted to the data the equation took this specific form, where for E_a, e_s is equivalent to e_0:

$$E_a=(e_s-e_a)^{0.88}(0.37+0.0041u_p). \tag{5}$$

The upper right-hand quadrant represents $Q_n\Delta$ as a function of mean daily air temperature (°F.) and solar radiation (Langleys/day). The lower right quadrant performs the function of adding $E_a\gamma$ to $Q_n\Delta$; the lower left

[1] In his derivation of equation (1), Penman uses $\Delta=(e_s-e_a)/(T_s-T_a)$ but defines Δ as de_s/dT_a. Penman's system of subscripts differs somewhat from the one used here. For this paper, symbols from [3] have been used.

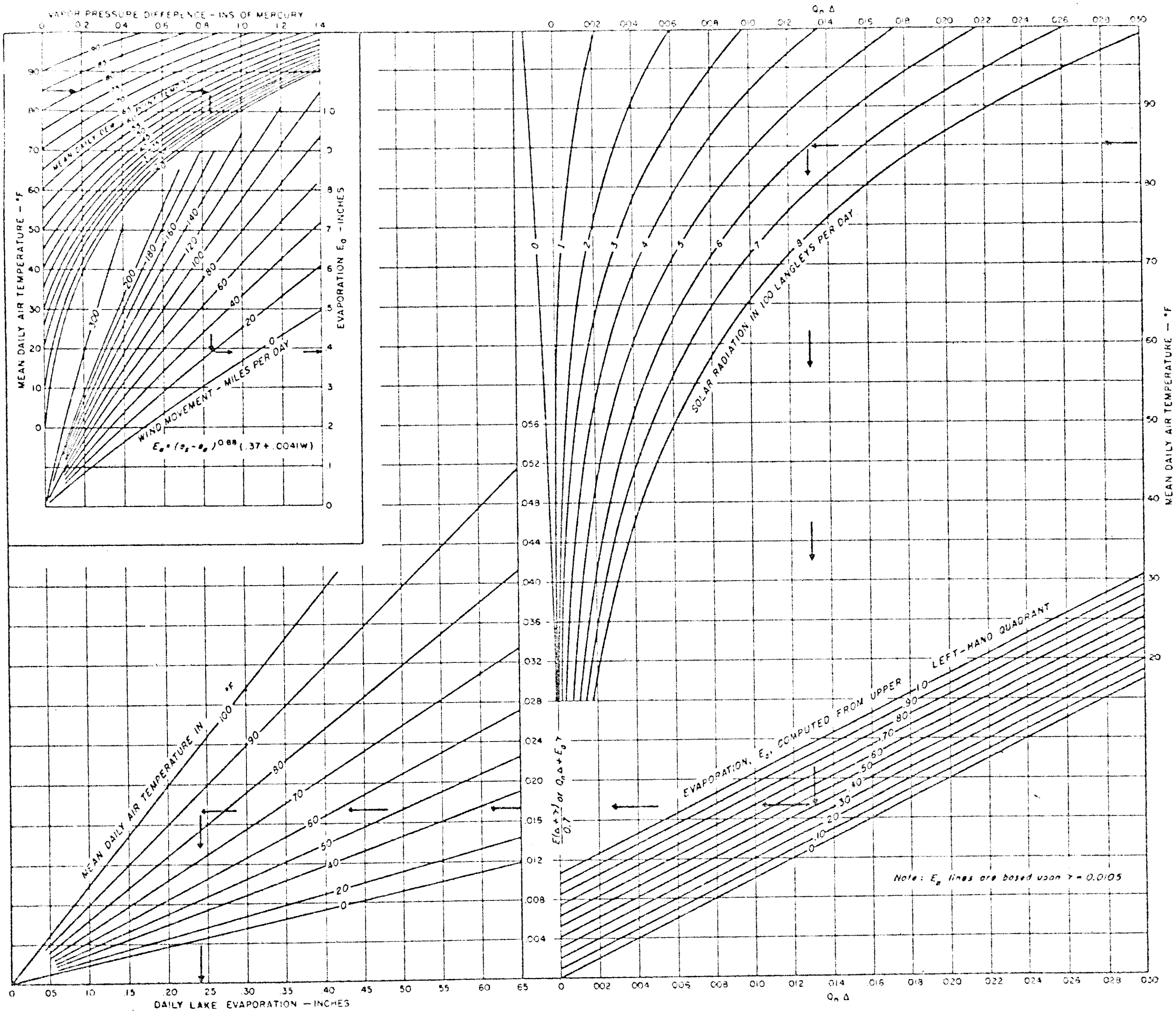

FIGURE 1.—Free-water evaporation as a function of meteorological factors. (From [3].)

quadrant multiplies $(Q_n\Delta+E_a\gamma)$ by $0.7/(\Delta+\gamma)$, where γ is a constant, and Δ is expressed in terms of air temperature.

