

*S = sunshine  
% possible hours*

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## INSOLATION AS AN EMPIRICAL FUNCTION OF DAILY SUNSHINE DURATION

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### ABSTRACT

Empirical relations between incident solar radiation received at the earth's surface and (1) percent of possible sunshine, (2) latitude, and (3) time of year are developed. These relations are combined into a graphical method for converting percent of possible sunshine into daily values of incident solar radiation for stations between latitudes 25° N. and 50° N. The method is tested on independent data from widely separated locations and a correlation coefficient of 0.97 between estimated and observed values is obtained.

### INTRODUCTION

Easily obtained estimates of the daily amount of incident solar radiation (insolation) received at the earth's surface, where observed values are unobtainable, have practicable application in the field of applied meteorology and hydrology. Such values have an immediate utility in the energy balance method of estimating heat and vapor transfer at snow, water, soil, or plant surfaces. This paper presents an empirical-graphical method of converting observed values of the percentage possible hours of sunshine into estimates of insolation.

The network of pyrliometer stations in the United States has expanded during recent years but the number of observations is not sufficient to define the areal distribution of insolation for short periods. There were 49 such stations in 1953 in the continental United States operated by the Weather Bureau, and 21 cooperative stations. A conversion of the percent possible sunshine obtainable from approximately 170 Weather Bureau stations into estimates of insolation would greatly improve the accuracy of the interpolated areal distribution.

Values of average theoretical direct solar radiation reaching the ground under cloudless conditions are available in the Smithsonian Meteorological Tables [1]. These computed values consider the following parameters: (a) solar constant, (b) radius vector of the earth, (c) zenith distance of the sun, (d) a transmission coefficient for the

atmosphere. The solar constant is considered to have a value of 1.94 ly/min and the zenith distance is a function of the latitude of the station, declination of the sun, and hour angle. For a particular station the quantity of solar radiation that is transmitted to the earth's surface, direct and diffuse, is a complicated function involving station elevation, character and amount of cloudiness, water vapor, kind and amount of pollutants—dust, smoke, etc. The percent of possible hours of sunshine, as determined by the sunshine recorder at the station, has been used as an indicator of the combined effect of these variables.

TABLE 1.—List of stations from which observations were obtained

Station	Lat. ° N	Station	Lat. ° N
Miami, Fla.....	25.8	Washington, D. C.....	38.8
Brownsville, Tex.....	25.9	Columbia, Mo.....	38.9
San Antonio, Tex.....	29.5	Indianapolis, Ind.....	39.7
Appalachicola, Fla.....	29.7	Lincoln, Nebr.....	40.9
Charleston, S. C.....	32.8	Cleveland, Ohio.....	41.3
Atlanta, Ga.....	33.5	Boston, Mass.....	42.4
Little Rock, Ark.....	34.7	East Lansing, Mich.....	42.8
Oklahoma City, Okla.....	35.4	Madison, Wis.....	43.1
Greensboro, N. C.....	36.0	Sault Ste. Marie, Mich.....	46.4
Nashville, Tenn.....	36.0	Bismarck, N. Dak.....	46.8

For use in developing the empirical method observations of percent possible sunshine  $S$ , insolation  $Q$ , and snow on the ground, for the years 1951, 1952, and 1953 were obtained from Weather Bureau records [2] for 20 stations, listed in table 1, in the continental United States.

