Continuous Hydrograph Synthesis with an API-Type Hydrologic Model

WALTER T. SITTNER, CHARLES E. SCHAUSS, AND JOHN C. MONRO

U. S. Weather Bureau, Silver Spring, Maryland 20910

Abstract. The U. S. ESSA Weather Bureau Hydrologic Research and Development laboratory has developed a complete hydrologic model utilizing an antecedent precipitation index (*API*) type rainfall–runoff relation to compute surface runoff. With increasing demand for continuous river forecasts as well as flood forecasts, it is necessary to have a model that will predict all components of flow as functions of observable independent parameters on a continuous basis. To formulate the model, existing and proved techniques were used where possible and new techniques developed as necessary. The model consists of four basic parts: a relation for computing ground-water recession, a method of computing the ground-water flow hydrograph as a function of the direct runoff hydrograph, an *API*-type rainfall–runoff relation, and a unit hydrograph. The rainfall–runoff relation is of the incremental type, yielding a runoff computation for each 6-hour period rather than computing the total storm runoff. This has been accomplished through the inclusion of a new parameter, retention index. Two important features of the model are the ease of adjusting parameters to observed flow and the sequential development of the four basic parts with a minimum of interaction.

INTRODUCTION

The U. S. ESSA Weather Bureau has for many years been engaged in a program of continuous river forecasting, utilizing a wide variety of hydrologic techniques to produce various types of forecasts. In large rivers the instantaneous discharge hydrograph is usually predicted by routing observed upstream flows and reservoir releases. Forecasts of total volumes of discharge during extended periods are based on analyses of anticipated precipitation and/or snowmelt. The response of individual headwater basins to storm events is predicted by the use of rainfall–runoff relations and unit hydrographs. Discharge of such basins during fair weather periods is arrived at by extension of ground-water depletion curves. What has been lacking is a purely objective means of predicting the flow from individual basins during periods when it consists of ground-water discharge combined with relatively small amounts of direct runoff.

The demands for river forecasts are continually increasing. These demands are for wider areal coverage as well as for improved precision in the low and medium flow ranges. To accommodate these demands the U. S. Weather Bureau has been evaluating various techniques for making continuous forecasts of the response of individual basins. Such evaluation must include a comparison of forecasts of storm events produced by existing procedures and by the continuous type model under consideration. To facilitate this comparison, the existing techniques were modified to embrace the concept of a continuous streamflow model. This modification, as effected, includes a new method of expressing ground-water discharge as a function of independent parameters. This combination of old and new techniques constitutes a complete hydrologic model that is the subject of this paper.

The results of the test of the model have been very encouraging. Although complete tests have been run on only two basins, the Monocacy River near Frederick, Maryland, and the French Broad River at Rosman, North Carolina, these tests indicate that this model may become a practical forecasting tool.

THEORY OF THE MODEL

General. All flow in any river channel is originally derived from precipitation. Individual particles of water, however, fall in different parts of the basin and reach the channel by a great number of routes. The travel may be above or below ground and may require months or years or no time at all. Consequently a detailed effort to categorize flow components could yield an almost unlimited number. The flow, however, is usually thought to consist of four components:

1. Channel precipitation: Rain falling directly on the surface of the stream.
2. Surface runoff: Water that falls on the basin surface and finds its way into the stream channel by means of overland flow.
3. Subsurface runoff (also called subsurface flow, interflow, or seepage into the stream): Precipitation that infiltrates the surface soil and moves laterally through the upper soil layers toward the stream channel. This may be pictured as a movement of air and water (unsaturated flow) above the ground-water level.
4. Ground-water runoff or ground-water flow: That part of discharge caused by percolation into the ground-water aquifer (saturated flow).

In runoff analysis there is no rational technique for completely and accurately delineating the various flow components that together define the hydrograph. Further, the decision as to how many components to recognize is somewhat arbitrary. It seems logical to define and treat as few as are necessary to obtain acceptable results. The procedure used in this study is to consider just two components, direct runoff and ground-water flow. Direct runoff consists of items 1, 2, and 3 above. Ground-water flow is defined in item 4.

The direct runoff component of the hydrograph is computed from precipitation by the use of an antecedent precipitation index (*API*) type rainfall–runoff relation and a unit hydrograph. As will be pointed out later, the rainfall–runoff relation has been modified somewhat, but the model computes this component of flow by basically standard techniques. The ground-water discharge hydrograph is represented as a function of the direct runoff hydrograph. The relationship between the two described below involves the use of the ground-water recession coefficient for the basin. The complete model then consists of four parts:

1. Rainfall–runoff relation.
2. Unit hydrograph.

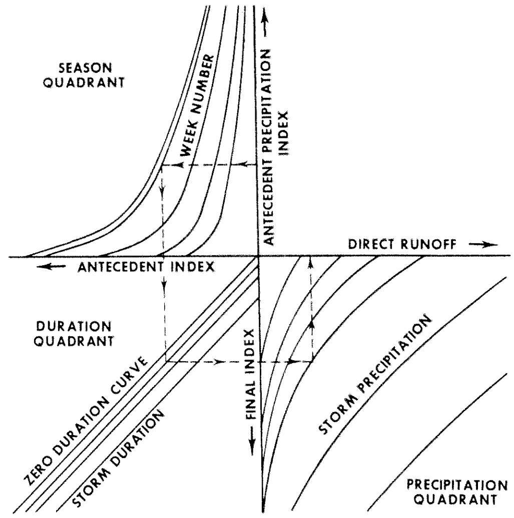

Fig. 1. Standard *API* type rainfall–runoff relation.

3. Relation for expressing ground-water hydrograph as a function of the direct runoff hydrograph.
4. Relation for evaluating the ground-water recession coefficient.

Rainfall–runoff relation. Direct runoff volume is determined within the model by using an *API* type rainfall–runoff relation [*Linsley et al.*, 1949, pp. 418–424]. In this relation (Figure 1) the *API* is used as an index to upper level soil moisture. It is a decay function of precipitation and reflects the precipitation regime for about one month prior to the event. In the season quadrant the *API* is combined with a seasonal parameter, week number, to produce an antecedent index (*AI*), which is intended to represent antecedent conditions completely. The duration quadrant applies a small adjustment based on storm duration to the *AI* and results in a final index (*FI*). The duration quadrant is usually assumed to be standard for all basins and simply applies an adjustment of +0.01 per hour duration. The precipitation quadrant expresses direct runoff as a function of *FI* and storm precipitation.