In the original work [3] the $Q_n\Delta$ quadrant was fitted graphically to the data, treating $Q_n\Delta$ as a single parameter, as contrasted with the analytical derivation of the other sections of the chart. For solution by computer it was necessary to derive the mathematical expression to approximate this quadrant. Several tests were made and the best success was with an exponential function of the form:

$$\log (Q_n\Delta+k)=(T+a)(c+f(R)) \tag{6}$$

from which a close fit was accomplished with this specific equation:

$$Q_n\Delta=\epsilon^{(T_a-212)(0.1024-0.01066 \ln R)}-0.0001 \tag{7}$$

where ϵ is the Naperian base and R is solar radiation in Langleys/day. The constant term, 0.0001, is significant, and particularly so when values of R and T_a are low.

The Clausius-Clapeyron equation is a convenient means to express vapor pressure (e_s, e_a, e_0) and Δ in terms of temperature:

$$e=\epsilon^{-k/(T+b)+c}=\epsilon^{c}\cdot\epsilon^{-k/(T+b)} \tag{8}$$

$$\Delta=\frac{de}{dT}=\frac{k}{(T+b)^2}\cdot\epsilon^{c}\cdot\epsilon^{-k/(T+b)} \tag{9}$$

where $k=7482.6$, $b=398.36$, and $c=15.674$.

By substitution, it is now a simple matter to express equation (3) in a form readily adaptable to modern high speed computers, and in terms of acceptable input data:

$$E_L=[\epsilon^{(T_a-212)(0.1024-0.01066\ln R)}-0.0001 +0.0105(e_s-e_a)^{0.88}(0.37+0.0041\,u_p)]\times [0.015+(T_a+398.36)^{-2}(6.8554\times 10^{10})\epsilon^{-7482.6/(T_a+398.36)}]^{-1} \quad (10)$$

Vapor pressure deficit (e_s-e_a) can be derived from air and dewpoint temperature input;

$$e_s-e_a=6.4133\times 10^5[\epsilon^{-7482.6/(T_a+398.36)}-\epsilon^{-7482.6/(T_d+398.36)}]. \quad (11)$$

Bosen [8] recently suggested an alternate means of expressing the vapor pressure and slope of vapor-pressure curve (Δ) in terms of air and dew point temperatures. Some saving in computer time should be anticipated with no significant loss in accuracy. He gives the following equation, where e_s is the saturation vapor pressure in inches of mercury, and T is the temperature in °F.:

$$e_s\simeq(0.0041T+0.676)^8-0.000019|T+16|+0.001316 \quad (12)$$

from which,

$$\Delta=\frac{de}{dT}\simeq 8(0.0041)\,(0.0041T+0.676)^7-0.000019, \quad T\geq -16°\text{F}. \quad (13)$$

With the suggested substitution, the lake evaporation equation becomes:

$$E_L=[\epsilon^{(T_a-212)(0.1024-0.01066\ln R)}-0.0001 +0.0105(e_s-e_a)^{0.88}(0.37+0.0041u_p)]\times [0.04686(0.0041T_a+0.676)^7+0.01497]^{-1} \quad (14)$$

and when $T_a\geq T_d\geq -16°$F.:

$$e_s-e_a=(0.0041T_a+0.676)^8-(0.0041T_d +0.676)^8-0.000019(T_a-T_d). \quad (15)$$

This and the pan equation have been programed and used extensively in the Hydrologic Services Division. Engelbrecht [6] used the lake equation in his soil moisture work with the IBM 650 and other computers. The Bosen equations effect a 5 to 10 percent saving in time on the Bendix G-15 computer. Other computers with relatively faster output facilities may be able to realize a relatively greater saving.

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