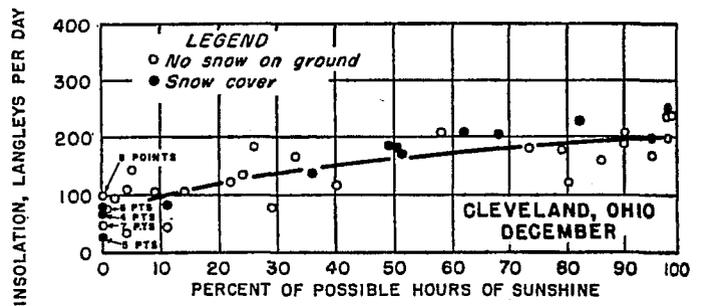
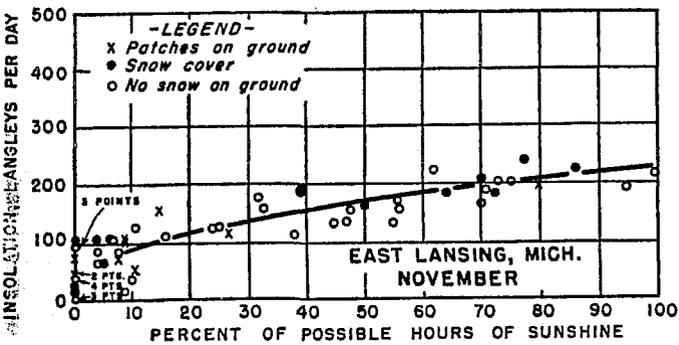
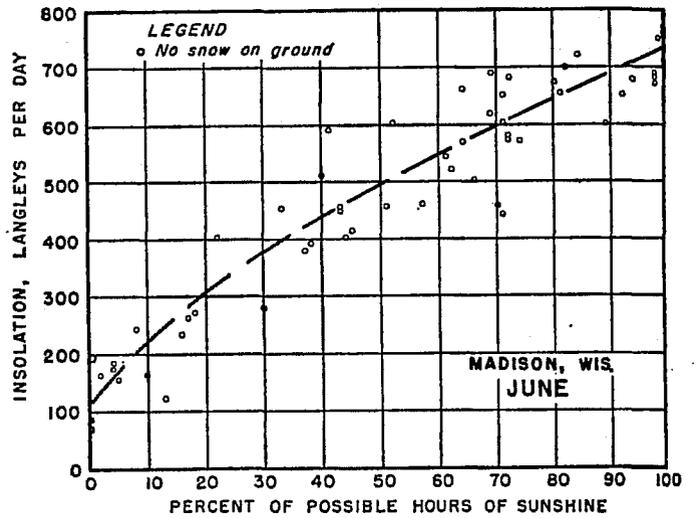
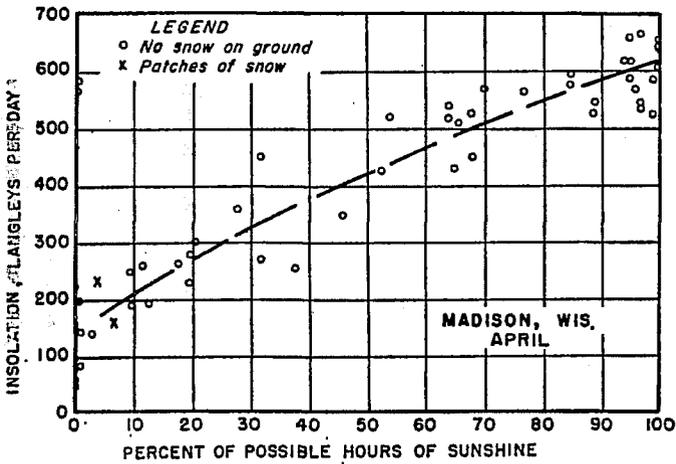
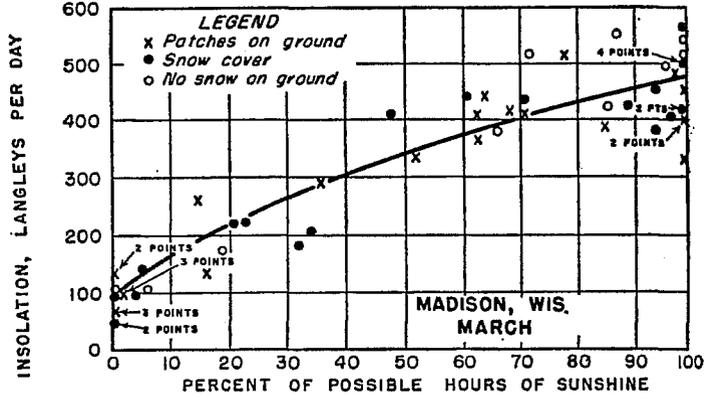
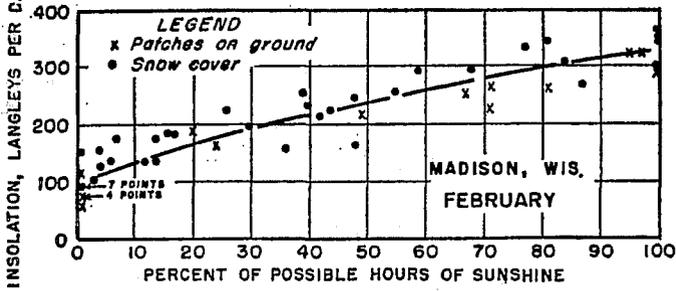
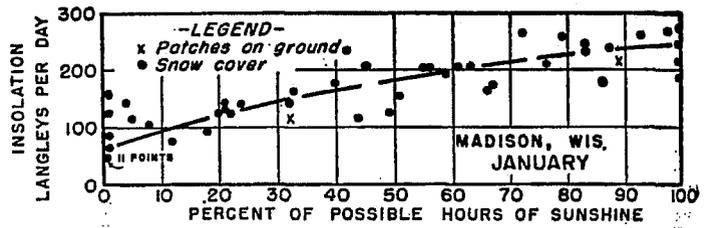
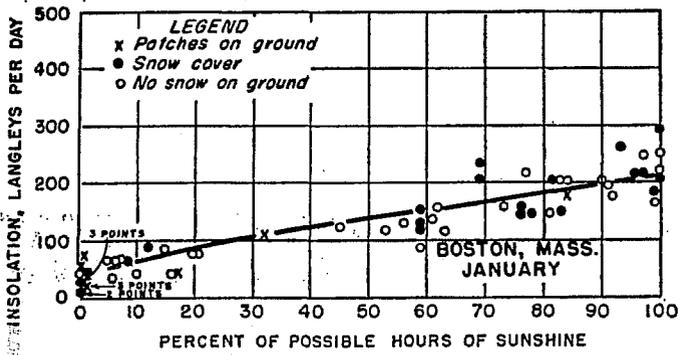