As described the relation determines the total direct runoff for an event of any duration in terms of total precipitation. In operational forecasting, however, six-hourly increments of runoff are usually required. The most common method of obtaining these increments is to determine an initial *AI* for the event, using the initial *API* and the week number. At the end of each six-hour period, the *AI* is used with the duration and total accumulated precipitation at that time to compute the total accumulated runoff. Successive figures of accumulated runoff are then subtracted to obtain runoff increments. Although such use of a total storm relation is quite consistent with the concept and with the fact that the relation is developed from total event data, a question occurs when an extended storm period is interrupted by one or more periods of little or no precipitation. Should one continue the computation as described above or break it and start over with new antecedent conditions, considering the subsequent precipitation periods as a separate event? The two methods will not give the same result, and there may be a significant difference. Making the choice always involves a high degree of subjectivity.

In using the *API* type of relation as part of a continuous model, this deficiency becomes of paramount importance. It is virtually necessary to have an incremental type of relation, that is, one in which the precipitation for each unit time period (six hours in this study) is converted to runoff on the basis of its own updated antecedent conditions. The procedure for each six-hour period must be identical to that for every other period and result directly in a six-hour runoff increment. Although the need for such a technique has long been felt, there are two problems connected with it.

The first problem is that the type of relation shown in Figure 1, regardless of the configuration of the curves, cannot be used incrementally as described above. The reason is evident from a discussion of Figure 2, which shows two precipitation regimes and the resultant *API* patterns. The direct runoff resulting from the one inch of precipitation falling in the first period of the fifth day is computed. In both cases the *API,* based on a daily recession factor of 0.9, is equal to 3.91 inches. Since all input parameters to the relation are identical in both cases, the relation must compute the same runoff in both cases. In Figure 2a the precipitation in question occurs after a continuous dry spell of 54 hours. Consequently before runoff starts, interception and depression storage losses must be satisfied. In Figure 2b, however, the subject period is the fifth period of a continuous storm which has already deposited 3.70 inches on the basin. Obviously basin retention capacity is largely satisfied, and the runoff from this inch of precipitation will be greater than from the corresponding inch in the previous case. Since the relation has no way of distinguishing between two such situations, it cannot in this form be used incrementally.

The second problem is one of development. Fitting the relation to a particular basin consists of correlating the independent variables for a number of events with the dependent variable, observed runoff, for each event. While the total runoff resulting from a precipitation event can be easily determined from the observed hydrograph, it is virtually impossible to apportion this quantity among the individual periods of the event. Consequently the development of an incremental relation would be expected to involve correlation with a dependent variable that is not observable.

The method which has been devised overcomes both of these problems. It involves the introduction of a new input parameter, retention index (*RI*). This is similar to the *API* but has a much lower recession factor and is therefore a short-term moisture index reflecting the presence of water in interception and depression storage. In Figure 2 the *RI* (dotted line), based

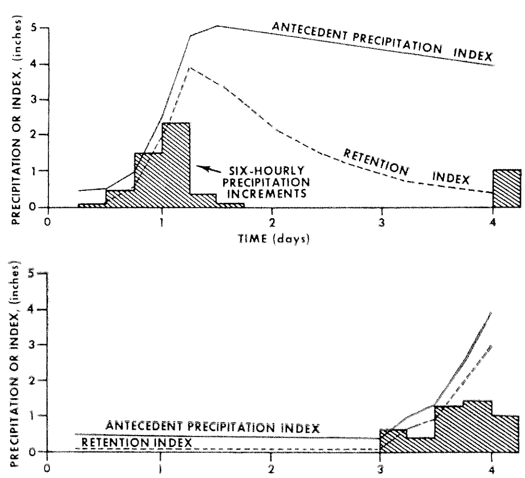

Fig. 2. Variation of antecedent precipitation index and retention index.

on a daily recession factor of 0.4, has a value at the beginning of the fifth day of 0.36 in the first example and 3.00 in the second. Consequently the RI can reflect the difference between these two situations, and if properly introduced into the relation, should make it possible to compute the correct runoff in both cases. Figure 3 shows how the RI is used. Since all events have been reduced to unit duration, the duration quadrant of Figure 1 is no longer needed and has been replaced by the RI quadrant. The total storm relation (Figure 1) is presumably capable of predicting the runoff from any six-hour event or from the first six hours of a longer event. In either case the duration quadrant would modify the AI by $+0.06$. The season quadrant of Figure 3 is identical to that of Figure 1 except that all of the curves have been shifted 0.06 to the left. The configuration of the curves in the precipitation quadrant has not been changed. Thus the relation will correctly predict the first runoff increment of the event if the RI quadrant equates the FI to the AI. If it is assumed that the RI at the beginning of any event is zero or close to it, then the zero RI curve must be a 45 degree line through the origin as shown. Since all RI values greater than zero must act to produce an FI smaller than the AI, all of the curves must lie above the zero curve. Since small amounts of precipitation can often satisfy retentive capacity and since further rainfall has little additional effect, the curves would be expected to exhibit decreased spacing for higher RI values as shown. If the curves are assumed to be straight lines, then the RI quadrant can be expressed by the formula

$$FI = AI(RA)^{RI} \tag{1}$$

where RA is a basin constant less than unity.

Since the season and precipitation quadrants can be developed on the basis of storm total parameters, all that is required to define the incremental relation is evaluation of the basin constant RA. The technique by which this is accomplished is described in the following section.

All curve families in the rainfall–runoff relation can be expressed analytically. A group of formulas to accomplish this is presented in the appendix.

Groundwater discharge hydrograph. As noted earlier, this component of channel flow, which originates as infiltrated water, is represented as a function of the direct runoff hydrograph. The close relationship between direct runoff and infiltration suggests such a function. If the ground-water flow hydrograph is considered to represent outflow from the ground-water aquifer, then it is reasonable to think in terms of an 'inflow to ground water' hydrograph, a composite of the inflow taking place throughout the basin. At any time when inflow is zero, the outflow follows a simple depletion pattern: that is

$$G_t = (Kg)^t(G_0) \tag{2}$$

where G_0 and G_t are the ground-water discharge values at time zero and time t and Kg is the ground-water recession factor. As the direct runoff approaches zero, the total discharge Q approaches the ground-water discharge G. If it is assumed that inflow to ground water I is a function of concurrent direct runoff discharge and that the ground-water and surface water divides coincide, then inflow must become zero at this point and equation 2 will apply. An expedient first assumption is that the relation between inflow to ground-water and direct runoff discharge may be represented by the simplest possible function, a linear one

$$I = Z(Q - G) \tag{3}$$

where Z is the ratio of the instantaneous value

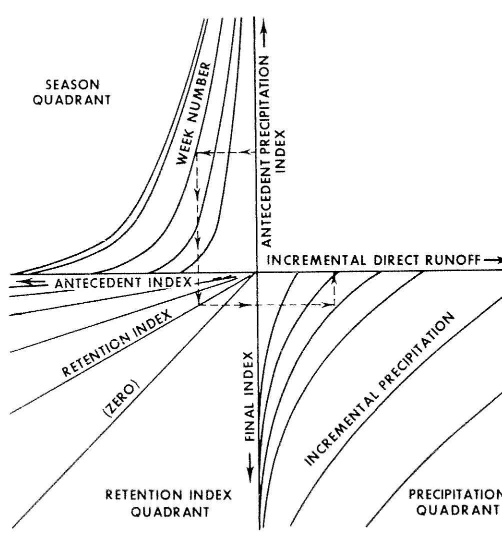

Fig. 3. Incremental rainfall–runoff relation.