FIGURE 1.—Representative scatter diagrams showing observed insolation as a function of observed percent of possible duration of sunshine, daily values.

DEVELOPMENT OF METHOD

The relation between insolation and the percent of possible sunshine was first investigated by plotting scatter diagrams for each station and month and fitting curves to the points by inspection. Figure 1 displays representative curves that resulted; the curvilinear relation corresponds to that obtained by Kimball [3]. The existence of snow cover has been reported [4, 5] to produce an increase in the diffuse radiation for overcast conditions. To determine the magnitude of this effect the points on the scatter diagrams were identified for those days having snow on the ground, patches of snow, or no snow. The scatter diagrams failed to show sufficient separation of the points to enable separate curves to be drawn to snow and no-snow cases; therefore they were considered as the same population in defining the curves. The values of  $Q$  for each station at 100, 80, 60, 40, 20, and 0 percent possible hours of sunshine were read from the curves of the scatter diagrams and  $Q/Q_0$  determined for each month for each station, where  $Q_0$  is the value read from the curve at 100 percent possible hours of sunshine.

In a study of the relation between average monthly values of  $Q$  and  $S$ , Fritz and MacDonald [6] found that for eleven stations between latitudes  $25^\circ$  and  $44^\circ$  N, a plot of  $Q/Q_0$  versus  $S$  could be fitted by a straight line  $Q/Q_0 = 0.35 + 0.61S$ . Their data contained  $S$  values ranging from 0.35 to 0.97. The correlation coefficient obtained was 0.88. Their work referred only to monthly average values of  $Q$  and  $S$  and is, therefore, essentially different from the work described in this paper. Similarly, Black, Bonython, and Prescott [7], using monthly mean data for 32 stations give the relation as  $Q/Q_0 = 0.23 + 0.48S$ .