of inflow to ground water to the concurrent instantaneous value of direct runoff discharge. Whether or not Z is constant cannot be determined at this time, but the following derivation does not depend on its being so. Later a functional relationship for Z will be determined empirically. If the value of Z is known, however, then the inflow to ground-water hydrograph may be computed with equation 3. This hydrograph, if suitably routed (simulating the movement of water through porous media), will yield the desired ground-water outflow hydrograph. If it is assumed that Muskingum routing [*Linsley et al.*, 1949, 502–503] with zero X (reservoir routing) will accomplish this, then all coefficients in the routing equation may be evaluated if the ground-water recession factor is known.

Referring to Figure 4 which shows a typical storage depletion curve, the discharge at any time t may be expressed as a function of that at a previous time a and the recession factor Kg

$$Q_t = Q_a(Kg)^{(t-a)} \tag{4}$$

During the differential period from time t to time $t + dt$, the change in storage $-dS$ is equal to $Q_t dt$. From this and equation 4

$$-dS = Q_a(Kg)^{(t-a)}\, dt \tag{5}$$

Considering the change in storage from time a to time b

$$\int_a^b -dS = Q_a \int_a^b (Kg)^{(t-a)}\, dt \tag{6}$$

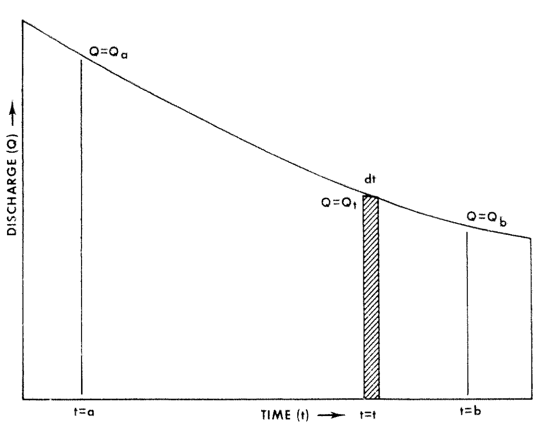

Fig. 4. Ground-water depletion curve.

Integrating

$$S \Big]_a^b = -Q_a \frac{Kg^{(t-a)}}{\ln Kg} \Big]_a^b \tag{7}$$

Applying limits

$$\begin{aligned} S_b - S_a &= -Q_a\left[\frac{Kg^{(b-a)}}{\ln Kg} - \frac{Kg^{(a-a)}}{\ln Kg}\right] \\ &= -\frac{Q_a(Kg)^{(b-a)}}{\ln Kg} + \frac{Q_a}{\ln Kg} \end{aligned} \tag{8}$$

By applying equation 4 at time a and substituting in equation 8

$$S_b - S_a = -\frac{Q_b}{\ln Kg} + \frac{Q_a}{\ln Kg}$$

or

$$Q_b = \left(S_b - S_a - \frac{Q_a}{\ln Kg}\right)(-\ln Kg) \tag{9}$$

The Muskingum storage equation with zero X equates storage with the product of outflow and the storage constant K. Since the outflow in this case is Q_b

$$S = K(Q_b) \tag{10}$$

As the routing involves only increments of storage, absolute storage volumes are not needed and are in fact indeterminate with this type of analysis. All that is required is the value of storage in reference to some arbitrary but constant level. Referring to equation 9, the quantity $(-S_a - Q_a/\ln Kg)$ is constant with respect to time and, although indeterminate, may be considered the datum value. The quantity $(S_b - S_a - Q_a/\ln Kg)$ then becomes the difference between the storage at time b and the datum value. This corresponds to the quantity S in equation 10. Equation 9 then becomes

$$Q_b = (S)(-\ln Kg) \tag{11}$$

Substituting in equation 10

$$S = (K)(S)(-\ln Kg)$$

Solving for K

$$K = -\frac{1}{\ln Kg} \tag{12}$$

For the routing problem at hand, Kg is the ground-water recession factor for the basin.

Having the storage constant K, the routing coefficients for a routing period of six hours (one-fourth day) are computed as follows:

$$C_0 = C_1 = \frac{1}{(8K + 1)} \tag{13}$$

$$C_2 = \frac{(8K - 1)}{(8K + 1)} \tag{14}$$

The routing equation then becomes

$$G_2 = (Z)(C_0)(Q_2 - G_2 + Q_1 - G_1) + (C_2)(G_1) \tag{15}$$

This gives an ordinate on the ground-water flow hydrograph G_2 in terms of the preceding ordinate G_1 and the differential quantity $(Q\text{-}G)$, which is an ordinate on the direct runoff hydrograph. Thus equation 15 may be used to generate the ground-water flow hydrograph if the direct runoff hydrograph is known. The equation may also be written

$$G_2 = \frac{(Z)(C_0)(Q_1 + Q_2) + (G_1)(C_2 - ZC_0)}{(1 + ZC_0)} \tag{16}$$

Equation 16 gives the ground-water hydrograph ordinate in terms of the preceding ordinate G_1, and points Q_1 and Q_2 on the total flow hydrograph. Thus the equation can be used to separate a hydrograph into its two components. While it is not used in this form in the model itself, it is used in the development of both the rainfall–runoff relation and the unit hydrograph.

The above hypothesis does not recognize the condition of depletion of ground-water supply to a point below that corresponding to zero channel inflow and is consequently applicable only to continuous streams. To use this approach with intermittent or ephemeral streams may well require some modification of the basic theory.

Relation for evaluating ground-water recession coefficient. The nature of the computation described above is such that the value of the coefficient Kg is critical. Consequently no attempt is made to use a constant value. Kg is considered to be primarily a function of discharge, having a value of unity at zero discharge and decreasing for higher flows. Since equal discharge values in different seasons probably result from different ground-water level configurations, provision is made for a seasonal variation in Kg.

Figure 5 is a schematic diagram of the complete model. All computations involved in the model can be performed by electronic computer.

DEVELOPMENT OF THE MODEL

Development of the model incorporates independent determination of its four basic relations prior to their combination into the composite model for final verification and adjustment. In this project all computations involved in the development procedure and the operation of the complete model were performed by a small scale electronic computer.

Ground-water recession coefficient. The first part of the model to be evaluated is a relation for expressing the ground-water recession coefficient as a function of ground-water discharge and week number. The daily coefficient is defined by

$$Kg = Q_2/Q_1 \tag{17}$$

where Q_1 and Q_2 are the discharges at some time on two successive days when there is no direct runoff. To derive the relationship a visual inspection of several years of mean daily hydrograph is made to select periods meeting this criterion.

Equation 17 is then solved for a very large number of pairs of discharge values. In practice, for the sake of expedience the mean daily

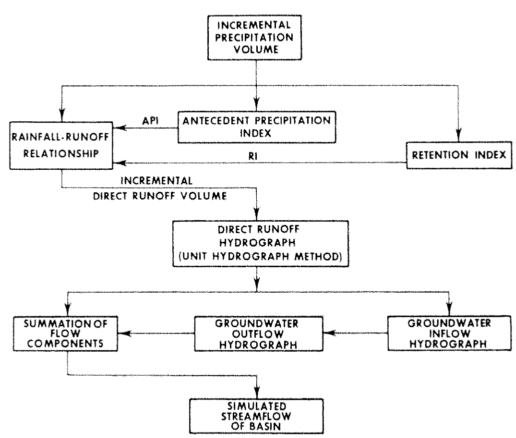

Fig. 5. Schematic diagram of *API* type hydrologic model.

values of discharge are used. Results are virtually identical to those which would be obtained by using instantaneous values. The computed values of Kg are then grouped by discharge and median values of Kg and Q_1 computed for each group. For low values of discharge, where Kg approaches unity, group medians give results superior to group averages. A curve through the points so defined represents the average relation between Kg and discharge. The seasonal parameter is then introduced by correlating the deviations of the individual events from the curve with week number. The resultant curve of week number versus deviation is applied as a linear function of discharge in such a way as to simulate a family of curves converging at zero discharge.

Ground-water flow hydrograph. Analyzing several years of mean daily streamflow data and applying equation 16, the ground-water flow hydrograph for the period can be generated, based on any assumed value of or relation for Z. In this application the routing period is one day, and the values of Q used in the equation are mean daily rather than instantaneous. The procedure results directly in a mean daily ground-water flow hydrograph which is virtually identical to that which would be obtained by working with instantaneous values of discharge. The adequacy of the trial value of Z cannot be fully evaluated since the actual ground-water flow hydrograph is not known. However, the manner in which it ties in with the recession of the total flow hydrograph following a rise is a good indication. If the value of Z is too small, the ground-water flow will consistently run below the total after it is obvious that direct runoff has ceased. If Z is too large, ground-water flow values exceeding total flow will result. Although it was found necessary in the study to make some minor revisions to the Z relation based on the output of the complete model, the above technique yielded results that, although tentative, closely approximated the final value.

As noted earlier, there is no theoretic reason for Z to be constant. To obtain a proper ground-water flow hydrograph, it was in fact necessary to adopt a variable ratio. In the one used, Z is a function of total discharge of the form

$$Z = ZA + ZB(Q) \tag{18}$$

where ZA and ZB are basin constants and Q is the total discharge. A third constant ZC is a limit which Z may not exceed. Some experimentation with other forms of relations took place, but that described gave the best overall results.

Rainfall–runoff relation. The development of the rainfall-runoff part of the model consists of developing a conventional total storm relation and then converting it to the incremental type by evaluating the coefficient RA in equation 1. To accomplish this, several trial values of RA are used. With each value all precipitation events are run through the relation, and the total of all computed increments for each event is compared to the observed total runoff. The errors for individual events are assembled into a summary containing average error, bias, maximum error, or any other meaningful parameter. The several values of RA are then plotted against each of the parameters and the best value of RA selected. It was found that all error analysis parameters tended to minimize at the same value of RA, lending credence to the general approach.

Theory dictates only that the RI recession factor be considerably less than that for the API (usually 0.9). It seems logical to expect that in practice the factor could be standardized, at least geographically, as the API factor has been. In this project, however, it was necessary to optimize both the recession factor and the constant RA, which is unique for a basin. Values of the daily recession factor used were 0.38 for the French Broad basin and 0.50 for the Monocacy.

It was found that in one of the test basins, the Monocacy River near Frederick, Maryland, results could be improved by considering RA a function of week number rather than a constant. To determine the relation for a seasonally variable coefficient, a value of RA is determined for each event such that the error for that event will be zero. These are then correlated graphically with week number. The application of the resulting curve in actual computation is accomplished by table look-up. The determination of the optimum RA for an individual event involves an iterative procedure that is complex but not formidable. During the process a sensitivity figure, the ratio of differential error to differential RA, is computed for each event.

These are used as weights in the correlation with week number.

As stated above, verification is based on the comparison of observed direct runoff for an event with the summation of the computed increments for that event. Such a comparison assumes that if the relation will consistently compute increments adding to the correct total, then the increments themselves must be correct. This assumption is quite logical. Since the incremental relation will compute the runoff from the first increment of any event equal to that resulting from application of the total storm relation, then an acceptably correct total for any two-period storm verifies the second increment. Similarly if the incremental relation correctly predicts the runoff for the first two periods, then the third period of any three-period storm is verified if the correct total is obtained. The reasoning may be extended in this manner to events of any duration. This logic assumes no bias in events of any particular duration category. Correlation of forecast error with duration is one of the tests which should be made in developing any rainfall—runoff relation. This procedure was followed with the relations for each of the test basins, and there was in fact no bias.

An interesting phenomenon was noted during the development process. Because it is possible for an increment of rainfall occurring late in a storm to produce virtually 100% runoff, the precipitation quadrant must be drawn in such a way as to indicate 100% runoff at zero *FI*. Since 100% runoff or any condition closely approaching it is not usually possible on a total storm basis, the season quadrant paired with such a precipitation quadrant will not be capable of producing an *AI* close to zero. The result is that the precipitation quadrant has an area (low *FI*) not used by the total storm relation and hence not defined in its development. The area is used in the incremental relation, however. A revision of the curves in this area, actually the definition of them, must take place during the conversion of the relation to the incremental type. This revision is easily accomplished once the need for it is recognized and understood. The important aspect of this is that the resulting relation more nearly approaches the ideal condition of being defined in all areas of all quadrants than a total storm relation. For this reason such a relation is expected to forecast properly a future event having initial conditions not encountered in the development and test data.