It has been suggested [8, 9, 10] that the relation between  $Q/Q_0$  and  $S$ , for daily values is

$$Q/Q_0 = k + S(1 - k) \tag{1}$$

where the reduction factor  $k$  is the ratio of total radiation with zero percent sunshine to total radiation with 100 percent sunshine. A refinement of the above relation may be represented by

$$Q/Q_0 = k + C_s(1 - k) \tag{2}$$

where  $C_s$ , the variable sunshine factor, is a function of  $S$ . The subscript  $s$  refers to the value of the factor at  $S$  percent sunshine. From the observed values of  $S$  and the values of  $Q/Q_0$  determined above,  $C_s$  was found empirically to be the same for all latitudes for a particular value of percent sunshine. The empirical relation of  $C_s$  to  $S$  is shown in figure 2. The circles are the mean values used to define the curve. On the other hand,  $k$  was found to vary slightly with season and considerably with latitude. The latitude variation is illustrated in figure 3 for mean annual values for the stations in table 1. The curve of figure 3 was fitted to the points by inspection. A  $k$  value of .40 for Fairbanks, Alaska, latitude  $64.8^\circ$  N. is not

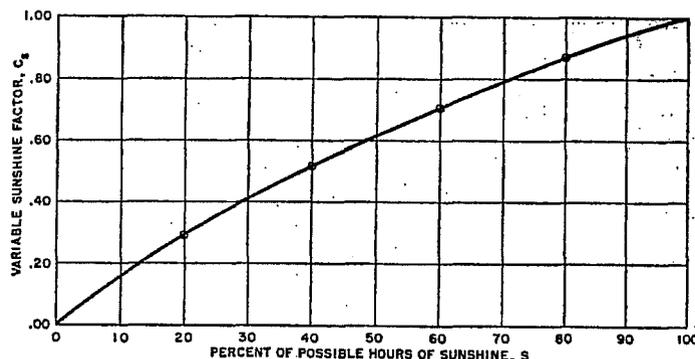


FIGURE 2.—Relation of the variable sunshine factor  $C_s$  to percent sunshine  $S$ .

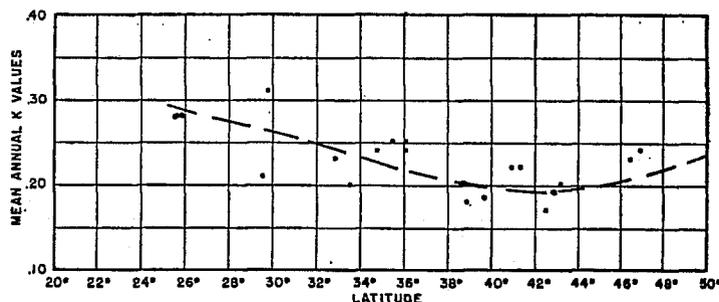


FIGURE 3.—Relation of mean annual values of  $Q/Q_0$  for zero percent sunshine (i. e.,  $k$ ) as a function of latitude for stations listed in table 1.

shown in the figure, but it was considered, as were some higher latitude values given by Kalitin [5], in determining the shape of the curve. The seasonal variation of  $k$  is shown in the table insert of figure 5 (zero correction with 80 or more percent sunshine).

The 100 percent sunshine data for March, June, September, and December for each station were plotted separately versus the station latitude in figure 4, and compared with calculated values of  $Q_0$  as obtained from the Smithsonian Tables [1]. The curves in figure 4 show the calculated values, made to fit the empirical data by appropriate choice of atmospheric transmission coefficients. The following coefficients were derived by trial and error: March 0.80, June 0.70, September 0.75, and December 0.85. The coefficients for the intermediate months can be interpolated. From these curves the curves labeled "latitude" of figure 5 were constructed. Kennedy [11], uses the formula  $I = I_0 a^m$  relating the insolation at the ground,  $I$ , to that exterior to the atmosphere  $I_0$ , the atmospheric transmission coefficient,  $a$ , and the solar air mass,  $m$ . From 2 years record at Fresno, Calif., and Lincoln, Nebr., he finds the transmission coefficient for clear days to be 0.91. Gerdel, Diamond, and Walsh [12], use this formula and assume an atmospheric transmission coefficient of 0.90 for all seasons to compute a set of "latitude" curves. These curves give around 50 ly/day less insolation than does figure 5.

As shown in figure 4, the  $Q$  values with 100 percent of possible sunshine vary only slightly with latitude at the summer solstice, approximating 740 langleys per day.

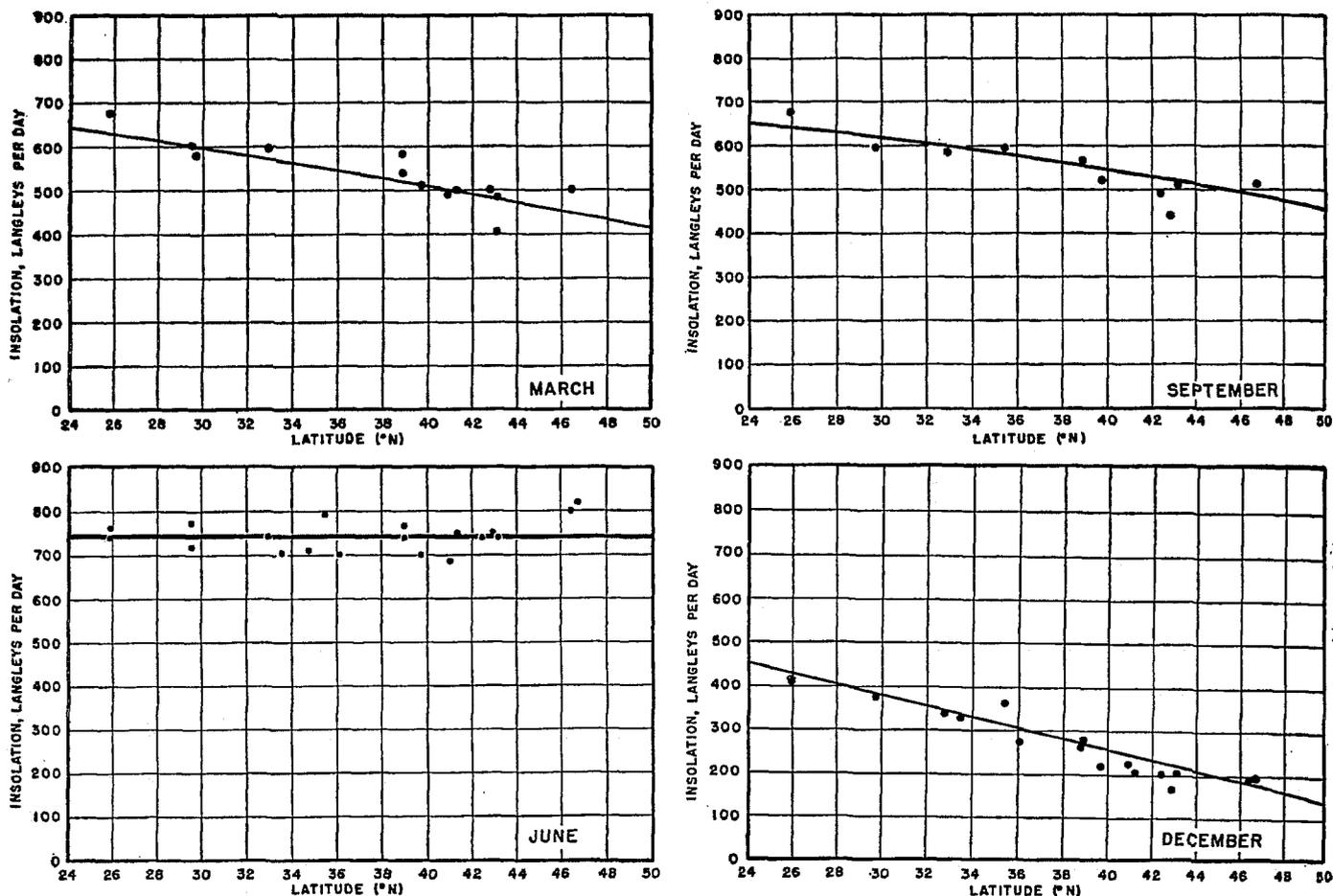


FIGURE 4.—Observed insolation for days having 100 percent sunshine, as a function of latitude and mean monthly values, during March, June, September, and December, for selected stations from list of table 1.