In both basins for which incremental relations were developed, these computed total storm direct runoff at least as well as the total storm relations.

Unit hydrograph. The best fit unit hydrograph for a basin would be one derived from all storms in the period of record. Such an analysis has never been practical because of the great amount of labor involved. In this study, comparable results were achieved by the use of a two-step process. A unit hydrograph was derived in the conventional manner, using several selected events. This was considered a first approximation. Once all model parts were defined, the model itself was used as the tool for adjusting the unit hydrograph. Using either the trial hydrograph referred to above or a subsequent approximation, several years of precipitation data were run through the complete model, and the results compared with the observed streamflow data. Such a comparison involves qualitative inspection of hundreds of storm events, large and small, and indicates unit hydrograph revisions reflecting a truly comprehensive sampling of the data.

Such a trial is fast and easy and can be repeated as many times as necessary to obtain the best fit. The authors do not know of any other technique for unit hydrograph development which permits as complete use of the data.

Sequential development. It will be noted from the foregoing discussion that the four basic parts of the model can be developed individually for a basin in the order specified. In the process the independent and dependent variables involved with each part can be identified and evaluated from hydrometeorologic records. Furthermore the development of each part is dependent upon values assigned to parts previously developed but not upon those to be developed. This permits a direct and definite development procedure. While iterative or trial and error processes are used to optimize some of the parts, there is no iteration among parts. This 'sequential development' capability is one of the model's pronounced advantages.

One minor deviation from this concept is the fact that the Z relation cannot be firmly determined before the complete model has been assembled. In the work so far, it has been found necessary to base some revisions to this formula on the final model output. Although this is coincident with the final revisions to the unit hydrograph, it is fairly easy to associate output errors with one part or the other. Refinements to the Z relation made at this time are small enough not to invalidate the rainfall–runoff relation already developed.

DISCUSSION AND RESULTS

The tests of the model involved two river basins. One is the Monocacy River near Frederick, Maryland. This is an 817 square mile basin located in the foothills of the Appalachians and in the North Central portion of the state. The elevation ranges from 230 to 1900 feet above sea level. The area is largely agricultural, and land cover is principally pasturage and deciduous trees. The mean annual precipitation is 40 to 45 inches, and the mean annual runoff 14.5 inches.

The other basin is the French Broad River at Rosman, North Carolina. The basin covers 68 square miles and is located in the southwest corner of the state, well up on the eastern slope of the Blue Ridge Mountains. The elevation ranges from 2200 to 6000 feet above sea level. Most of the area is unused and covered with stands of deciduous trees. The soil zone is shallow and highly permeable. The mean annual precipitation is 70 to 80 inches, and the mean annual runoff is 43.9 inches.

Instrumentation in the test basins is somewhat better than that usually encountered in operational forecasting. It is felt that for purposes of research and model testing, atypical instrumentation is desirable, although the results of the tests are to some extent superior to those obtained operationally.

During the winter, snow is quite common in the Monocacy basin and falls occasionally in the French Broad basin. This project was not concerned with the computation of snowmelt or with methods of treating the resultant water. Where snow existed, however, it could not be ignored, and it was dealt with in a rational but very rudimentary manner. The procedure was to adjust the precipitation record on the basis of temperature. Each period of precipitation was categorized as liquid or solid. If solid, it was deleted from the record and added to snow cover. This cover was melted on the basis of temperature and the melt figures inserted into the precipitation record. The result was a record which carried solid precipitation at the time it melted rather than at the time it fell. This record was used as model input. The Monocacy record was treated in this manner, but that of the French Broad basin was not. The object was not to reproduce accurately all winter rises but simply to keep the moisture accounting computations from getting badly out of phase.

As noted the model provides acceptable output using only one input parameter, precipitation. This is important since other hydrometeorologic data, such as potential evapotranspiration, are usually not available from sites representative of basins being forecast.

Any hydrologic model contains a great many coefficients and parameters. The concept of the model can be such that these are actual measures of physical quantities, or they may be indices to those quantities. The *API* model is of the latter type. The distinction involves a rather important aspect. Any model, to serve a useful purpose, must be fitted to a basin by determining the values of the various coefficients. There are two basic methods for doing this. One involves use of measured values of the basin input–output quantities and a procedure for adjusting coefficients to fit. The other consists of a theoretic determination of the coefficients based on measurable physical characteristics of the basin itself. With a highly rational model where the coefficients are of the actual measure type, the fitting process usually involves parts of both methods. The index type model, however, is restricted to the first method. The ability to acquire information about the coefficients of a rational model without using hydrometeorologic records is a great advantage in some applications. If significant changes in the physical characteristics of a basin have been made recently or are being anticipated, the manner in which these affect the hydrologic characteristics can be quantitatively estimated. In certain types of planning activities, this capability is needed. The basin changes being referred to are hydrologic (land

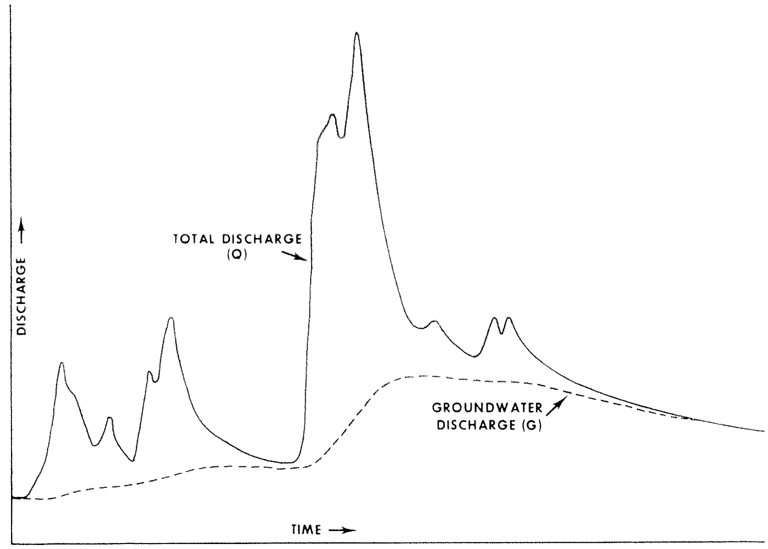

Fig. 6. Typical hydrograph separation.

use and urbanization) as opposed to hydraulic (dams and storage reservoirs).

In operational river forecasting, the ability to alter a model theoretically to reflect such changes is seldom needed. In natural basins of the size commonly forecast, these changes usually come about so slowly that their effect can be gleaned from hydrologic records. The model's ability to make use of existing forecast procedure is a great advantage in adapting it to areas of present forecast responsibility.

A necessary feature of any forecast model is the ability to adjust model parameters at any time to correspond to observed streamflow. In this model, because of the simplicity of its concepts, this adjustment can be made quite easily.

A number of observations made during the study are of interest. The concept of ground-water discharge as a function of direct runoff apparently gives adequate results. Correct evaluation of total discharge verifies the accuracy of the two components of which it consists. In addition, however, the computed ground-water flow hydrograph itself agrees nicely with a logical concept of how this component should appear. Figure 6 shows a portion of the French Broad instantaneous hydrograph separated into the two components. Scales have been left off since what is of interest is the relative shape of the two curves during a typical rise.

The foregoing discussion discloses a number of features of the model which, in a forecasting tool, are distinct advantages. One of the most important qualities in a forecast model, however, is the ability to reproduce accurately an observed hydrograph. In this, the model compares very favorably with all other known methods of stimulating streamflow. Figures 7–10 show one year of discharge record for each of the test basins with both observed and simulated discharge plotted. Here, as in the evaluation data which follow, verification is based on mean daily values of both observed discharge and model output.

Visual examination of plotted hydrographs is a highly reliable method of evaluating the accuracy of model output, although it is almost completely subjective. Consequently an attempt was made to compute some meaningful statistical summaries. Unfortunately, there is no single statistical test or group of tests which is truly comprehensive. Nor are there any standards with which statistical results might be compared. The two tests described below are thought to be informative, but no rigid interpretation of the results can be made.

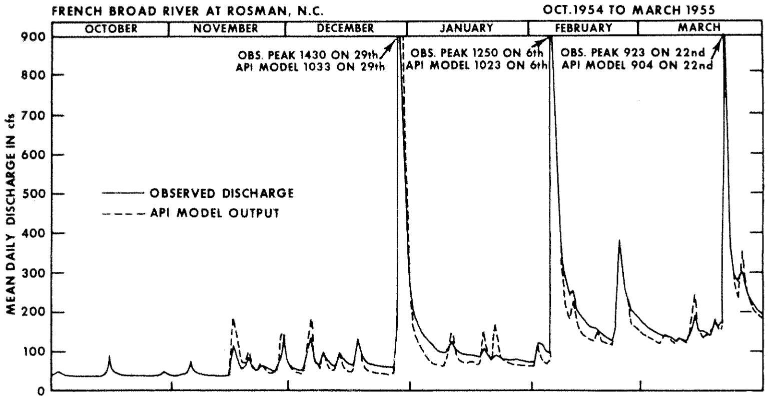

Fig. 7. Sample hydrograph simulation, French Broad River.

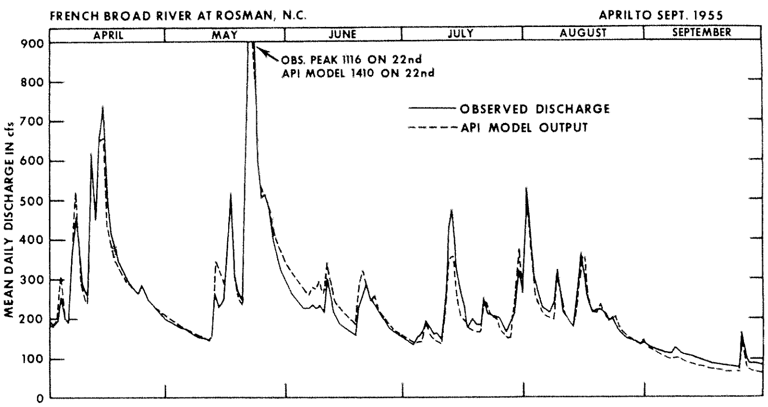

Fig. 8. Sample hydrograph simulation, French Broad River.

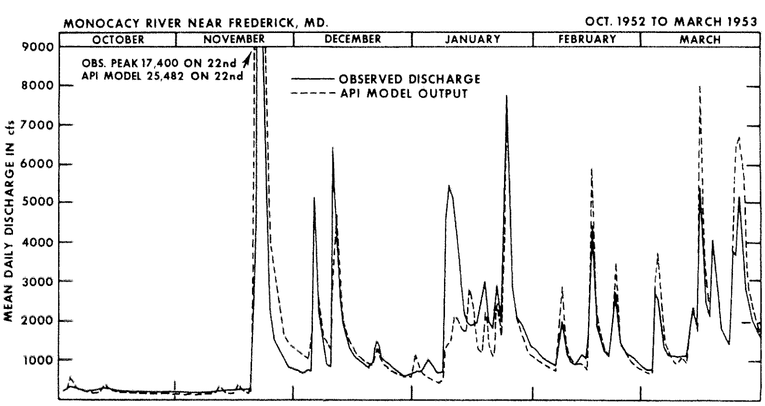

Fig. 9. Sample hydrograph simulation, Monocacy River.

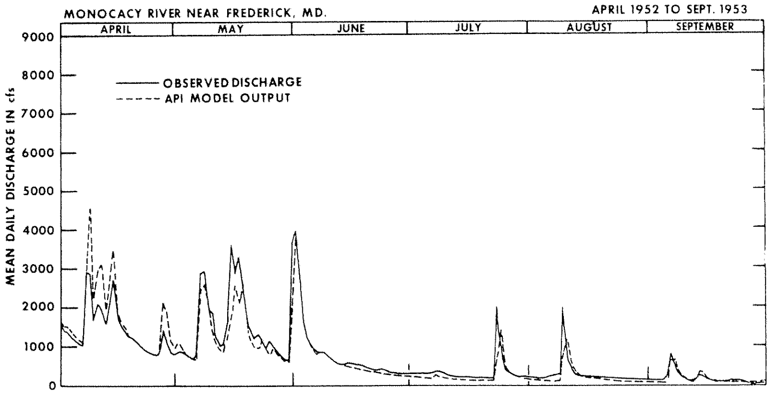

Fig. 10. Sample hydrograph simulation, Monocacy River.

The first test involves computation of the error in the computed mean daily discharge figure for each day in the period of study. The summary of the errors is presented in Figure 11 in the form of a frequency distribution graph. This is a plotting of error as abscissas against the percent of events having less than that error as ordinates.