This is mostly because the increase in daylight hours with latitude counterbalances the decrease in solar altitude. The curve of  $k$  as a function of latitude, figure 3, is the curve for  $Q/Q_0$  for zero percent sunshine in figure 5. These values of  $k$  together with those of  $C_s$  from figure 2 were used in equation (2) to determine the reduction factor variation with latitude for the remaining curves of the other percent sunshine values shown in the right side of figure 5.

The empirical relations between insolation  $Q$  received at the earth's surface and (1) percent of possible sunshine  $S$ , (2) latitude, and (3) time of year thus have been combined graphically in figure 5 to provide a working chart for estimating daily values of insolation.

#### TEST OF METHOD

The empirical relationship of  $S$  and  $Q$  as developed has been tested on independent data from widely separated locations: Salt Lake City, Utah, 1952; Seattle, Wash., 1952; Madison, Wis., 1950; Atlanta, Ga., 1953; Appalachicola, Fla., 1953; and Portland, Maine, 1950. For this test, data used were for the 1st, 10th, and 20th day of each month of the year. For a few of the days these data were

missing. A total of 207 cases were estimated from the graph of figure 5. As an evaluation of the estimates obtained from the relation, the usual correlation test was made of values estimated from the graph, versus observed values. These are plotted in figure 6. The correlation coefficient obtained was 0.97, with a standard error of estimate of 36 langley's on a daily basis. This corresponds to about 170 langley's on a monthly basis. (If seasonal trend is removed the correlation coefficient drops to 0.84.)

Much of the residual scatter of the estimated insolation values results from the variability in the character of clouds and other restricting phenomena, and their time of occurrence. Also, for a particular station the  $Q$  estimated from the  $S$  data may show a systematic difference from the observed  $Q$  values due to local or regional diminution of radiation attributable to factors such as haze and industrial pollutants in the atmosphere. Station correction factors would produce some improvement and could be obtained for all stations with  $Q$  observations, and isolines constructed for adjustment in the values of  $Q$  obtained from the empirical method [10]. Difficulty would arise, however, in interpolating  $S$  values to dis-



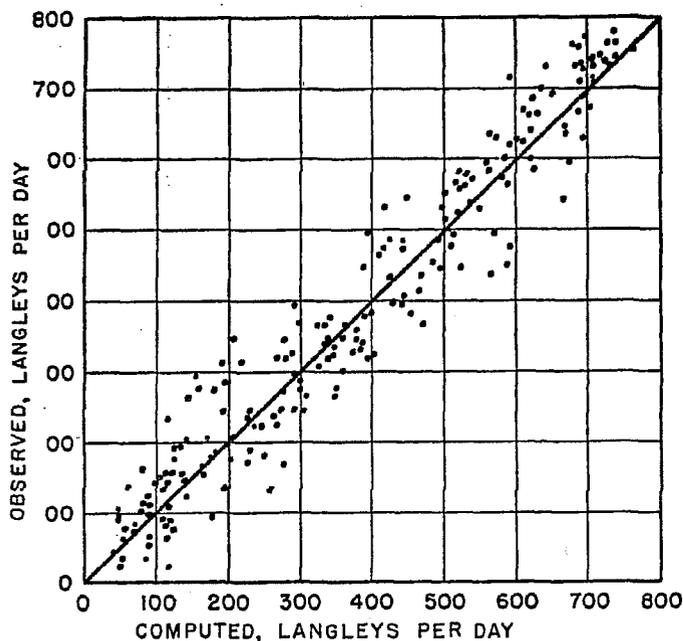


FIGURE 6.—Computed versus observed insolation for independent data.

tinguish large-scale regional influences from local influences such as metropolitan haze.

The relation presented affords a convenient method of converting percent of possible hours of sunshine into insolation values for stations located between latitude  $25^{\circ}$  N. and  $50^{\circ}$  N. An elevation correction was expected to be necessary [3, 8]; however, the estimates for Salt Lake City (elevation 4,260 feet) show no bias. Tests indicate that the use of percent possible hours of sunshine is a better indicator of actual insolation than either diurnal temperature functions or sky cover observations in tenths of sky covered by clouds, or areal interpolation among the radiation stations, even when this interpolation is restricted to synoptic situations where the stations used for the interpolation appear to be under similar meteorological influences.

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