The second test is designed to simulate actual forecast conditions. The change in discharge from a given date to some date in the future is compared with the change forecast by the model. The difference is the error. It is expressed both in cfs and in percent of the true discharge. All discharge figures are mean daily. The error is computed for periods of 24, 48, and 72 hours. The following example illustrates the method:

Date	Observed discharge	Model Output
10	50	59
11	46	49
12	72	70
13	78	83
14	67	65

To simulate forecasts made on the tenth for 24, 48, and 72 hour periods, the model forecasts changes of −10 cfs, +11 cfs, and +24 cfs. The observed changes are −4 cfs, +22 cfs, and +28 cfs, resulting in errors of −6 cfs, −11 cfs, and −4 cfs. Expressing these errors as percentages of the observed discharge at the end of each forecast period results in −13%, −15%, and −5%. In actual forecasting as opposed to continuous modeling, the discharge at the beginning of the forecast period would be known and the model output adjusted to agree with it. The

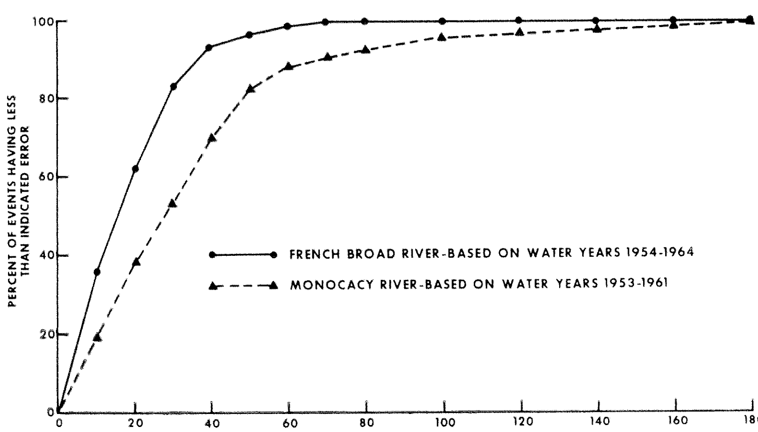

Fig. 11 Frequency distribution of errors in model output.

forecasts therefore would be in error by precisely the amounts shown. Starting on the eleventh, the errors are −5 cfs, +2 cfs, and −5 cfs, or −7%, +3%, and −7%.

Computations of the type described have been made for every day in the period of record for both basins. A summary of the results is presented in Tables 1 and 2. Figure 12 is a frequency distribution plot similar to that of Figure 11 but based on the errors in the 24-hour forecast. The graph is restricted to one forecast period in the interest of clarity. Since basins of the size used in the study reach their crests about one day after the beginning of direct runoff, the most important portion of the hydrograph is the first day following the forecast or a revision to it. Consequently, the 24-hour forecast figures are the most meaningful of the three computed.

Twenty years of streamflow data were simulated in the study, nine in one basin and eleven in the other. At selected times, the model output was adjusted to 'tie' it to observed streamflow. This was done experimentally, and the effect of such a 'tie-in' was found to extend for varying periods into the future, depending on circumstances. The results presented above were obtained by starting with the observed discharge on the first day of the period of record and then running for nine or eleven years with no tie-ins and no input other than precipitation. Interestingly, there was no tendency toward long term divergence from the observed hydrograph. That is, the eleventh year was no better or worse than the fifth or the first, and the quality of each year was not significantly different from what it would have been had there been a tie-in at the beginning of that year. Although it is the nature of the computation to impose upper and lower limits on the output, it was still somewhat surprising to see it faithfully following not only storm peaks but also the long term variation in base flow after ten years of 'free wheeling.'

SUMMARY

A hydrologic model has been devised that simulates basin response on a continuous basis. The model consists of four basic parts: a ground-water recession coefficient relation; a relation for computing Z, which is a coefficient in a formula expressing ground-water flow as a func-

TABLE 1. French Broad River at Rosman, North Carolina. Statistical Summary of Errors in Forecast of Change in Discharge (errors expressed in cfs)

Class Interval			Standard Error			Average Error			Bias		
Range (cfs)	No. of Events	Percent of Total	24-hr.	48-hr.	72-hr.	24-hr.	48-hr.	72-hr.	24-hr.	48-hr.	72-hr.
0–40	42	1	1	1	2	0	1	1	0	0	0
41–80	444	11	6	12	14	3	5	6	0	−1	−1
81–160	1313	33	18	25	29	8	12	14	−1	−2	−3
161–320	1560	39	34	47	51	16	24	27	−1	−1	−1
321–640	550	14	69	89	90	42	57	60	+3	+13	+13
above 640	106	2	221	207	235	153	143	158	+30	0	+18

Errors Expressed in Percent of Observed Discharge at End of Forecast Period

Class Interval			Standard Error			Average Error			Bias		
Range (cfs)	No. of Events	Percent of Total	24-hr.	48-hr.	72-hr.	24-hr.	48-hr.	72-hr.	24-hr.	48-hr.	72-hr.
0–40	42	1	3	4	7	2	3	5	0	0	−1
41–80	444	11	10	17	20	4	8	10	0	−2	−2
81–160	1313	33	15	21	25	7	10	11	−1	−2	−2
161–320	1560	39	14	19	20	7	10	11	0	0	0
321–640	550	14	15	19	20	9	13	14	0	+2	+2
above 640	106	2	22	19	24	16	14	16	+5	+2	+3

TABLE 2. Monocacy River near Frederick, Maryland. Statistical Summary of Errors in Forecast of Change in Discharge (errors expressed in cfs)

Class Interval			Standard Error			Average Error			Bias		
Range (cfs)	No. of Events	Percent of Total	24-hr.	48-hr.	72-hr.	24-hr.	48-hr.	72-hr.	24-hr.	48-hr.	72-hr.
0–100	113	3	24	46	34	8	17	18	2	−1	0
101–200	766	23	59	87	163	25	39	58	1	−2	−12
201–400	812	25	126	220	208	61	105	111	0	−6	+8
401–800	685	21	322	390	441	151	206	228	3	+12	+17
801–1600	510	16	420	711	944	242	390	491	−64	−39	−8
1601–3200	265	8	986	1392	1319	639	880	903	+73	+243	+233
3201–6400	97	3	2321	2695	2761	1791	2010	2102	+99	−35	−141
above 6400	36	1	2926	3199	3674	2320	2524	2839	+25	−1177	−1486

Errors Expressed in Percent of Observed Discharge at End of Forecast Period

Class Interval			Standard Error			Average Error			Bias		
Range (cfs)	No. of Events	Percent of Total	24-hr.	48-hr.	72-hr.	24-hr.	48-hr.	72-hr.	24-hr.	48-hr.	72-hr.
0–100	113	3	26	50	38	10	19	21	+3	0	0
101–200	766	23	37	53	96	16	25	37	0	−1	−8
201–400	812	25	44	76	72	21	36	39	0	−2	+2
401–800	685	21	55	66	74	26	35	39	0	+2	+3
801–1600	510	16	35	59	78	21	33	42	−5	−3	−1
1601–3200	265	8	44	66	61	29	40	41	+1	+10	+10
3201–6400	97	3	57	65	67	42	47	50	+4	+2	0
above 6400	36	1	32	34	38	26	28	30	−1	−13	−17

tion of direct runoff discharge; a rainfall–runoff relation; and a unit hydrograph.

The coefficient Z referred to above enables the ground-water component of channel flow to be computed as a function of the direct runoff hydrograph, using a linear routing procedure that simulates the natural lag characteristics of ground-water movement. During periods of pure ground water, the computation yields a recession curve mathematically identical to the ground-water recession.

The antecedent precipitation index (API) type rainfall–runoff relation as used by the U. S. Weather Bureau in operational river forecasting has been modified to operate on an incremental basis. A retention index RI has been added to reflect the degree of saturation of interception and depression storage. It decays rapidly in comparison to the antecedent precipitation index.

A unique method for unit hydrograph optimization was used in the study. Using the model itself as a tool, the computation of hundreds of storms on a continuous basis provided the data for evaluation of the trial unit hydrographs.

The model generates two flow components, ground-water flow and direct runoff discharge, and uses only precipitation as an input parameter. Standard types of forecast procedure are used for a portion of the analysis. Although the model was devised for the purpose of comparing this procedure with conceptual models, it appears that it may be a practical forecasting tool itself. Other applications are likely.

The model is completely digital and all computations can be performed by machine.

APPENDIX

U. S. Weather Bureau river forecast centers have for a number of years been using electronic computing equipment for solving API type rainfall–runoff relations, and a number of digitizing approaches have been devised. The formulas presented below comprise the method which was used in this study.

To formulate the season quadrant, the two boundary curves are defined by polynomials 1 and 2, there being differing degree expressions for the segments above and below unity *API*.

If the *API* is equal to or less than unity

$$AX = 1 - API$$

$$AMX = SA_1 + SA_2(AX) + SA_3(AX)^2$$

$$AMN = SB_1 + SB_2(AX) + SB_3(AX)^2 \tag{A1}$$

If *API* is greater than unity

$$AX = 6 - API,$$

but is equated to zero if negative.

$$AMX = SC_1 + SC_2(AX) + SC_3(AX)^2 + SC_4(AX)^3 + SC_5(AX)^4$$

$$AMN = SD_1 + SD_2(AX) + SD_3(AX)^2 + SD_4(AX)^3 + SD_5(AX)^4 \tag{A2}$$

AMX and *AMN* are the maximum and minimum *AI* values that may result from a particular value of *API*.

A twelve ordinate harmonic equation 3 is then used to express the actual *AI* as a function of the computed boundary values and the date of the event.

$$SE = SG_1 \cos(WK) + SG_2 \cos(2WK) + SG_3 \cos(3WK) + SG_4 \cos(4WK) + SG_5 \cos(5WK) + SG_6 \cos(6WK)$$

$$SF = SH_1 \sin(WK) + SH_2 \sin(2WK) + SH_3 \sin(3WK) + SH_4 \sin(4WK) + SH_5 \sin(5WK)$$

$$SI = SJ + SE + SF \tag{A3}$$

WK is the week number divided by $(52/2\pi)$ and is defined by equation 4 below.

$$WK = 0.0172[30.36(M - 1) + D] \tag{A4}$$

M and *D* are the month and day corresponding to the event. The adjustment $(52/2\pi)$ causes the parameter *SI* above to exhibit exactly one cycle as the week number varies from 1 to 52 and expresses the position of the particular week curve between the two boundary curves.

AI is computed using equation 5 below

$$AI = AMN + SI(AMX - AMN) \tag{A5}$$

The season quadrant is therefore represented by 28 basin constants. They are: $SA_1 - SA_3$, $SB_1 - SB_3$, $SC_1 - SC_5$, $SD_1 - SD_5$, $SG_1 - SG_6$, $SH_1 - SH_5$, and *SJ*.

The *RI* quadrant, as noted in the text, is ex-

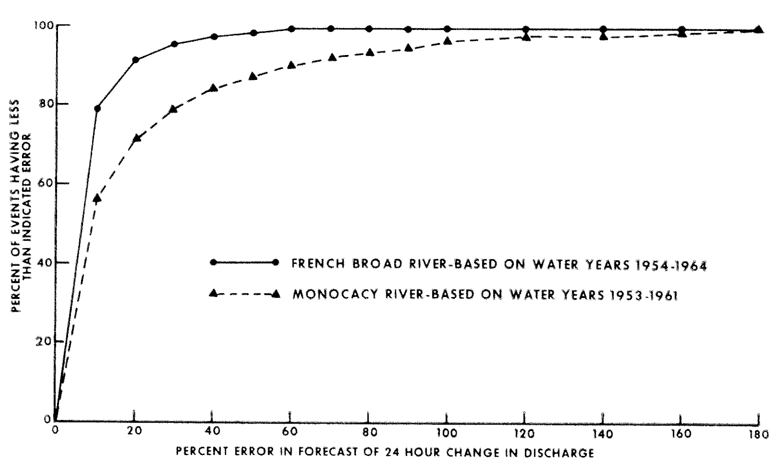

Fig. 12. Frequency distribution of errors in forecast of 24-hour change in discharge.

pressed by equation 6

$$FI = AI(RA)^{RI} \tag{A6}$$

involving one basin constant RA.

In the precipitation quadrant, two intermediate parameters PF and PG are expressed (7) as functions of the FI and five basin constants PA, PB, PC, PD, and PE.

$$\begin{aligned} PF &= PA + PB(FI) \\ PG &= PC + PD(FI)^{PE} \end{aligned} \tag{A7}$$

The incremental runoff RO is then given in terms of the incremental precipitation P by formula 8.

$$RO = [P^{PF} + PG^{PF}]^{1/PF} - PG \tag{A8}$$

REFERENCE

Linsley, R. K., M. A. Kohler, and J. L. H. Paulhus, *Applied Hydrology*, McGraw-Hill, New York, 1949*a*.

(Manuscript received October 23, 1968; revised May 19, 1969.)